Answer to Question 6 (Dr Forster's 'Nice' Structured Proof Exercises)

James Baker (jeb200)

April 4, 2013

1	$\forall x.\exists y.R(x,y)$	Assumption
2	actual sk_1	Assumption
3	$\exists y.R(sk_1,y)$	$\forall -$ Elimination $(1,2)$
4	actual $sk_2, R(sk_1,sk_2)$	Assumptions
5	actual sk_3	Assumption
6	$(sk_1 = sk_3) \vee \neg(sk_1 = sk_3)$	Excluded Middle
7	$sk_1 = sk_3$	Assumption
8	$R(sk_1,sk_2)$	Line 4
9	$R(sk_3,sk_2)$	$= -$Substitution $(8,7)$
10	$sk_1 = sk_3$	Assumption
11	$sk_2 = sk_2$	Reflexivity
12	$sk_1 = sk_3 \Rightarrow sk_2 = sk_2$	$\Rightarrow -$Introduction $(10-11)$
13	$R(sk_3, sk_2 \wedge (sk_1 = sk_3 \Rightarrow sk_2 = sk_2)$	$\wedge -$ Introduction $(9,12)$
14	$R(sk_1,sk_2) \wedge R(sk_3,sk_2) \wedge (sk_1 = sk_3 \Rightarrow sk_2 = sk_2)$	$\wedge -$ Introduction $(8,13)$
15	$\exists y_2.(R(sk_1,sk_2) \wedge R(sk_3,y_2) \wedge (sk_1 = sk_3 \Rightarrow sk_2 = y_2))$	$\exists -$ Introduction (14)
16	$\neg(sk_1 = sk_3)$	Assumption
17	$\exists y.R(sk_3,y)$	$\forall -$ Elimination 1
18	actual $sk_4, R(sk_3,sk_4)$	Assumptions
19	$R(sk_1,sk_2)$	Line 4
20	$R(sk_3,sk_4)$	Line 18
21	$sk_1 = sk_3$	Assumption
22	$\perp$	$\neg -$ Introduction $(16,21)$
23	$sk_2 = sk_4$	$\perp -$Elimination (22)
24	$sk_1 = sk_3 \Rightarrow sk_2 = sk_4$	$\Rightarrow -$Introduction $(21-23)$
25	$R(sk_3,sk_4) \wedge (sk_1 = sk_3 \Rightarrow sk_2 = sk_4)$	$\wedge -$ Introduction $(20,24)$
26	$R(sk_1,sk_2) \wedge R(sk_3,sk_4) \wedge (sk_1 = sk_3 \Rightarrow sk_2 = sk_4)$	$\wedge -$ Introduction $(19,25)$
27	$\exists y_2.(R(sk_1,sk_2) \wedge R(sk_3,y_2) \wedge (sk_1 = sk_3 \Rightarrow sk_2 = y_2)$	$\exists -$ Introduction (26)
28	$\exists y_2.(R(sk_1,sk_2) \wedge R(sk_3,y_2) \wedge (sk_1 = sk_3 \Rightarrow sk_2 = y_2))$	$\exists -$ Elimination $(17, 18-27)$
29	$\exists y_2.(R(sk_1,sk_2) \wedge R(sk_3,y_2) \wedge (sk_1 = sk_3 \Rightarrow sk_2 = y_2))$	$\vee -$ Elimination $(6, 7-15, 16-28)$
30	$\forall x_2.\exists y_2.(R(sk_1,sk_2) \wedge R(x_2,y_2) \wedge (sk_1 = x_2 \Rightarrow sk_2 = y_2))$	$\forall -$ Introduction $(5,29)$
31	$\exists y_1.\forall x_2.\exists y_2.(R(sk_1,y_1) \wedge R(x_2,y_2) \wedge (sk_1 = x_2 \rightarrow y_1 = y_2))$	$\exists -$ Introduction (30)
32	$\exists y_1.\forall x_2.\exists y_2(R(sk_1,y_1) \wedge R(x_2,y_2) \wedge (sk_1 = x_2 \Rightarrow y_1 = y_2))$	$\exists -$ Elimination $(3, 4-31)$
33	$\forall x_1.\exists y_1.\forall x_2.\exists y_2.(R(x_1,y_1) \wedge R(x_2,y_2) \wedge (x_1 = x_2 \Rightarrow y_1 = y_2))$	$\forall -$ Introduction $(2-32)$
34	$\forall x.\exists y.R(x,y) \Rightarrow \forall x_1.\exists y_1.\forall x_2.\exists y_2.(R(x_1,y_1) \wedge R(x_2,y_2) \wedge (x_1 = x_2 \Rightarrow y_1 = y_2))$	$\Rightarrow -$Introduction $(1-33)$