A Model Tripos Question

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In a propositional language p, q, $\ldots$ are the atomic formulæ; $\neg p$ etc. are **negatomics**; a **literal** is a formula that is an atomic or a negatomic; A, B are complex formulæ. The **dual** $\hat{A}$ of a propositional formula A is the result of replacing every literal in A by its negation. (Notice that $\hat{A}$ is *not* usually the same as $\neg A$!)

A formula A **self-dual** if it is logically equivalent to its own dual: that is to say that $A \longleftrightarrow \hat{A}$ is a tautology. (For example: p `XOR` q is self-dual—'p' and 'q' being literals—even tho' A `XOR` B is not self-dual in general.)

Show—by considering disjunctive normal forms or otherwise—that whenever A is a self-dual formula there is a formula B such that A is logically equivalent to $B \longleftrightarrow \hat{B}$.