2017 p3 q11H

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Problem reduction exercises are always hacky and *ad hoc* ("ad hack" the wags say). There are a few tricks you can try but none of them really amount to a technique. You just have to snoop around looking for unlocked windows.

This particular problem reduction exercise is this: given a machine that, given n, will go PING! if W_n is decidable (we know it is semidecidable), use it to answer questions of the kind: "Is W_m finite?" To do this tweaking we have to be able to do the following, given m, come up (computably!) with n s.t. W_n is recursive iff W_m is finite. So: if W_m is *in*finite then W_n must fail to be decidable. What is your favourite example of a semidecidable set that is semidecidable but not decidable? As Imre would say: "Switch brain off and do the obvious thing". Yes, $\mathbb{K}$. Duh.

So, given m, i compute n as follows. I get a volcano V (a machine running in parallel with itself that emits numbers without being asked) that emits members of W_m. While this is going on i am trying to compute members of $\mathbb{K}$. I do this by running $\{k\}(k)$ for lots of k in parallel. Which k? Well, at each stage i am running $\{k\}(k)$ in parallel on all the k that are below the largest number emitted by V so far. This process i have described is parametrised by m and so represents a function from $\mathbb{N}$ to $\mathbb{N}$. The set of numbers i get is of course semidecidable and is W_n for some n, and, yes, i can compute this n from m.

By assumption i have a machine $\mathfrak{M}$, wot i have trafficked from Eastern Europe and am keeping in inhumane and degrading conditions in a mouldy and rat-infested attic, where I use it to answer questions of the form "Is W_n a decidable set?" I now force open the jaws of my machine and insert the number n that i got from the preceding paragraph. What might W_n be? It might be finite, and if it's finite, well, it's finite. But it might be infinite. But if it's infinite it must be $\mathbb{K}$, and so is not decidable! So if $\mathfrak{M}$ says that W_n is recursive it must be that it is finite, but that means that W_m was finite!