These are corrections to the notes used in the 2014 course. If you are using the older notes then you should download the newer notes.

Page 10, Exercise 4.4

A clarification

(i) Suppose that $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are continuous. Show that the map $(f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ given by $(f, g)(x, y) = (f(x), g(y))$ is continuous.

Page 14, Exercise 5.4

$C([a, b])$ should be $C([0, 1])$. Similar correction page 79.

Page 14, Exercise 5.7 Replace part (ii) by

(ii) If V is a vector space over $\mathbb{R}$ and ρ is a metric derived from a norm, show that the one point sets $\{\mathbf{x}\}$ are not open in this metric.

(iii) Deduce that the discrete metric d on the vector space V cannot be derived from a norm on V.

Page 19, Exercise 7.9 (i) Last sentence

If $f,\ g : X \to Y$ are continuous show that the set

$$\{x \in X\ :\ f(x) = g(x)\}$$

is closed.

Page 26 Single 'in' at end of send paragraph of Section 10.

Page 33 Replace twice 'page 19' by 'page 99'.

Page 35

Remove rogue fullstop to get 'definition of connectedness which'

Page 45 Exercise 16.7

Last part should be numbered (iii)

Page 45 Exercise 16.8 (ii)

Find a set A of $\mathbb{R}$ with the usual topology such that A, $\operatorname{Cl} A$, $\operatorname{Int} A \operatorname{Cl}$ and $\operatorname{Cl} \operatorname{Int} \operatorname{Cl} A$ are all distinct.

Page 45 Exercise 16.8 (iv)

Starting from a set A

Page 50 End of Exercise 16.21. Last part (vi), not (vii).

Page 50 In Exercise 16.22 (ii)

‘conditions (1) to (4) of part (i)’

Page 50 In Exercise 16.23 Update to 2014, 20014,

Page 53 In Exercise 17.9 (ii) remove extra {

Page 54 In Exercise 17.7 (iii). First displayed equation.

$$\{np \,:\, n \in \mathbb{Z}\}$$

Page 54 In Exercise 17.10, third line
‘subsets of $\mathbb{R}$’ not ‘subsets of E’

Page 57 In Exercise 17.22 add the condition ‘f and g continuous’.

Page 58 In Exercise 17.22, extra bracket

$$t \mapsto \big(\phi(t), \psi(t)\big)$$

Page 58 In Exercise 17.22, extra bracket

$$t \mapsto \big(\phi(t), \psi(t)\big)$$

Page 59 In Exercise 17.27, extra bracket

$$(X, d)$$

Page 64 In statement of Exercise 1.1 (i). ‘Identify $f^{-1}(\{1\})$’ (not $f^{-1}(\{1,2\})$’)

Page 70, final formula
Reverse inequality to get

$$\|f+g\|_1 = \int_a^b |f(t)+g(t)|\,dt \leq \int_a^b |f(t)|+|g(t)|\,dt = \|f\|_1 + \|g\|_1,$$

Page 78 Exercise 5.3, third displayed formula

$$n \geq N \Rightarrow \rho\big(f(x_n), f(x)\big) < \epsilon.$$

Page 82 Lemma 7.3 (ii), last paragraph full stop should be comma.

and $V' \subseteq U \subseteq A$, then $V' = U$.

Page 83 Second paragraph of the solution of Exercise 7.9

Now $f^{-1}(U)$ is open as is $g^{-1}(V)$

Page 84 Last sentence of solution of Exercise 7.9 (ii). Replace f by $\tilde{f}$ throughout.

Then

$$|\tilde{f}(p_n) - \tilde{f}(2^{-1/2})| + |\tilde{f}(q_n) - \tilde{f}(2^{-1/2})| \geq |\tilde{f}(p_n) - \tilde{f}(q_n)| = 1,$$

so $\tilde{f}$ cannot be continuous.

Page 89 There is a random ']' after **Lemma 10.5** More importantly the last two sentences but one of the proof of Lemma 10.5 should read

Choose $N > \delta^{-1}$. We can find an $e \in E_N \subseteq E$ with $u \in B(e, 1/N)$, so

$$e \in B(u, 1/N) \subseteq B(u, \delta) \subseteq U.$$

Page 107 In the proof of of Lemma 14.3 interchange the headings 'if' and 'only if'.

Page 19 Exercise 17.9 is correct as it stands but I think a better way of looking at it involves a rewrite

Sometimes this idea works (see, for example, part (ii) of Exercise 7.9 and sometimes it does not (see, for example, part (iii) of Exercise 7.9). When it does work, this is very powerful technique.

Exercise 0.1 (Exercise 7.9)**.** *(i) Let (X, τ) be a topological space and (Y, d) a metric space. If $f, g : X \to Y$ are continuous show that $f(x) = g(x)$ for all $x \in X$.then the set*

$$\{x \in X \, : \, f(x) = g(x)\}$$

is closed.

(ii) Let (X, τ) be a topological space and (Y, d) a metric space [Footnote Exercise 9.7 gives an improvement of parts (i) and (ii).]. If $f, g : X \to Y$ are continuous and $f(x) = g(x)$ for all $x \in A$, where A is dense in X, show that $f(x) = g(x)$ for all $x \in X$.

(iii) Consider the unit interval $[0, 1]$ with the Euclidean metric and $A = [0, 1] \cap \mathbb{Q}$ with the inherited metric. Exhibit, with proof, a continuous map $f : A \to \mathbb{R}$ (where $\mathbb{R}$ has the standard metric) such that there does not exist a continuous map $\tilde{f} : [0, 1] \to \mathbb{R}$ with $\tilde{f}(x) = f(x)$ for all $x \in A$.

We modify Exercise 9.7 accordingly.

Exercise 0.2 (Exercise 9.7)**.** *Prove Exercise 7.9 (i) and (ii) with '(Y, d) a metric space' replaced by '(Y, σ) a Hausdorff topological space'.*

The proof of part (iii) corresponds to the proof of part (ii) in the old notes and the proofs of parts (i) and (ii) now run as follows.

(i) Let

$$E = \{x \in X \,:\, f(x) = g(x)\}.$$

We show that the complement of E is open and so E is closed.

Suppose $b \in X \setminus E$. Then $f(b) \neq g(b)$. We can find open sets U and V such that $f(b) \in U$, $g(b) \in V$ and $U \cap V = \varnothing$. Now $f^{-1}(U)$ is open as is $g^{-1}(U)$ so $b \in f^{-1}(U) \cap g^{-1}(V) \in \tau$. But $f^{-1}(U) \cap g^{-1}(V) \subseteq X \setminus E$. Thus $X \setminus E$ is open and we are done.

(ii) Let E be as in (i). We have $A \subseteq E$ and E closed so $X = \operatorname{Cl} A \subseteq E = X$ and $E = X$.

Page 77 In answer to Exercise 4.11 third displayed formula, replace $\|$ by $\|_2$ to get

$$B(\mathbf{x}, r) = \{\mathbf{x}\} \cup B_E(r - \|\mathbf{x}\|_2).$$