Network models in banking

1. How do banks benefit by forming 'links' with one another, and what are the properties of networks that are thereby created?
2. Once a network has formed, might this increase the risk that 'bank failure' is 'infectious'?
3. Can the risk of such infection be perceived locally?
   Can some sort of *local regulation* provide sufficient safeguard, or is some sort of *global regulation* required?

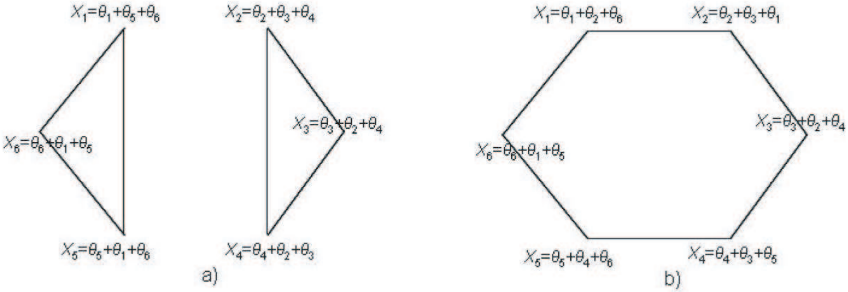
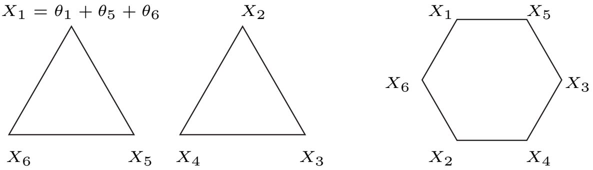
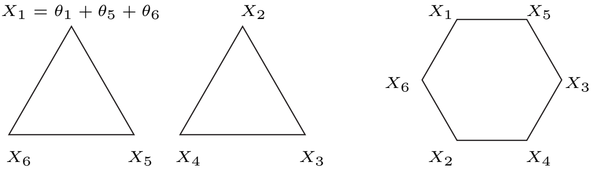


Fig. 1: a) Clustered network; b) Efficient network.



(a) (b)

We wish to minimize $E[\eta \,|\, \eta \geq 1]$, where $\eta = \sum_i 1_{\{X_i < 3r\}}$.



(a) (b)

We wish to minimize $E[\eta \,|\, \eta \geq 1]$, where $\eta = \sum_i 1_{\{X_i < 3r\}}$.

$$E[\eta \,|\, \text{bank } i \text{ fails}) = \sum_j P(X_j < 3r \,|\, X_i < 3r)$$

(a) is better than (b) since

$$P(X_j < 3r \,|\, X_i < 3r) = \frac{P(X_i < 3r \text{ and } X_j < 3r)}{P(X_1 < 3r)}$$

$$
\begin{aligned}
P(X_i < 3r \text{ and } X_j < 3r)& \\
&= P(\theta_i + \theta_{i_2} + \theta_{i_3} < 3r \text{ and } \theta_j + \theta_{j_2} + \theta_{j_3} < 3r)
\end{aligned}
$$

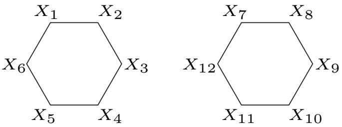
This is least when $|\{\theta_{i_2}, \theta_{i_3}\} \cap \{\theta_{j_2}, \theta_{j_3}\}|$ is small

Lesson: try to reduce prevalence of common neighbours.

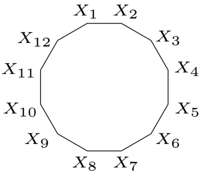
‘triangles are bad’.

But which of these is better?

(b)



(c)



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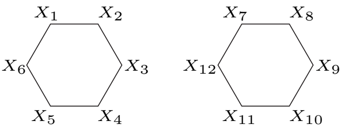

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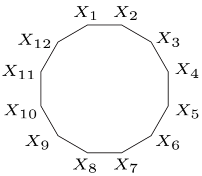

In both (b) and (c) each node has 2 other nodes with which it shares 1 common neighbour, and $n-3$ nodes with which it shares no common neighbours. So $E[\eta \mid X_i < 3r]$ is the same.

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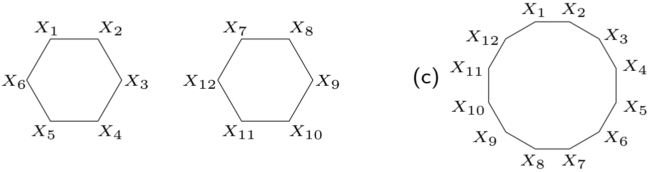

In both (b) and (c) each node has 2 other nodes with which it shares 1 common neighbour, and $n-3$ nodes with which it shares no common neighbours. So $E[\eta \,|\, X_i < 3r]$ is the same.

What about $P(\eta \geq 5 \,|\, X_i < 3r)$?

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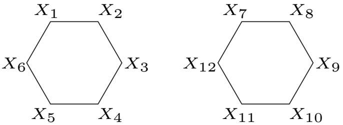
(b)



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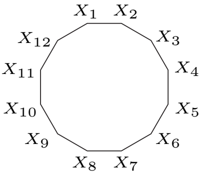

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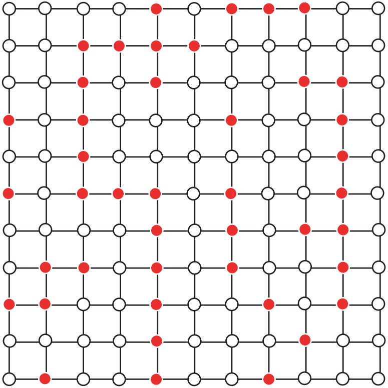
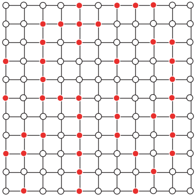
What about $P(\eta \geq 5 \,|\, X_i < 3r)$?

(c) is better — it has fewer pairs of nodes who have neighbours who share a common neighbour.
Risk of an extreme bad event like '≥ 5 failures' depends knowledge of graph connectivity that is not locally observable.

Suppose n^2 banks are arranged in a $n \times n$ square lattice.



Each bank can adopt a high risk strategy, or **low risk strategy**.



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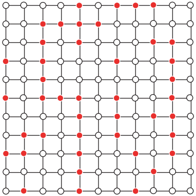

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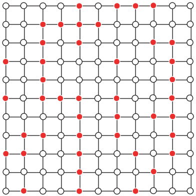

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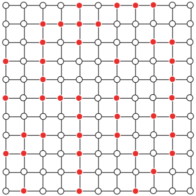
   with probability $1 - \theta$ it does not fail, but then:
   (a) if any neighbour of i fails, then i also fails.



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   with probability $1 - \theta$ it does not fail, but then:
   (a) if any neighbour of i fails, then i also fails.
   (b) if all 4 neighbours of i do not fail, then i obtains R, $R > r$.

What might be a Nash equilibrium strategy?

Assume

$$(1-\theta)^5 R < r < (1-\theta) R \,.$$

- All banks $\rightarrow$ low risk is not an equilibrium.
  Bank i benefits by switching to high risk strategy since

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  Bank i benefits by switching to low risk strategy since

$$r > R(1-\theta)^5\,.$$

Consider a mixed equilibrium in which bank i adopts a high or low risk strategies with probabilities p and $1-p$, respectively. This must satisfy

$$\begin{array}{ccc} \text{low risk} & & \text{high risk} \\ r & = & (1-\theta)(1-p+(1-\theta)p)^4 R \end{array}$$

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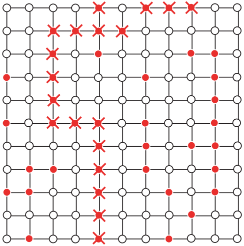
So the Nash equilibrium is the mixed strategy with

$$p = \frac{1}{\theta}\left[1 - \left(\frac{r/R}{1-\theta}\right)^{1/4}\right].$$

This varies from 1 to 0 as r/R varies from $(1-\theta)^5$ to $(1-\theta)$.

What is the probability of a banking crisis?

1. All banks adopting risky strategies in the 'top row' fail. (Perhaps 'top row' banks made sub-prime loans.)
2. No bank in any row below fails of its own accord.



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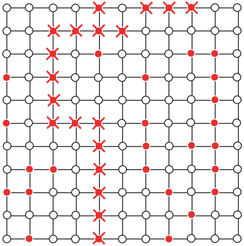

What is the probability ϕ that failure of top row banks causes some bank failure in every row below?

$\phi = P(\text{bank failure in every row} \mid \text{failure throughout top row})$

Obviously, ϕ depends on p, the probablity with which banks are adopting the high risk strategy.

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Surprisingly,

$$\phi(p) \approx \begin{cases} 0 & p < 0.593 \\ 1 & p > 0.593 \end{cases}$$

Recall that p depends continuously on $(r/R)/(1-\theta)$. Thus if $(r/R)/(1-\theta) \approx 0.593$ then a small change in r, R, or θ can 'flip' the whole banking system from 'safe' to 'risky'.