1. Show that if a group $G$ contains an element of order six, and an element of order ten, then $G$ has order at least 30.

2. Show that the set $\{1, 3, 5, 7\}$ forms a group under multiplication modulo 8. Is it isomorphic to $C_2 \times C_2$ or $C_4$?

3. How many subgroups does the quaternion group $Q_8$ have? What about the dihedral group $D_8$?

4. Let $H$ be a subgroup of a group $G$. Show that there is a (natural) bijection between the set of left cosets of $H$ in $G$ and the set of right cosets of $H$ in $G$.

5. What is the order of the Möbius map $f(z) = iz$? What are its fixed points? If $h$ is another Möbius map what can you say about the order and the fixed points of $hfh^{-1}$? Construct a Möbius map of order 4 that fixes 1 and $-1$.

6. Show that $\mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}^2; (t, (x, y)) \mapsto (e^t x, e^{-t} y)$ defines an action of $(\mathbb{R}, +)$ on $\mathbb{R}^2$. What are the orbits and stabilisers of this action? There is a differential equation that is satisfied by each of the orbits. What is it?

7. Suppose that $G$ acts on $X$ and that $y = g \cdot x$ for some $x, y \in X$ and $g \in G$. Show that $\text{Stab}_G(y) = g \text{Stab}_G(x) g^{-1}$.

8. Suppose that $Q$ is a quadrilateral in $\mathbb{R}^2$. Show that its group of symmetries $G(Q)$ has order at most 8. For which $n$ is there a $G(Q)$ of order $n$? *Which groups can arise as a $G(Q)$ (up to isomorphism)?

9. Let $G$ be a finite group and let $X$ be the set of all its subgroups. Show that $(g, H) \mapsto gHg^{-1}$ defines an action of $G$ on $X$. Show that for $H \in X$, $|\text{Orb}_G(H)| \leq |G/H|$. Deduce that if $H \neq G$ then $G$ is not the union of all conjugates of $H$.

10. Show that $D_{2n}$ has one conjugacy class of reflections if $n$ is odd and two conjugacy classes of reflections if $n$ is even.

11. Let $G$ be the group of all symmetries of a cube. Show that $G$ acts on the 4 lines joining diagonally opposite pairs of vertices. Show that if $l$ is one of these lines then $\text{Stab}_G(l) \cong D_6 \times C_2$.

12. Show that every group of order 10 is cyclic or dihedral. Suppose that $p$ is any odd prime. *Can you extend your proof to groups of order $2p$?

13. Let $G$ be a finite abelian group acting faithfully on a set $X$. Show that if the action is transitive then $|G| = |X|$.

14. Let $p$ be a prime. By considering the conjugation action show that every group of order $p^2$ is abelian. Deduce that there are precisely two groups of order $p^2$ up to isomorphism.