RESEARCH REPORT FOR LIFTOFF FELLOWSHIP 2006

TERUYOSHI YOSHIDA

My research interest is in the generalization of global/local class field theory using arithmetic geometry, in particular the Langlands correspondence and the arithmetic geometry of Shimura varieties. This year I have been focusing on the cohomological (algebro-geometric) realization of the local representation-theoretic phenomena in this branch of algebraic number theory.

0.1. **Non-abelian Lubin-Tate theory.** A fundamental result in the non-abelian generalization of class field theory in the local case is the local class field theory for GL_n, proved by Harris-Taylor [HT] and Henniart [He]. In particular, Harris-Taylor proves the realization of this correspondence in the ℓ-adic vanishing cycle cohomology groups of the deformation space of height n formal group. This realization was conjectured by Deligne-Drinfeld-Carayol [Ca], and is called non-abelian Lubin-Tate theory. Precisely speaking, Harris-Taylor computed the alternating sum of this cohomology groups, and the cohomology groups for each degree were recently determined by Boyer [B]. All of these works make use of the global method – they start from the known cases of global Langlands correspondences over CM-fields, realized in the ℓ-adic cohomology groups of certain kinds of unitary Shimura varieties (studied by Kottwitz [K] and Clozel [Cl]; this is a non-abelian generalization of the theory of complex multiplication for abelian varieties), and proves the compatibility of local and global Langlands correspondences by reducing to the case where it is known (abelian cases and the ones which are automorphically induced from them). A big question in this field is whether we can prove/reconstruct the non-abelian Lubin-Tate theory by using only local methods. My paper [Yo1] addresses this question in the depth 0 case, where we can relate the vanishing cycle cohomology groups with the Deligne-Lusztig theory.

0.2. **Weight spectral sequence and Hecke correspondences.** In the aforementioned work of Harris-Taylor [HT], they show the local-global compatibility of Langlands correspondence (in the cases where they can construct the global correspondence – conjugate self-dual, regular algebraic, and square-integrable at one place) up to semisimplification, at all places prime to ℓ. In my joint work with R. Taylor, we compute the monodromy of the local Galois representations occuring in the ℓ-adic cohomology of the unitary Shimura varieties in question, and show the compatibility for the whole of Frobenius-semisimplified Weil-Deligne representation. There we reduced the question to the case where the local representaion has fixed vectors by the Iwahori subgroup, and studied the unitary Shimura varieties with Iwahori level structure, which has a strictly semistable model over the ring of integers. There we only needed to show the degeneration of the weight spectral sequence, therefore we ignored the action of local Hecke algebra and counted the dimensions. In my subsequent work [Yo2], I am determining the actions of local Hecke algebra by direct local computations involving intersection theory on semistable schemes, using the techniques of [Sa]. This leads to a more direct proof of local-global compatibility in this case, and reveals the representation theoretic structure in the weight spectral sequence. The computation of the part corresponding to the vanishing cycles for semistable

schemes is completely local in nature, suggesting the local proof of non-abelian Lubin-Tate theory in the Iwahori level; combining it with [Yo1], we expect to have the arithmetic-geometric understanding of non-abelian Lubin-Tate theory in all of the tamely ramified case.

0.3. **Local class field theory via Lubin-Tate theory.** Even in the wildly ramified case, the Galois representations induced from a character (1-dimensional representation) of the Galois group of a finite extension covers a siginificant class of Galois representations. Therefore it is important to have a better understanding of the 1-dimensional case (classical Lubin-Tate theory), the only case where we have a satisfactory local proof, and look for the possibility of relating it to the non-abelian case. In my paper [Yo3] which is a simplification of the proof given in Lubin-Tate [LT] and Iwasawa [Iw], I proved the base-change property of local Artin-maps directly in terms of Lubin-Tate groups, without recourse to the local Kronecker-Weber theorem. It will be important to understand this in more geometric terms.

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