I suggest you work on these exercises. They will help you understand better the necessary background from algebraic geometry. Problems are chosen from the following well-known book.


Chapter 1: Ex 1.2, 1.5, 1.6, 1.7, 1.8, 1.12

Comment or Hint

1.5. The example 2.4 on p.59 of Silverman deals with the same elliptic curve.

1.12. This exercise is concerned with rationality properties. It is important to understand how Galois groups act on points, rational functions and maps between varieties.

Chapter 2: Ex 2.3, 2.5, 2.6, 2.7, 2.8, 2.13

Comment or Hint

2.3. Note that a non-constant map \( \varphi : \mathbb{P}^1 \rightarrow \mathbb{P}^1 \) is always given by a rational function defined on \( \mathbb{P}^1 \), namely an element in \( \bar{K}(\mathbb{P}^1) \)

2.6. Compare the group law defined here with Composition Law 2.1 on p.55 of Silverman. Here the group law doesn’t involve choice of projective coordinates. For this exercise, you can get some hint by reading Example 6.10.2 on p.139 of Hartshorne’s Algebraic Geometry.