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Thomas Forster

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Ryan Jenkinson has a rather good idea. The language is regular iff its reverse is regular. So it will suffice to show that the reverse of this language is not regular. What is the reverse? You sit there in your darkened room (as in the thought-experiment in section 1.2 of `https://www.dpmms.cam.ac.uk/~tf/cam_only/languages-and-automata.pdf`) watching characters from $\{0, 1\}$ coming at you. As long as you are seeing nothing but 1s you are happy. When you receive the first 0 you recall the number of 1s you have seen, n, say. Then, if $n \in \mathbb{K}$ you go into a permanently happy state. If $n \notin \mathbb{K}$ then you go into a permanently *un*happy state.

The question is: "Is this language regular?" The way to answer it is to check whether or not the number of states you might be in is infinite. Clearly if you want to know whether or not the number of 1 you have seen so far is a member of $\mathbb{K}$ then the only thing you can do is keep counting the 1s as they come in. With only finitely many states you can keep track of the residue mod p and stuff like that, but of course nothing like that will do!

More to the point, suppose we had a DFA $\mathfrak{M}$ that recognised this language. Then we could solve the `HALT`ing problem, as follows. Given n, a candidate for membership of $\mathbb{K}$, form the string 1^n0 and feed it to $\mathfrak{M}$. $\mathfrak{M}$ will go PING!!! iff $n \in \mathbb{K}$. Clearly there can be no such $\mathfrak{M}$.