COMPUTER SCIENCE TRIPOS Part IB 2013
Paper 6
Computation Theory (AMP)

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March 7, 2021

(a)

(i) What does it mean for a λ-term to be a β-normal form? Defining the sets of canonical (C) and neutral (U)λ-terms by the grammar

$$C ::= \lambda x.C \mid U$$
$$U ::= x \mid UC$$

show that a λ-term is a β-normal form if and only if it is canonical.

[5 marks]

(ii) Carefully stating any standard properties of β-reduction, explain why a λ-term reduces to at most one β-normal form (up to α-equivalence). [4 marks]

(iii) Give an example of a λ-term that does not reduce to any β-normal form. [2 marks]

(b)

(i) Define what it means for a closed λ-term F to represent a partial function $f \in \mathbb{N} \to \mathbb{N}$.

[4 marks]

(ii) The composition of partial functions $f, g \in \mathbb{N} \rightharpoonup \mathbb{N}$ is the partial function $g \circ f = \{(x, z) : (\exists y)(x, y) \in f \wedge (y, z) \in g\} \in \mathbb{N} \rightharpoonup \mathbb{N}$. Suppose F represents f, G represents g, and f and g are totally defined. Show that $\lambda x.G(Fx)$ represents $g \circ f$.

[2 marks]

(iii) Give an example to show that $\lambda x.G(Fx)$ need not represent $g \circ f$ when f and g are not totally defined.

[3 marks]

(a) (i)

"Neutral". I knew i'd seen this expression somewhere, somewhere local, but the notes i read it in probably allude to material no longer examinable. I found the following file that explain is `https://www.cs.cmu.edu/~fp/courses/15814-f18/lectures/Notes-15814-f18.pdf`