The Naked Lecturer

T. W. Körner

April 29, 2013

Now that I have your attention, I would like to to write about mathematics lecturing. If readers are annoyed by my confusing the pronouns 'you', 'one', 'she' and 'he', they should remember that I think of myself as addressing an audience of mathematicians and non-mathematicians. The non-mathematicians are rather shadowy creatures, but the mathematicians have clear characters and include both men and women.

1 Lecture Courses

For most mathematicians lecturing is part of their job. A few mathematicians grudge every moment of teaching as a moment taken from their research. Of course, teaching may occasionally aid research. Conway lectured on the construction of the real numbers starting with naive set theory. giving a different version of the standard constructions each year. I suspect that he would not have produced theory of surreal numbers if he had not given those lectures. When I lectured on elementary functional analysis, I would always close the section on Baire's category theorem by saying that, almost certainly, many variations on the ideas remained to be exploited. Later I came across one such variation. Even so, it is reasonable to suppose that, in general, the more effort devoted to teaching, the less effort is devoted to research.

In spite of this, most mathematicians do not regret the time they spend teaching. In part, this reflects a feeling that, having been well taught ourselves, it is our duty to give back something of what we owe. In part, it reflects the fact that research is, on the whole, a lonely occupation and teaching is a social one. In Eastern Europe under communism, politically suspect mathematicians might be moved from teaching universities to research only institutes with the move been intended as a punishment and felt as such. Mainly, I think, mathematicians like to lecture for the reasons outlined in *Surely You're Joking, Mr. Feynman!*. If you only do research, then a year

without a good idea is a wasted year. If you do research and teaching, then, no matter how the research has gone, you will have done something useful. Finally, most people, even mathematicians, have an extrovert side. 'The smell of the grease paint and the roar of the crowd' is irresistible even when translated into 'the feel of the chalk and the scratch of the pen'[1].

I was a student at Cambridge in the 1960's. Some of my lecturers belonged to an older generation. Later we learnt that many of them had done exciting things during the war, but, at the time, they just seemed incredibly old and staid. On the whole, this older generation approached their lectures by producing as perfect a set of notes as possible and then writing them out on the blackboard. An extreme example of this system was given by J. C. Burkill whose course on measure theory was a word for word transcription of his book *The Lebesgue Integral*. In retrospect, I consider this book to be an excellent introduction to the subject, but I still do not think that the lectures added much to my understanding.

The younger lecturers reacted against this older style in a variety of ways. Instead of giving the same perfected course each year, they preferred three year stints on the principle that 'the first year you learn, the second year you teach, the third year you embroider and the fourth year is worse'. Some of them produced elaborate printed notes in order that the students could concentrate on the lectures without being distracted by the business of taking notes. So far I was concerned, this was a failure, either because taking notes actually helps concentration, or because I found it hard to read notes and listen at the same time, or for some other reason. This experience must have been widespread since few lecturers of my generation produce such notes and many of those who do only give them to the students *after* the relevant lecture.

On the whole, the lecturers who left the greatest impression on me were those like Varopoulos, Conway and Swinnerton-Dyer both for their evident swagger (who can forget Swinnerton-Dyer arguing that 'nature cannot be so unkind as to allow this result to be false') and the fact that they lectured without notes. Of course, 'making an impression' is not quite the same as 'giving a good course' or 'teaching well', but it is an excellent start[2].

Later, I spent the third year of my PhD at Orsay (one of the Parisian universities). On arrival, I was told that I had been put down to give a seminar in four weeks' time on a paper of Kahane. Naturally, the seminar

[1]There is claimed to be a survey in which 94 percent of professors said they were better teachers than the average faculty member on campus. I have been unable to track down such a survey but it sounds horribly plausible.

[2]It can be argued that the purpose of lecturing is not so much to teach as to provide the audience with a set of stories to tell their children.

would be in French and without notes. Since I did not know Kahane's paper and my French was limited to four years in school and a few further hours in language labs, the four weeks were rather nerve wracking but, so far as I was concerned, the seminar went fine[3] (the audience may have felt differently) and the thrill I experienced made me a convert to noteless lecturing.

Watching a lecture by Bolobás or a similar master of the art of lecturing without notes is like watching a magician give a stage performance. The lecturer may not be naked but, at least, strips down to the undergarments to show that nothing is hidden from from the audience. There is a roll of drums and then 'look ladies and gentlemen no artificial aids' a delicate inequality is conjured from thin air or some intractable theorem wrestled to the ground.

Why is this performance (for it is a performance), so valuable? I think there are two reasons. The first is the moral effect. Mathematics students faced with a new result have a natural tendency to believe that it is too hard for anybody to understand properly. If you copy out a proof on to the board or flash up the proof on a projector, the implicit message is that the proof is too hard for you to do anything but copy it out word for word. If you produce the proof without notes, the implicit message is that the proof is so easy that that is not worth making a fuss about.

There is a second reason for this style of lecturing. Mathematics is not a collection of facts but of processes. A slide show (and what is a computer presentation but a slide show) of theorems and their proofs is like walking through a museum full of stuffed animals. Only by watching you actually proving the results can your audience see the animals live in their native habitats.

The non-mathematician may ask how a lecturer can possibly remember 50 minutes of mathematics. The answer is that she does not. Many proofs are entirely routine and can be constructed on the fly. Most of the remainder require one, or, at most, two, ideas and, once those are understood, the rest of the proof is again routine. A mathematics lecture is not like a classical symphony but like a jazz improvisation starting from a small number of themes[4].

Boswell record Johnson's opinion that

> People have now-a-days got a strange opinion that everything should be taught by lectures. Now I cannot see that lectures can do so much as reading the books from which the lectures

[3]The secret to lecturing in a foreign language is to learn the first five minutes by heart and then rely on adrenaline.

[4]Unfortunately names are facts and not processes. If I need to use the name of a mathematician during a lecture I write it on my cuff or an equivalent surface.

> are taken. I know nothing that can be best taught by lectures except where experiments are shown. You may teach chemistry by lectures:– you might teach the making of shoes by lectures.

I believe that the making of proofs (as opposed to the discovery of new mathematics) is like the making of shoes. It is better taught by watching someone make a shoe in front of you than by trying to figure it out from books.

Even if the stage magician's act looks impromptu, it will be better if it is rehearsed. Varopoulos depended on his native wit and knowledge to carry him through his lecture. Since he has ample supplies of both, five lectures out of six would be splendid bravura performances and the sixth would misfire completely. Conway would ask for the syllabus five minutes before a lecture and then lecture brilliantly, but this reflected the fact that, over the years, he had reflected deeply on almost every topic to be met in the undergraduate course.

The rest of us need to prepare in advance. (Some lecturers like Beardon produce a beautiful set of notes and then lecture without consulting them, but few aspire to this standard.) When I started as a lecturer, I used to try out each lecture in an empty lecture room. Nowadays, I am too self conscious and lazy to do this (although I suspect it represents best practice) and, instead, I work through the next day's lecture the evening before. Only the most self-confident mathematician will find the eigenvalues of a 3×3 matrix in public without having gone through the calculation privately several times before.

It is also vital to have an undisturbed ten minutes before the lecture. It does not matter if you do not think about the lecture. It does matter that you do not think about anything else. I have discussed this with colleagues and we all agree that, however well you prepare in advance, going directly from a committee meeting or similar occasion to give a lecture is a recipe for disaster.

A high wire act is not a real high wire act unless the performer might fall off. What happens when you fall off? The first lecture of my first course ran into such difficulties that I had to repeat the entire lecture next time and, the worst that can happen having happened, I no longer fear it[5]. The key advice for a lecturer who has got lost in an exposition is 'when you find yourself in a hole stop digging'. Tell the audience that you need to think,

[5]The students for that course included Terry Lyons, Jonathan Partington, Kieth Carne and Richard Pinch. For the rest of the courser, whenever there was a gap in my reasoning one of them would point it out and another would explain how it should be filled. It was an exhilarating experience but such audiences are rather rare.

step back from the blackboard and reflect. If you see your way clear, return to the blackboard and continue. If you cannot see your way clear or you have failed in a second attempt, tell the audience what the problem is and that you will return to the proof in the next lecture. Some lecturers feel that it is important to give the correct proof at the blackboard (on the principle that, when you fall off a horse, you should immediately remount) but my experience is that, once things go wrong, they tend to continue that way and I prefer to write out the proof before the next lecture and hand out copies to the audience.

Mathematicians and those well on the way to being mathematicians understand that mathematics is difficult and that, from time to time, things will go wrong. The advice of the previous paragraph only applies to advanced courses. Beginning students are unforgiving. I once started a course with the words 'I am not the world's greatest expert in this subject' and never fully recovered the confidence of the audience. 'It is one of the first duties of a professor' writes Hardy 'to exaggerate a little both the importance of his subject and his own importance in it.' Fortunately, beginning students automatically assume that you are one of the world's greatest experts in your subject, so no lying is necessary. Since the mathematics taught to beginning students is necessarily elementary, it is highly unlikely that you will get lost but, if this happens, bluff ('I have given you the general idea, so go away and try and fill in the details. If you can't, I will give them next time') may be better than confession.

Failure at the blackboard may be nothing more that 'blackboard blindness' but may indicate a gap in your own understanding. After many years of presenting the Hahn–Banach theorem smoothly and successfully, I gave a lecture in which I got totally confused. but, as a result, I feel I now understand the workings of the theorem much better.

What are the disadvantages of lectures given without notes. The first is that the students notes will not be as perfect as if you copy or project previous written notes onto the board and the students copy those notes. There will be more small errors and proofs invented on the fly will not have the finished elegance of those written out beforehand. There is no doubt that weaker students value complete and accurate notes more than anything else. (The problem is that, having got complete and accurate notes, weaker students can do nothing with them.)

Those who agree with the weaker students that the main purpose of lectures is to produce complete and accurate notes must answer the traditional question 'Did Gutenberg live in vain?'. Unless we believe that a university education may be summarised as 'take notes, learn notes, pass exam, forget notes' we should include among our educational objectives that students

should learn to use libraries and consult books (or failing that, that they should consult Wikipedia with its many excellent mathematics articles).

Let me repeat what I said earlier. Mathematics is not a collection of facts but of processes. You cannot learn to ride a bicycle or play the violin from lectures. Instead you watch others riding bicycles or playing the violin and try to imitate them. You learn by long and painful practice (in the case of the bicycle painful to yourself, in the case of the violin painful to others). In the same way, you can only learn mathematics by doing exercises. It is possible (but quite hard) to learn mathematics without lectures by just reading books and doing exercises. It is possible (though, in my view, slightly unsatisfactory) to learn mathematics without books by just attending lectures and doing exercises. It is impossible to learn mathematics by just attending lectures and reading books.

I believe that lectures are only a small part of a mathematical education and that students learn most of their mathematics in other ways. I think that what students gain from lectures is the picture of a mathematician at work. By watching how she approaches a proof or how she always an example and a counterexample for each definition they are initiated into the mathematician's mode of thought. The cold perfection of books needs to be supplemented by the vision of mathematics as a living thing subject to human error.

The second disadvantage of lecturing without notes is organisational. Even if you get the class to help you number theorems consecutively, neither you nor your class will know the 'the number of the theorem that we proved last Friday where we dropped the condition on differentiability '. (Incidentally, lecturers should constantly bear in mind that the only person in the room who has been paying full attention all the time is the lecturer herself.) In addition you may suffer from notational drift by which the function f of the previous lecture becomes the function g of today. For this reason, I now mainly use a modified version of the noteless lecture in which I give the students a skeleton set of notes consisting of the statements of the definitions and theorems of the course and lecture following those notes.

2 Seminars

So far I have been concerned with lecture courses. What about seminars and similar occasions? The first thing to say is that the shorter the time given, the harder the task. If you lecture for 24 hours, you can build up a rapport with your audience. Ideally, you can move towards a more conversational ambiance in which they do not hesitate to answer your questions or to ask

their own. Even if this is not possible, you will be able to adjust your lecturing style (speed, sophistication, number and type of examples, ...) to match their reactions. If you have only sixty minutes matters are much harder[6] and it may be impossible to rectify the choice of an inappropriate level. (I have been to very many seminars where the speaker has pitched the level too high and very few where the speaker has pitched the level too low.)

When I discussed lecture courses, I made it clear that I considered them as a fairly minor part in education of our students. In the same way I think that seminars can only play a minor role in the education of their elders. In lecture courses we spend a long time trying to explain well understood things to our students with only limited success. In seminars we spend a short time trying to explain complicated and badly understood things to each other. Since, on the most optimistic view, we are only slightly more capable than our students it is foolish to expect too much.

What do I expect, or at least hope for, when I attend a seminar? In general I hope to understand the first few minutes, because the lecturer is be telling me things I already know. In general, I expect not to understand most of the rest, since the lecturer will be talking about things that I do not know and therefore cannot understand. But, during the few instants of changeover from one phase to the other, I hope to gain some insight into the subject of the seminar which I cannot gain from the books and papers bound to the formality of written mathematics. A good seminar talk will tell the audience what problem is being considered, where it comes from, why it is important and give some hint of what ideas are used to attack it[7]. If the speaker manages to get anything further across the audience should consider this an uncovenanted bonus.

A *colloquium* is intended for an entire department. Most of the audience will not have studied your branch of mathematics since they were students. If you pitch your talk at the level of second year students that is probably the right level.

The non-mathematician may observe that, if my account of seminars is correct, then most of a seminar audience spends most of its time not understanding what the speaker is saying. It is a matter of observation that most non-mathematicians who fail to understand something in a talk blame

[6] I once attended an AMS meeting with 10 minute lectures. I think that the speakers did a splendid job, but I have no idea how they did it.

[7] Some seminar speakers think that their job is to present a long list of theorems. Analytic number theorists seem particularly prone to to the style 'In 1953, X proved that the growth was no greater than $(\log x)^{2/3}$ but in 1957 Y improved this to $(\log x)^{20/31}(\log\log x)^{1/5}$'. The Russian school enlivens the recital with details of priority disputes.

the lecturer or, less usually, themselves[8]. Mathematicians are used to not understanding things and do not see that there is any blame to assign.

One of the leading British mathematicians once told me that he experienced a near breakdown in his third year of university. Up to then, he had understood everything in the lecture courses but this was no longer true. All his friends had gone through the same process much earlier, so he had to deal with this by himself. He argued that everyone had to deal with this crisis at some time and what mattered was not the timing of the crisis but how you cope with it.

Mathematicians are very good and polite audiences. One of my finest memories is of a conference dinner in Finland held in a farmhouse. At the end of the dinner, the local dramatic society performed a twenty minute skit in Finnish dealing with recent Finnish politics. The audience of Japanese, French and German mathematicians watched with rapt attention and applauded noisily at the end.

When I talked about lectures, I said that they were rarely useful for research. This is not true of seminars about one's own work. Writing a paper is a bit like assembling a piece of complicated machinery. You have to concentrate on making sure that every part is free of defects and links correctly to its neighbours. Preparing a seminar forces you to take a step back and look at the piece of work as a whole. What are the principles behind the design of the machine? What obstacles have you overcome? What obstacles remain which prevent you from using the machine to overcome other problems?

If there is an expert on your topic in the audience, then you have the satisfaction of addressing her in person rather than through a paper which she may or may not read. She may find it much easier to grasp your underlying ideas in the context of an informal seminar than if she has to hack her way through the undergrowth of formal proof in your paper. Occasionally she may ask a question or make a remark which changes your ideas. (This does not happen very often. It has only happened to me on a handful of occasions and I have only understood the point being made sometime later. But those handful of occasions have played a major rôle in my research.)

Unfortunately, if there is an expert in the audience, it is very difficult to avoid addressing your talk to her and ignoring the rest of the audience. This is always rude and often unproductive since the expert may not be as expert as you think. (However low in the mathematical hierarchy you may feel yourself to be, you are probably the world expert on your own work.)

[8]This links with the belief common in the cultured classes that anything can be explained in five minutes.

Speaking from repeated experience, I know that this is a very difficult trap to avoid. I suggest that you adopt a rigid policy of addressing the general audience for, say, the first half of the seminar and only then take the presence of the expert or experts into account.

There is a another type of seminar which is so foreign to the Anglo-Saxon tradition that I know of it only through other people's accounts. This is announced as *X's Seminar*. Professor X chooses the speaker, subject and, in some cases, the audience. The speaker's job is to explain her topic to Professor X. The seminar speaker talks, with frequent interruptions from Professor X and his favourite pupil, until either Professor X is convinced that the whole thing is trivial or wrong, in which case he informs the speaker of the fact, or until Professor X has explained the topic to the lecturer. It was said that political meetings in the old Soviet Union resembled mathematics seminars but mathematics seminars resembled political meetings. It is clear that, if you have the right Professor X, the recipe produces a memorable and productive event event. It is also clear that the right Professor X is a rather rare character.

Proportionally, I have given many more bad seminars than bad lecture courses. Partly this is due to the extra difficulties of the seminar form, but there is a further problem. A lecture course is a regular event and it is not hard to cultivate a regular habit of preparation. In contrast, having promised to give a talk in a couple of months, it is natural to forget about it or simply procrastinate until there is simply not enough time to prepare properly. Usually you can wing it successfully, but not always.

Mathematicians may also be asked to give 'popular lectures'. If the audience is under eighteen, there may well be an element of compulsion in their decision to attend. If the audience consists of adults, they have chosen to come and they are either convinced or wish strongly to be convinced that mathematics is interesting and useful. In either case your sole duty is to entertain. Speaking personally, such lectures lack the edge which comes from the possibility of disaster and the satisfaction of avoiding that disaster. It may (indeed it does) sound pompous but I cannot imagine any audience for a popular lecture which would give me the same thrill as I got from giving the first lecture to beginning undergraduates at Cambridge, knowing that, in all likelihood, the audience contained a student whose name would be remembered when mine is forgotten.

There is a problem with general talks attributable to the spirit of the age. Some years ago, I attended a conference for mathematicians and engineers. The mathematicians felt insulted by the engineers' flashy computer presentations which were clearly used and reused for many occasions. The engineers felt insulted by the blackboard lectures of the mathematicians which were

clearly just thrown together for that particular conference. PowerPoint and its relatives make it possible for people to talk in public who previously lacked the confidence to do so[9]. Unfortunately this means that many audiences (not, however, mathematicians) expect a PowerPoint presentation and are disappointed when they do not get it.

No mathematician who reads this will be surprised by my recommendation (valid for the year 2009), that if a computer presentation is required, they should use Beamer. There are three reasons in increasing order of importance. Firstly Beamer is 'open source' and, because mathematics as practised for the last 3000 years has been 'open source', mathematicians are ideologically attracted to such programs. Secondly, PowerPoint is not designed to communicate mathematics (or, indeed, according to Edward Tufte, to communicate anything at all). Thirdly, and most importantly, Beamer is a LaTeX system and LaTeX is now the language of mathematical printing.

Finally, a small book could be written about the problems of lecturing in other people's lecture rooms, starting with those rooms designed by an architect who once met a man who had an aunt who had actually attended a lecture[10]. Among the many things that may not be provided are coloured chalks, white chalk that does not crumble in your hand, any chalk at all, whiteboard markers that actually mark the board, whiteboard erasers, overhead projectors, transparencies for overhead projectors, working pens for overhead transparencies, computer projection systems compatible with your computer and computers which can understand your memory stick. It is also a rule that all computer systems will fail mysteriously ten minutes into any talk. If you are prepared for such problems, they may not materialise. If you are unprepared, they almost certainly will[11]. Make sure that you see the lecture room well in advance and check all the equipment you use. Insist that your host's unlimited hospitality includes leaving you alone for ten minutes before the talk to collect your thoughts.

[9]Whether the sum of human happiness is increased thereby is another matter. The pleasure of the PowerPoint presenter is often bought at the expense of the audience.

[10]The following advice is intended for any architect who reads this. Remember the reason lecture theatres are so called is that they are *theatres* with the lecturer as the star. Look at the lecture theatre at the Royal Institution in London to see how such a *theatre* should be built.

[11]A historian friend of mine who was told that the room for her talk was fully equipped for computer presentations. When she arrived with her memory stick it turned out out that this meant it had a portable projection-screen.

3 Reflections

My discussion has probably annoyed non-mathematicians, mathematicians and educationalists. However it has annoyed them in different ways.

To the non-mathematician the tone of the discussion probably appears insufferably arrogant. But any working mathematician must have a touch of arrogance. If you tackle an unsolved problem you must believe that you can succeed where others have failed. You may think this is because you are cleverer than they were, or because you know more, or because you have a new approach or just because you are prepared to work harder and longer (it quite possible for a problem to be quite well known but never to have been seriously attacked). Note that this arrogance is of a limited nature. There is a very old story of two men being chased by a bear. One of them shouts 'Its no good, you can't outrun the bear' and the other shouts back 'I'm not trying to outrun the *bear*'. When you try to solve a problem you do not have to outrun all the people who could have attacked the problem, only those that have.

Lecturing is very different from research but unless you believe that you know things that your audience does not know, that those things are worth knowing and that you can teach them to your audience, it is not clear why you should be lecturing to them. Modesty is out of place in front of the blackboard.

Professor M is not only one of the deepest mathematicians in the world but also when one of the most modest. When he lectures he apologises to the audience for the triviality of what he is going to present. In his anxiety not to bore the audience he lectures faster and faster with more and more apologies. It is said that when he went on a lecture tour of US universities he was followed by another lecture who gave a second lecture to explain Professor M's original lecture.

One of the many paradoxical elements in teaching is that whilst students who accept nothing that you say will do badly, students who accept everything you say will not do particularly well. However, since good students mainly educate themselves outside the lecture room, you need not worry that you will influence them too much[12] .Eventually they will outgrow you. The clever ones will make this clear to you but the very clever ones will listen to you as respectfully as ever.

Mathematicians will be annoyed by the fact that I have not dealt with the problem of lecturing to those unwilling to learn. This reflects the fact

[12]And, if you do influence them, it will be in unpredictable ways. One of my former students told me that he '... will never forget the way you told us to think, don't calculate' whilst another remembered the way I told him that '... calculation is the way to truth'.

that (as they have been muttering through clenched teeth) 'The lines have fallen to me in pleasant places'. It also reflects the fact that I have very little advice to give. In the rare cases when your audience is willing to learn from others but not from you, it may be worth asking yourself why you are teaching them what you are teaching them. If it is going to be useful to them, take time out to explain to them why it is going to be useful to them. If it is not, then perhaps you should be teaching them something else.

Educationalists will be annoyed that I have not made any reference to their labours. Like most mathematicians, I remain unconvinced that they have, as yet, much to offer. They are welcome to put this down to arrogance (see above) and intellectual laziness but they would do better to reflect on the reasons we give for our views.

(1) Much of what is quoted as educational research is mere expression of opinion. Much of the rest hardly rises above what Feynman calls cargo cult science (see *Surely You're Joking Mr Feynman!* again).

(2) The only outcomes of education that we can measure (student satisfaction, recall of material after two months and so on) are unsatisfactory proxies for whatever outcome it is that we actually wish for.

(3) Education is not a generic process. The education required to produce a literary critic, a chemist, a violinist or a mathematician is very different. It is now customary in British Universities to provide courses on lecturing to new lecturers[13]. Often such courses include the advice that you should only put three sentences on a blackboard at at a time. So far as mathematicians are concerned, one might as well advise mountaineers to avoid steep slopes or surgeons only to operate on the exceptionally healthy.

(4) Many good mathematicians are also good lecturers. (It is only youthful innocence that causes our students to believe that those who lecture badly do so because they are great thinkers.) After all, good research usually demands insight and clarity and these virtues provide a very good foundation for good lecturing. However, if it is necessary to make a choice, most mathematicians would prefer someone with something to say but who says it badly to some one with nothing to say who says it brilliantly. The better mathematician trumps the better lecturer.

Having said all this, I do believe that a lecturer who thinks about the process of lecturing will lecture better than one who does not. By taking thought, a dreadful lecturer can become a bad lecturer, a bad lecturer can become a mediocre lecturer, a mediocre lecturer a good lecturer and a good

[13]My younger colleagues suggest that I should include advice to those who organise such courses that, other things being equal, it might, perhaps, be better if courses on lecturing are given by people who can lecture.

lecturer an excellent one. If you go to a lecture or seminar which you enjoy you should ask yourself why it went so well. If you go to a lecture or seminar which you disliked, ask yourself what the speaker did wrong. If you give a lecture that goes particularly well or particularly badly ask yourself why. (Often the answer is obvious and unhelpful, but not always.)

There are some simple pieces of advice[14]. Look at the clock from time to time. Try not to end in the middle of a proof. (You will only have to start again at the beginning next time. Either finish early or waffle a bit about what is coming.) Start each lecture by summarising where you have got to. Finish each lecture by saying what you have done. Pause from time to time to allow the audience to catch up on its note taking and to catch its intellectual breath. You can produce such a pause either by stopping speaking (very hard, if you decide to be silent for 30 seconds, you will need to time yourself by the watch, otherwise you will restart after 10 seconds) or by talking about non-essential matters like the history of the problem or the traffic on the way to work[15]. After lectures, walk to the other end of the room to check that your blackboard writing is legible[16]

It is not entirely surprising that our custom of giving students a morning of advice on attending lectures before they have actually attended one, sometimes fails in its intended purpose. I suspect that the advice would be more effective if it was given after two weeks of lectures, when the students actually know what a lecture is. In the same way, I doubt if lecturing training is very effective if it is given to those who have not lectured before. If I were a vice-chancellor [17] I would substitute occasional meetings with plenty of cream cakes[18] to allow lecturers to talk about problems and exchange hints of the type given in the previous paragraph.

Those who see mathematicians as irredeemably conservative should reflect that university mathematical education does change with time and the path to the modern mathematics departments runs from the École Polytechnique in 1800 through Göttingen in 1910. They should also reflect that the system is an almost ideally Darwinian one. The most successful teachers will have the most successful students and they will teach, with substantial variation, in the way they were taught. If a consistently better teaching method

[14]Complicated pieces of advice are no use. A swimming trainer once told me that it took 24 hours of practice to change any aspect of a swimmer's style. There is just too much going on in a lecture to allow you to follow anything but the simplest advice.

[15]Dr R endeared himself to generations of Cambridge students by complaining about his dreadful hangovers.

[16]I once asked Conway the best way to finish a lecture course. He replied 'Early'.

[17]'Which, thank the lord, I'm not, sir.'

[18]The cream cakes are particularly important.

appears it should spread rapidly.

Mathematicians have their heroes. When I was a young mathematician at Orsay, the word went round that Gelfand had been allowed out by the Soviet authorities to visit Paris and would give a lecture at 4pm. The lecture room was filled to capacity and beyond by mathematicians from all over Paris with the most eminent in the front row[19] and late arrivals sitting on the steps or standing at the back. There was a storm of applause and Gelfand (yes, the Gelfand whose name figured so prominently in my fourth year studies, yes, Gelfand himself) arrived visibly tired from his long journey but visibly delighted to be where he was, shaking hands enthusiastically with the entire front row. The lecture was beautifully delivered in excellent English but at a level of abstraction well beyond me. All I remember is a an elegant formula suddenly appearing and Gelfand saying that 'A concrete formula like this reassures us that we must be on the right track'. Those who think that the only purpose of lectures is to communicate knowledge will not understand why Gelfand's lecture is a treasured memory.

[19]When I was young, I thought that the old sat in the front to demonstrate their eminence. Now that I am older I realise their choice has more to do with the state of their eyes and ears.