CORRECTIONS TO NAIVE DECISION MAKING

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This correction page (last modified August 30th 2022) is based on corrections by John Haigh, Robert MacKay, Nigel White and Matthew Towers to whom many thanks

Page 4, line -3 Reverse inequality sign.

Page 5, line 2 Reverse inequality sign.

Page 6 In (iii) reverse both inequality signs.

Page 15 Second para of section, reverse inequality sign.

Page 53 Exercise 2.4.13, 5th line delete second 'in'.

Page 58 Exercise 2.4.24 First sentence. 'Players pay the banker 1/40 of a unit to take part in a game of Simplejack. In this game a large pack of cards containing ...'

Page 65 Fifth line of footnote 23 'will be discussed'

Page 67 Example 2.5.16, first displayed inequality should read

$$\Pr\left(\left|\frac{X_1 + X_2 + \cdots + X_n}{n}\right| < \delta\right) \geq 1 - \epsilon.$$

Page 69 Let $X_{jk} = 1$ if A_k chooses the jth grotto.

$$Z = \sum_{k=1}^{n} Y_1 X_{1k} + \sum_{k=1}^{n} Y_2 X_{2k} + \cdots + \sum_{k=1}^{n} Y_m X_{mk}.$$

Page 74 Kelly recommends $t = 1/12$. Same correction line 1 of Exercise 2.6.7.

Page 75 First displayed formula

$$\frac{11}{12} \times \left(\frac{11}{12} + \frac{11}{5}\frac{1}{12}\right) = \frac{121}{120}$$

The figures in the next paragraph could be changed a little but remain of the correct order.

Page 75 Exercise 2.6.7, line 1. 'This part requires'

Page 77 Exercise 2.6.10. I fell asleep at the wheel. Should read

I bet a fixed amount t. Thus my fortune $X_{j+1}(t)$ after the $j+1$th throw is given by

$$X_{j+1}(t) = \begin{cases} X_j(t) + tu & \text{if the } j\text{th throw is heads,} \\ X_j(t) - t & \text{if the } j\text{th throw is tails.} \end{cases}$$

Page 79 Exercise 2.7.1 Space after (i)
The second (ii) should be (iii)
Correct suffix in second displayed equation to give

$$F(u_1, u_2) = p_1 \log p_1 + p_2 \log p_2 + p_1 \log u_1 + p_2 \log u_2.$$

Page 81 Exercise 2.7.2 part (iv) should begin
(iv) In the remaining Case C, when $p_1 u_1 > 1$ and

$$\frac{1}{u_1} + \frac{1}{u_2} + \cdots + \frac{1}{u_n} > 1,$$

show that

Page 90 Exercise 3.2.6 (ii). The final term of the sum should be $(m-1)^2 x^{m-2}$.

Page 104 First sentence of third line 'Thus g' is increasing.'

Page 109 Exercise 3.5.10 displayed equation should read

$$c_p(|a|^p + |b|^p) \geq |a|^p + |b|^p$$

Page 110 First line. I have the ranges of summation in a twist. Should be

$$p_i = \sum_{j=1}^{m} \pi_{ij},\ q_j = \sum_{i=1}^{n} \pi_{ij}$$

Page 121 Lemma 4.3.4 (iv)'. I was asleep at the wheel. Should read
(iv) If $a \equiv a'$ and $b \equiv b'$ then $a + b \equiv a' + b' \mod n$.

Page 124 Proof of Lemma 4.3.10 (ii), third line should read.

$$y \equiv v_1 y_1 + v_2 y_2 \equiv v_1 \times 0 + v_2 \times 1 \equiv v_2$$

Page 155 Case (iv)

$$A_1 = \{X_1 < X_2, X_3, X_4, X_5\},$$
$$A_2 = \{X_2 < m\},\ A_3 = \{X_3 < m\},\ A_4 = \{X_4 < m\},\ A_5 = \{X_5 < m\}$$

Page 159 Exercise 5.3.9 (ii) second sentence should read:-
Show that the probability that we stop at the last card is at most t and the probability that none of the k largest cards are turned over before the mth card is at most $(1-t)^m$.

Page 161 Last line should read

$$An \log n \geq \log n! \geq Bn \log n$$

Page 170 Paragraph after Exercise 5.5.7.
What happens if we try to work out the shortest paths between every pair of towns?

Page 173 Exercise 5.5.18 (i) Replace the meaningless 'length a decreasing cycle' by 'a distance decreasing cycle'.

Page 179 Exercise 6.1.7. Last sentence replaced by

If the process does terminate, will pairing it produces necessarily be stable?

Page 186 (Thanks to David Pauksztello) 5th line down 'everybody else's preferences $B > C > A$'

Page 192 Replace 'C beats A' by 'A beats C'

Page 200 Third complete paragraph, second line. Replace 'choosing row 1 heads with' by 'choosing row 1 with'

Page 205 Second line of first complete paragraph

We shall say that Rowena adopts strategy $\mathbf{p}$ if she chooses row i with probability p_i and that Calum adopts strategy $\mathbf{q}$ if he chooses column j with probability q_j. *Page 217* Exercise 7.6.2. First sentence should read

Suppose that George has chosen $z > x_0$ and Fred knows the value of z.

Page 218 Exercise 7.6.2.

In (ii) replace $y < x_0$ by $z < x_0$

In (iii) replace $y = x_0$ by $z = x_0$

Page 228 Exercise 8.2.4, penultimate sentence.

'may be (x_1, y_1) itself'

'for all $(x, y) \in \tilde{K}$'

Page 228 Exercise 8.2.2 (ii). The displayed alternatives should read

(a) If $(x, y) \in K$ and $x = 0$ then $y \leq 0$.

(b) If $(x, y) \in K$ and $y = 0$ then $x \leq 0$.

Page 238 Penultimate line. Displayed formula should read

$$\beta(\mathbf{p}^*, \mathbf{q}) \leq \beta(\mathbf{p}^*, \mathbf{q}^*)$$

Page 243 First sentence should begin

Almost exactly the same arguments as before show that there are exactly three Nash equilibrium points $(p, q) = (1, 1)$ with expected value to Albert of 1 and to Bertha of 0, $(p, q) = (0, 0)$ with expected value to Albert of 0 and to Bertha of 1 and $(p, q) = (2/3, 1/3)$

page 254 Matthew Towers informs me that my erratum was in error and suggests

(5) If C misses both, then, by (2), we know that B will fire at A. If B hits A, then the result is a duel between C and B in which C and B fire alternately and C has first shot so the probability of C winning is

$$\frac{cb}{b + c - bc}.$$

If B misses A then A will shoot at B and hit him, so C must then shoot at A and will win the game if and only if he hits A.

$$\begin{aligned}Pr(C \text{ wins if he misses first shot}) &= Pr(C \text{ wins if } B \text{ hits } A)b + Pr(C \text{ wins if } B \text{ misses } A)(1\\ &= \frac{cb}{b+c-bc} + c(1-b)\\ &= c\frac{b^2c - b^2 - 2bc + c + 2b}{b+c-bc}\end{aligned}$$

Thus C has a better chance of winning if he misses the first shot (and should therefore *deliberately miss*) if

$$c\frac{b^2c - b^2 - 2bc + c + 2b}{b+c-bc} > c\frac{1-b}{b+c-bc},$$

which simplifies to

$$b(3-b) + (b-1)^2c > 1.$$

Page 254 Replace first part of Exercise 9.2.3 as follows

Exercise 9.2.3 (i) Show that, if $3b - b^2 > 1$, then, whatever the value of c, player C should always deliberately miss with his first shot. Show that, if $b^3 - 3b^2 + 4b < 1$ C should always try to hit B with his first shot. Explain to a non-mathematician why, if B is strong, C should deliberately miss his first shot and, if B (and therefore C) is very weak, why C should try to hit A with his first shot.

Page 259 In the statement of Lemma 9.3.3 delete the numbers (i) and (ii) and replace the starting words 'We have' in the last sentence by 'Conclude that'.

Page 267 In the diagram there should be arrows from THH to HHT and from HTT to TTH

Page 268 For those with good eyesight, last line first P_{TH} should be p_{TH}.

Page 274 Exercise 9.5.5 in table.

$$B(A,S) = \frac{1}{1-p}$$

*P*age 267 The game **HHH** was invented by Walter Penney (Journal of Recreational Mathematics, October 1969, p. 241) and is referred to as Penney's game. Strong apologies.

*P*age 274 There is an error in the last line which affects the following paragraph

We now observe that

$$B(I,N)-B(I,P)=\frac{2-p}{1-p}-3=\frac{2p-1}{1-p},$$
$$B(I,N)-B(I,O)=\frac{2(1+p)-3}{1-p^2}=\frac{2p-1}{1-p^2},$$
$$B(I,P)-B(I,O)=-\frac{3p^2}{1-p^2}+\frac{p}{1-p}=\frac{p(1-2p)}{1-p^2}.$$

Thus

$$B(I,N)>B(I,P) \text{ for } p>1/2,\ B(I,P)>B(I,N) \text{ for } 1/2>p,$$
$$B(I,N)>B(I,O) \text{ for } p>1/2,\ B(I,O)>B(I,N) \text{ for } 1/2>p,$$
$$B(I,O)>B(I,P)) \text{ for } p>1/2,\ B(I,P)>B(I,O) \text{ for } 1/2>p.$$

Looking at these results, we advise Sonia to play the strategy 'never press' (or, what turns out to be exactly the same strategy, 'do the same as Tania') whenever $p\geq 1/2$, to follow the strategy 'do the opposite of Tania' when $1/2\geq p\geq 0$. (If $p=1/2$ there is a free choice between the two recommended strategies.)

*P*age 274 Last line on page should read

$$B(I,P)-B(I,O)=-\frac{3p^2}{1-p^2}+\frac{p}{1-p}=\frac{p(1-2p)}{1-p^2}.$$

*P*age 283 Third displayed formula from the bottom of the page.

$$t=1 \text{ or } t=\frac{1-p}{p}.$$

*P*age 283 Exercise 10.1.4 (i) displayed formula should read

$$q_n=A+B\left(\frac{1-p}{p}\right)^n$$

*P*age 285 Exercise 10.1.5 (ii)

Can you suggest three distinct strategies which are as good as bold play?

*P*age 285 Exercise 10.1.6 first line

stake of 1 dollar

*P*age 286 Second displayed equation

Show, by induction, or otherwise that

$$p_r(f)=p_r(k2^{-r}) \text{ if } k2^{-r}\leq f<(k+1)2^{-r}$$

for integers k with $0\leq k\leq 2^r$.

*P*age 290 Exercise 10.2.7 (ii) line 3 delete second 'better' (or, if you prefer, first 'better' but not both).

*P*age 290 Third line of second paragraph after Exercise 10.2.8. replace ‘there’ by ‘their’.

*P*age 290 Expression

$$10\,000 \times \mathbb{E}(\text{expected number of months to bankruptcy}),$$

should be replaced by

$$10\,000 \times \text{expected number of months to bankruptcy},$$

*P*age 291 Last three sentences before Exercise 10.2.10 should read:-

The probability of bankruptcy in a particular month is $(1-p)^8$ and so, by Lemma 9.3.3, the expected number of times I enter the casino is $1/(1-p)^8$. Thus the expected return from an investment of 2 560 000 is

$$10\,000 \times (1-p)^{-8}.$$

Taking a typical value of $p = .49$ we get an expected return of about 2 184 935 dollars.

*P*age 291 Exercise 10.2.10

(i) What is the expected return if $p = 1/2$? Why should we expect this?

(ii) What is the return if p takes the extremely favourable value $p = .495$?

*P*age 297 Exercise 10.3.11 There are two part (ii)s. Renumber appropriately.

The first (ii) should read

(ii) Write down the general solution of

$$(E-I)u_n = \binom{n}{r-1}.$$

*P*age 302 Three lines after ★ undisplayed equation

‘$(1-p)u_{n+2} - u_{n+1} + ue_n = 1$’ should be ‘$(1-p)u_{n+2} - u_{n+1} + pe_n = 0$’

*P*age 303 Last line should read

$$f_n = \frac{1-p}{1-2p}\left(\frac{1-p}{p}\right)^N \left(1 - \left(\frac{p}{1-p}\right)^n\right)$$

*P*age 304 First displayed formula should read

$$f_N = \frac{1-p}{1-2p}\left(\left(\frac{1-p}{p}\right)^N - 1\right).$$

*P*age 304 Exercise 10.4.7 First paragraph, after first displayed equation add ‘with r an integer’ to read

‘and $Y_0 = r$ with r an integer, explain’

*P*age 307 Exercise 10.4.10 (i) first displayed equation

$$e_m = \frac{\sum_{j=m}^{n} j p_j}{1 - q\sum_{j=0}^{m-1} p_j}.$$

*P*age 307 Exercise 10.4.10 (i) last paragraph first sentence second line. Reverse inequality to get '$e_{m+1} \leq e_m$'.

*P*age 307 Exercise 10.4.1, second paragraph, second sentence '$V_0 = p/q$'.

*P*age 308 The statement

In the UK National Lottery 50% of the cost of each ticket is returned to the buyers as prizes.

should be modified by the insertion of the word 'roughly'.

*P*age 313 Exercise 10.5.10 first sentence should read

(i) Show that $|x - \log(1+x)| \leq x^2$ for $1/2 > |x|$.

*P*age 313 Exercise 10.5.10 (iv). Factor of n missing. Should read

Show that

$$\frac{2n}{N(n)^2}\left[\log\left(1+\frac{a}{n}\right) + \log\left(1+\frac{2a}{n}\right) + \dots \log\left(1+\frac{N(n)a}{n}\right)\right] \to a.$$

Deduce that

$$\left[\left(1+\frac{a}{n}\right) \times \left(1+\frac{2a}{n}\right) \times \left(1+\frac{3a}{n}\right) \times \cdots \times \left(1+\frac{N(n)a}{n}\right)\right]^{2n/N(n)^2} \to e^a$$

as $n \to \infty$.

*P*age 318 L. F. Richardson not J. F. Richardson!

*P*age 322 Footnote clarification.

and the elder Bush

*P*age 323 Exercise 11.1.1 (iv). At end of first sentence insert 'in a row'.

*P*age 323 Middle

The probability of throwing ten heads in a row is now $(1/2)^{10}$, so, perhaps, it would be good idea.

*P*age 329 4th line down. 'and apply (iv)' [not (i)]

*P*age 330 Exercise 11.3.3 (vi). Third displayed equation inequality should (of course) be reversed.

$$\Pr\left(\left|\sum_{j=1}^{n} X_j\right| \leq y\right) \geq 2\exp\big(-y^2/(2A)\big).$$

*P*age 331 Exercise 11.3.4, displayed formula should read

$$\Pr\big(|Y_n - nq| \leq n\epsilon\big) \leq 2\exp\big(-n\epsilon^2/2\big).$$

*P*age 332 Exercise 11.4.1, second line. Insert 'the' to get 'both the following properties'.

*P*age 333 Exercise 11.4.2, second paragraph first line. Replace 'reject' by 'accept'.

*P*age 334 Exercise 11.4.4 (iv). $R(a, p)$ instead of $R(P, a)$

*P*age 356 Exercise B6 (iii) last sentence. Exercise B4 (not B.6).

*P*age 352 Exercise C5 (ii) line one. Replace doubled 'a' by a singleton.

*P*age 361 Exercise C.4 (ii) second paragraph line 1. Remove 'of'.

Bibliography The Martin Gardner book [23] is

The 2nd Scientific American Book of Mathematical Puzzles and Diversions

index Should contain reference to Penney to go with correction page 267.

*P*age 368 ref [41]: J. Maynard Smith. (oops, thanks to Charles Goldie)