1. Set Theory without the Axiom of Foundation . . . .
Dr. T. E. Forster

The independence of the axiom of foundation can be easily established. Various axioms have been proposed which contradict foundation and these should be discussed and the consistency of a few proved.

Asking oneself whether or not one believes the axiom of foundation prompts one to reflect on what one thinks sets are and what a formalised set theory is for. Various people have advocated set theory without foundation as a way of modelling other phenomena which exhibit illfoundedness. Examples are situation semantics (Barwise and Perry *et al.*) and Robin Milner's work on concurrency. The appropriateness of this use of set theory is controversial and could be discussed. Typically the antifoundation axioms provoked by these motivations do not involve the existence of a universal set, and their consistency is generally unproblematic. In contrast set theories with a universal set have deep and poorly-understood connections with Type theory and Polymorphism.

I would be prepared to discuss this matter with interested students and supply copies of relevant literature.

Relevant Courses

Essential:Set Theory and Logic

References

Recommended reading:
P. Aczel: Non-well-founded sets. CSLI 1988
T.E.Forster: Set theory with a universal set (2nd ed^n) OUP 1995
. . . and references therein.
Barwise and Moss: Vicious Circles