Non-linearities at black hole horizons

Béatrice Bonga - 13 October 20

Based on arXiv:2306.11142

[Neev Khera, Ariadna Ribes Metidieri, BB, Xisco Jiménez Forteza, Badri Krishnan, Eric Poisson, Daniel Pook-Kolb, Erik Schnetter, Huan Yang]

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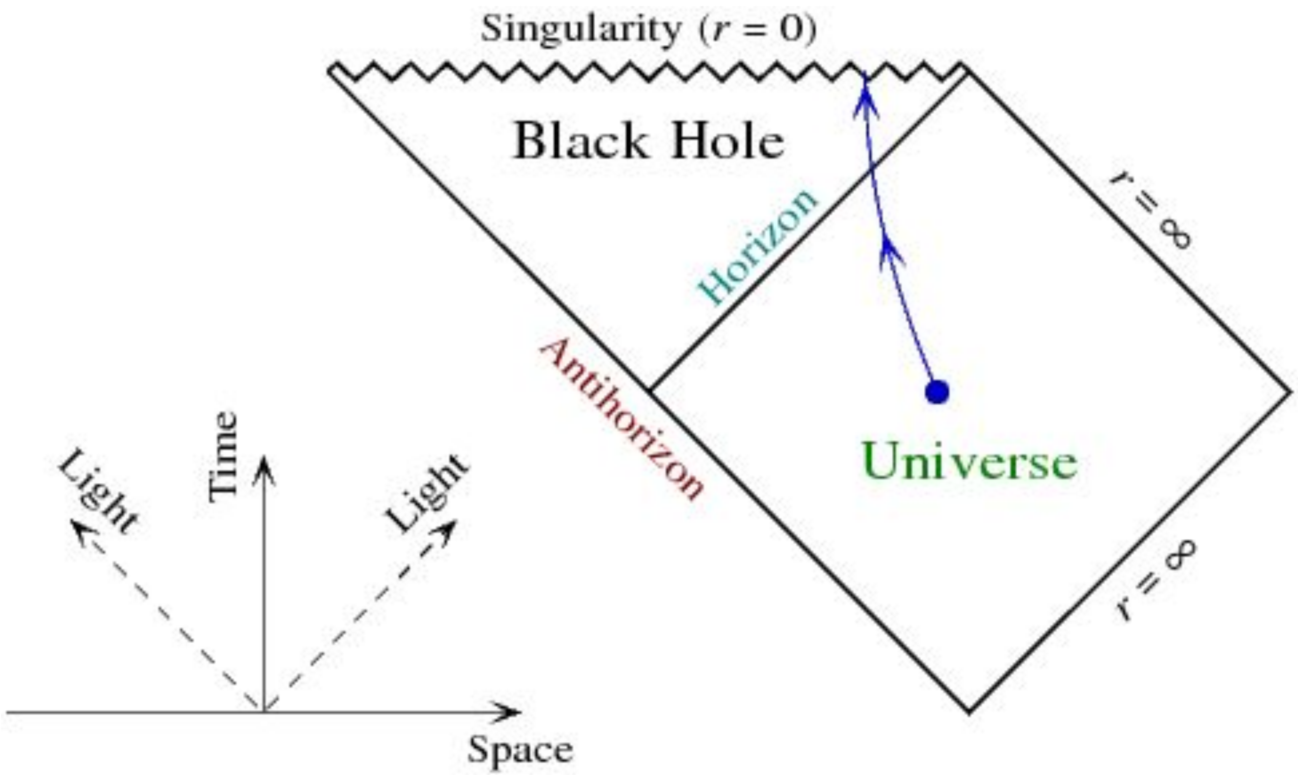
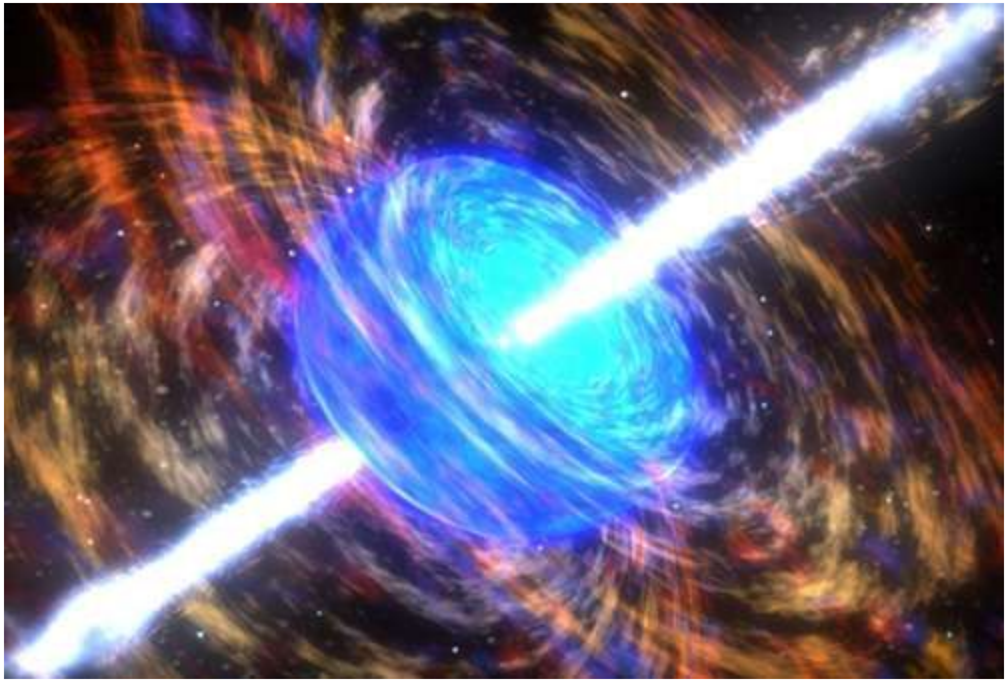
Why care about horizons?

Observations are @ null infinity



Electromagnetic observations and their sources



Gravitational waves…

are interesting because of their origin!

Corollary:

QNMs are interesting because they are emitted by black holes.

Non-linearities?

Perturbation theory on Kerr

$$\Psi^{(1)} \sim A^{(1)}_{\pm,lmn}(r) e^{-i\omega_{\pm,lmn} t + i\phi_{\pm,lmn}} \, {}_2Y_{lm}(\theta, \varphi)$$

$$\mathcal{O}\Psi^{(1)} = 0$$

$$\mathcal{O}\Psi^{(2)} = \mathcal{S}(h^{(1)}, h^{(1)})$$

$$\Psi^{(2)} \sim A^{(2)}_{\pm,lmn}(r) e_2^{-i\omega^{(2)}_{\pm,lmn} t + i\phi_{\pm,lmn}} Y_{lm}(\theta, \varphi)$$

$$\omega_{lmn \times l'm'n'} = \omega_{lmn} + \omega_{l'm'n'}$$

Amplitude relation

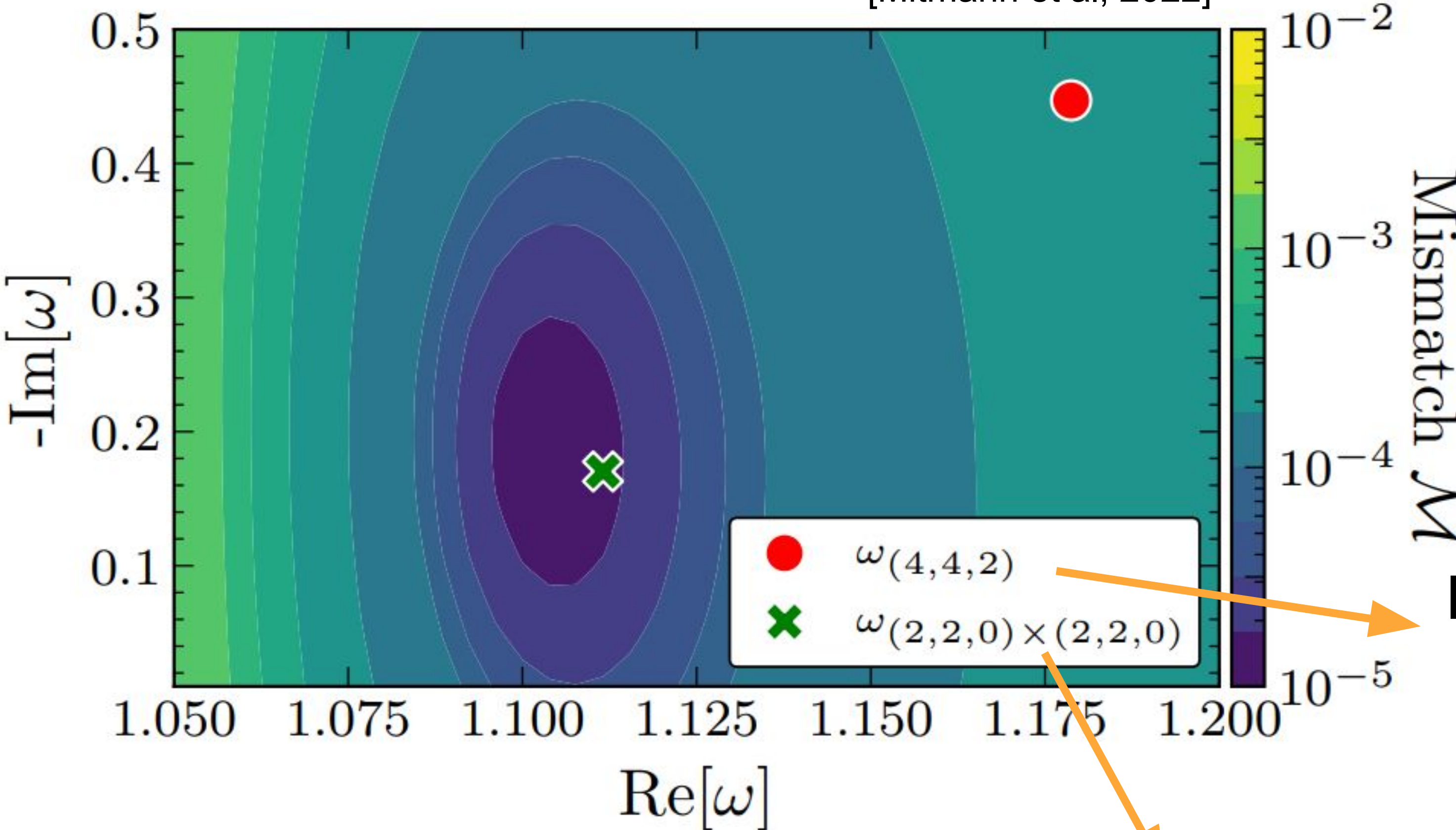
$$\mathcal{O}\Psi^{(2)} = \mathcal{S}(h^{(1)}, h^{(1)})$$

$$A^{(2)}_{lmn} Y_{lm} \sim \Sigma \overbrace{\boxed{f(r;M)}}^{\text{background}} \overbrace{\boxed{A^{(1)}_{lmn} A^{(1)}_{l'm'n'}}}^{\text{initial data}} \underbrace{Y_{lm} Y_{l'm'}}_{\sim G_{lm\times l'm'} Y_{lm}}$$

$$A^{(2)}_{lmn\times l'm'n'} = c_{lmn\times l'm'n'}(M, a) A^{(1)}_{lmn} A^{(1)}_{l'm'n'}$$

Non-linear model preferred @ infinity

[Mitmann et al, 2022]



Prediction linear order

Prediction quadratic order in perturbation theory

So why do I think this is exciting?

Implications for observations:

$$h^{obs} = h^{linear} + h^{non-linear}$$

but frequencies are "finger-printed" with an order in perturbation theory!

Also true @ black hole horizon?

Horizon should be more non-linear, but not too crazy

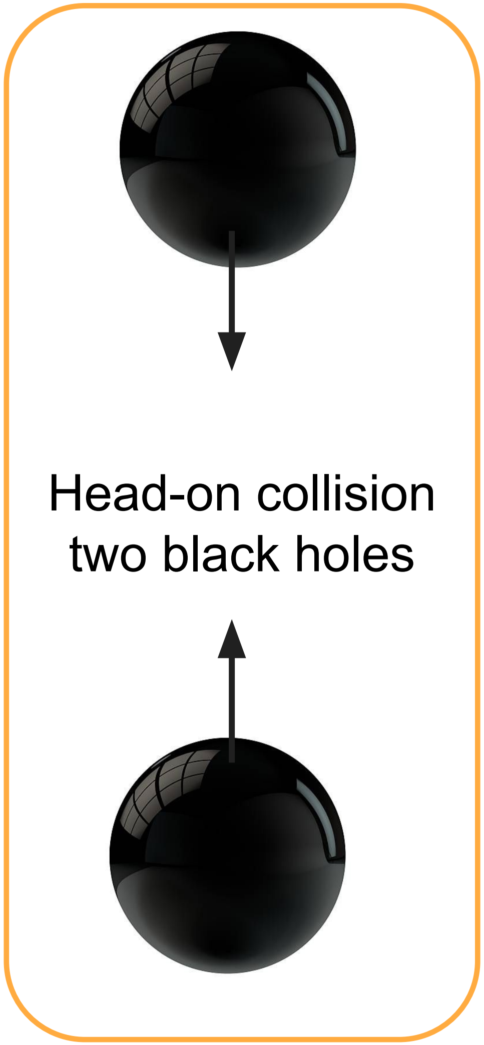
→ easier to find quadratic QNMs

Horizon is strong field regime →hopeless to try to find any QNMs

Disclaimer

All results are based on fitting observations.
No theoretical derivations (yet)....

Two sets of simulations using the Einstein Toolkit

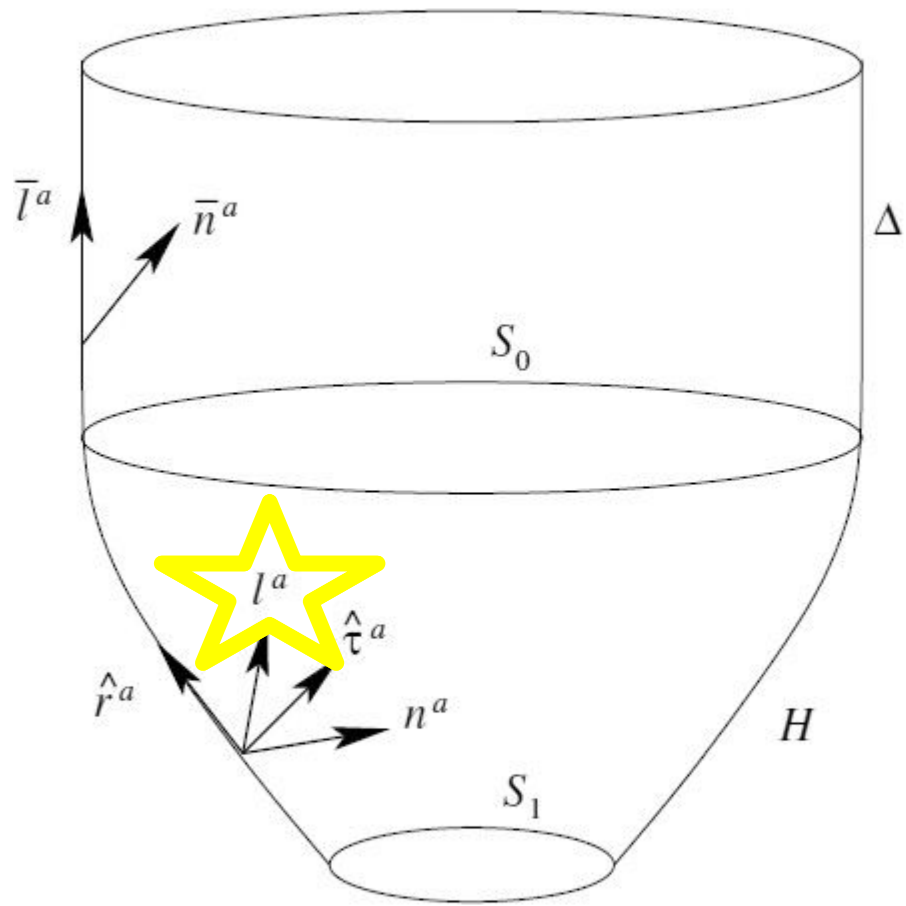


(1) Resulting BH is non-rotating
(2) Axisymmetric simulations ⟶ no m=0 modes
(3) High resolution near horizon (but poor near infinity)

- Unboosted: m1,m2,P=0
- Boosted: m1=m2, P≠ 0

⇨ linear amplitudes 10x bigger

Shear at the horizon



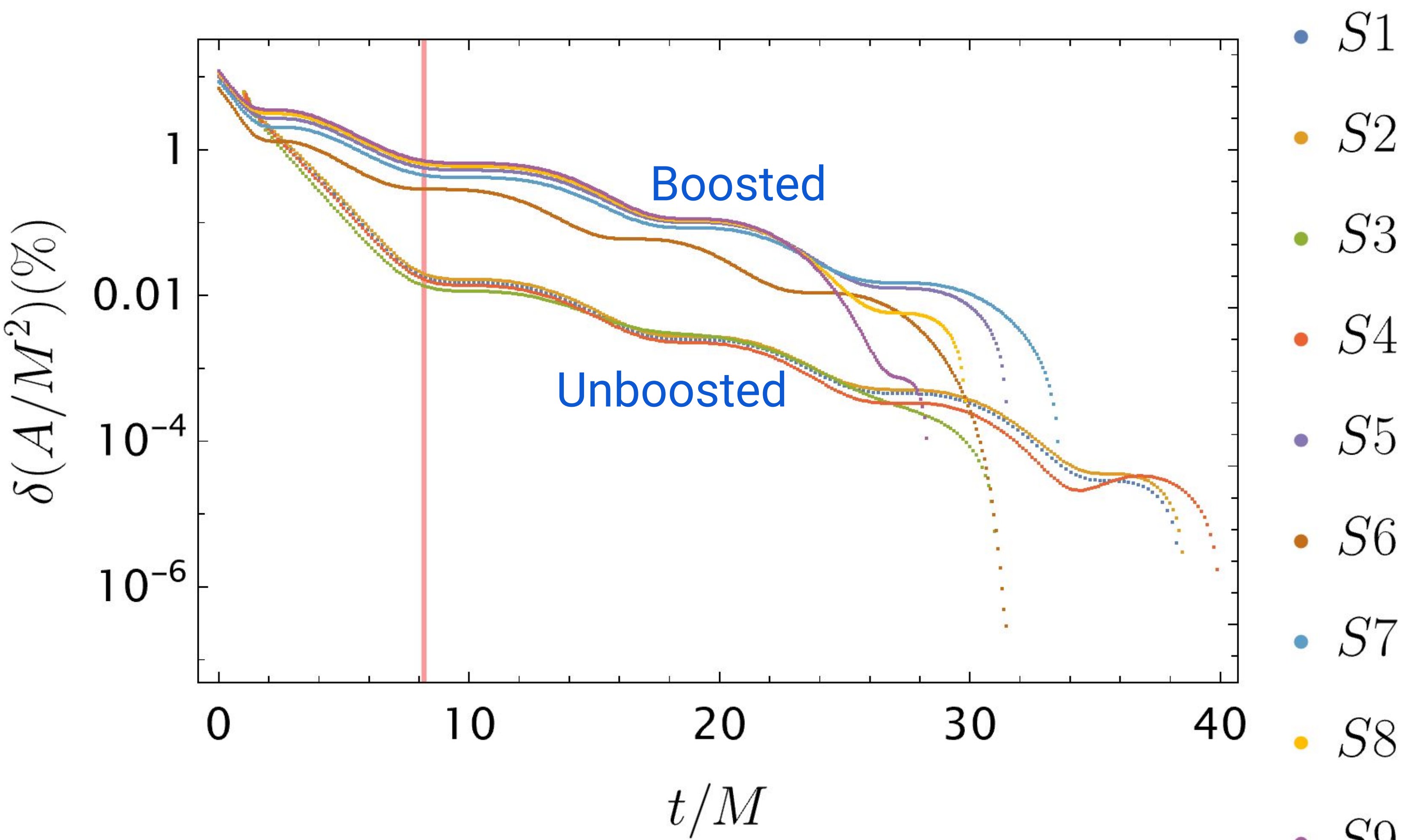
Choice of time

Time ⟷ Definition of frequency

Disclaimer: We simply use the simulation time.

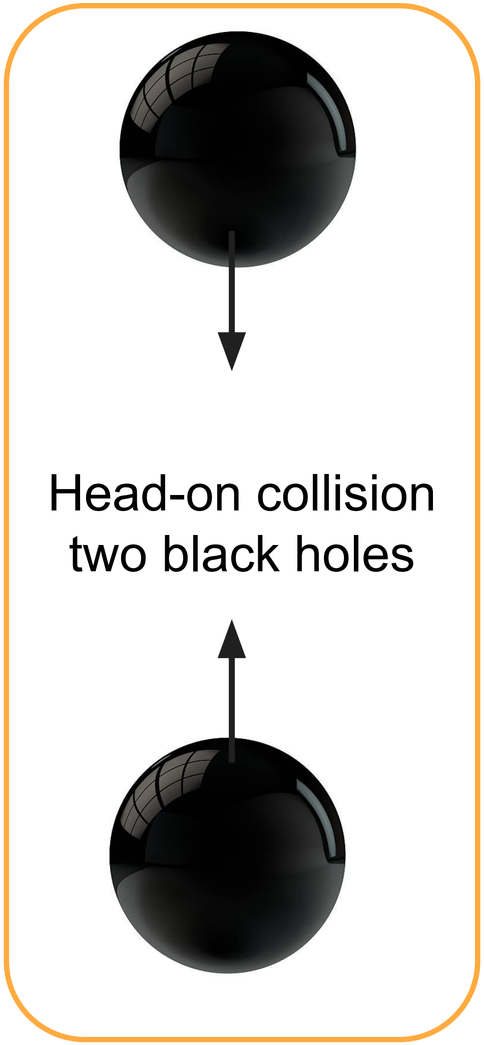
Same issue at infinity!

Ringdown: Mass changes ≤ 1 %



We take $t_{ringdown}$ = 8.2 M

Two sets of simulations using the Einstein Toolkit



- Unboosted: m1,m2,P=0
  - ⇨ model with 4 tones
- Boosted: m1=m2, P≠ 0
  - ⇨ linear amplitudes 10x bigger
  - ⇨ model with 3 tones

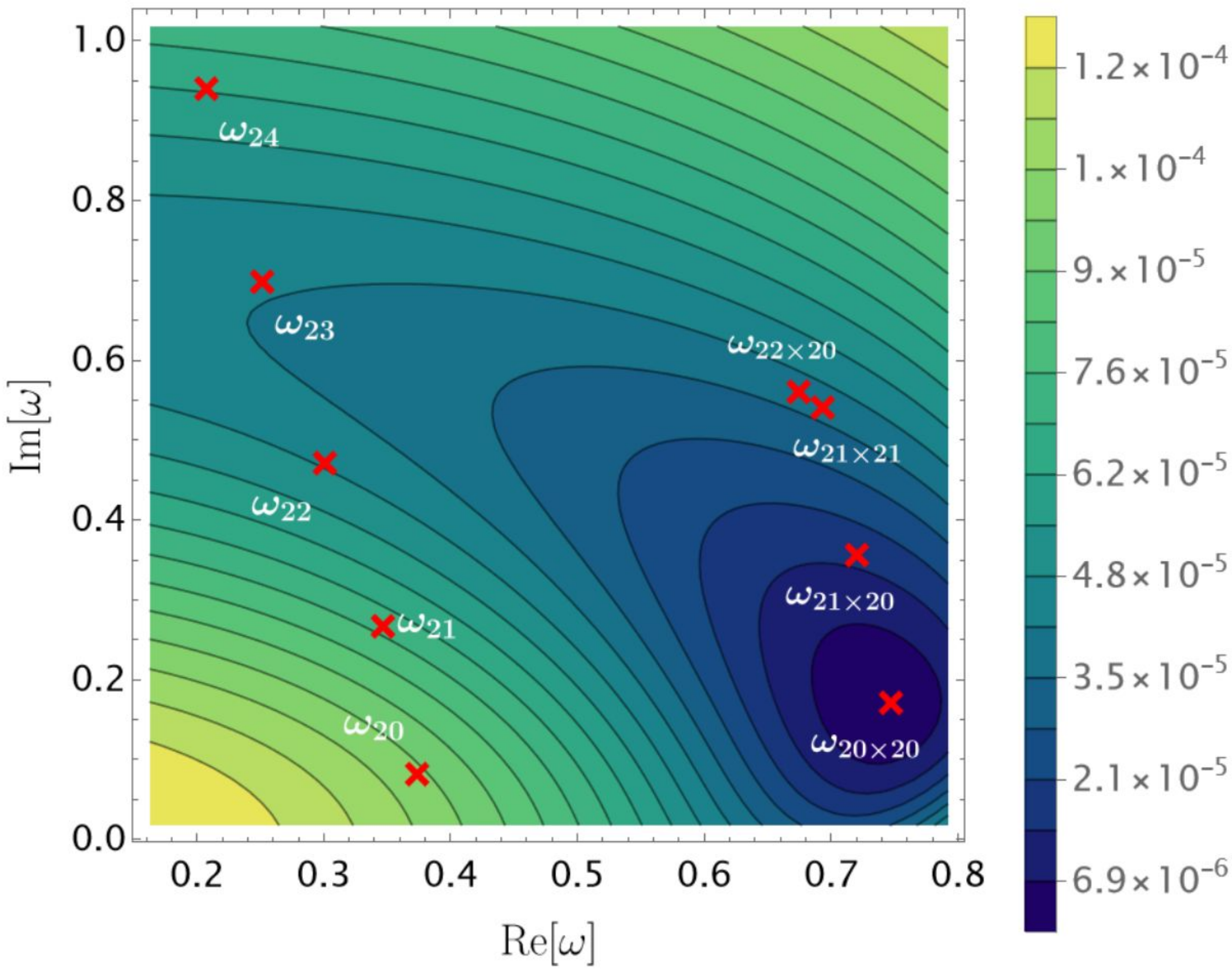
S7: boosted

Equal mass → $l=2,4,6,\ldots$ are only non-zero.

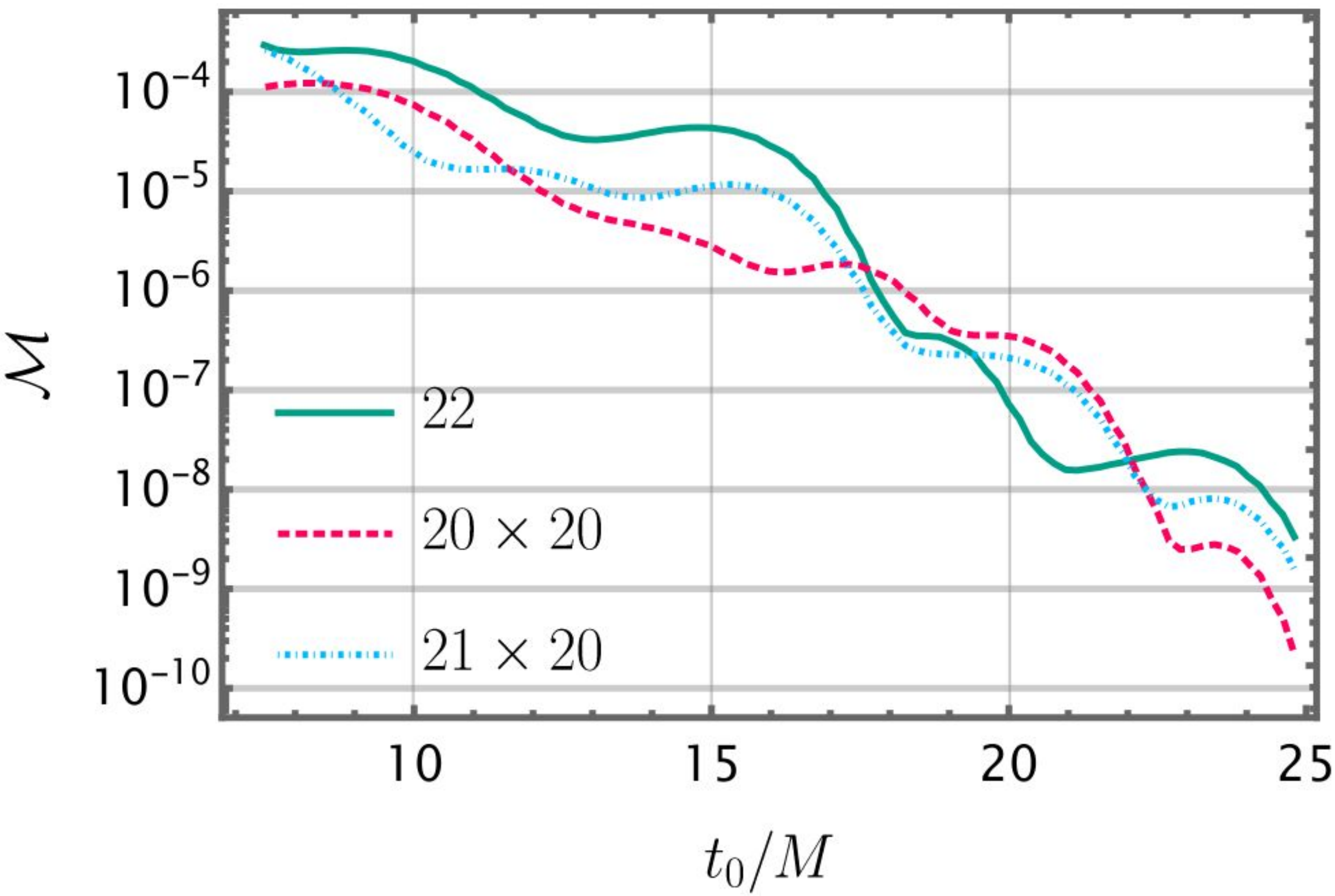
Notation: $\omega_{lmn} \rightarrow \omega_{ln}$

For $l=2$, possible quadratic modes are $\omega_{20 \times 20}$ and $\omega_{20 \times 40}$ + possible versions with overtones.

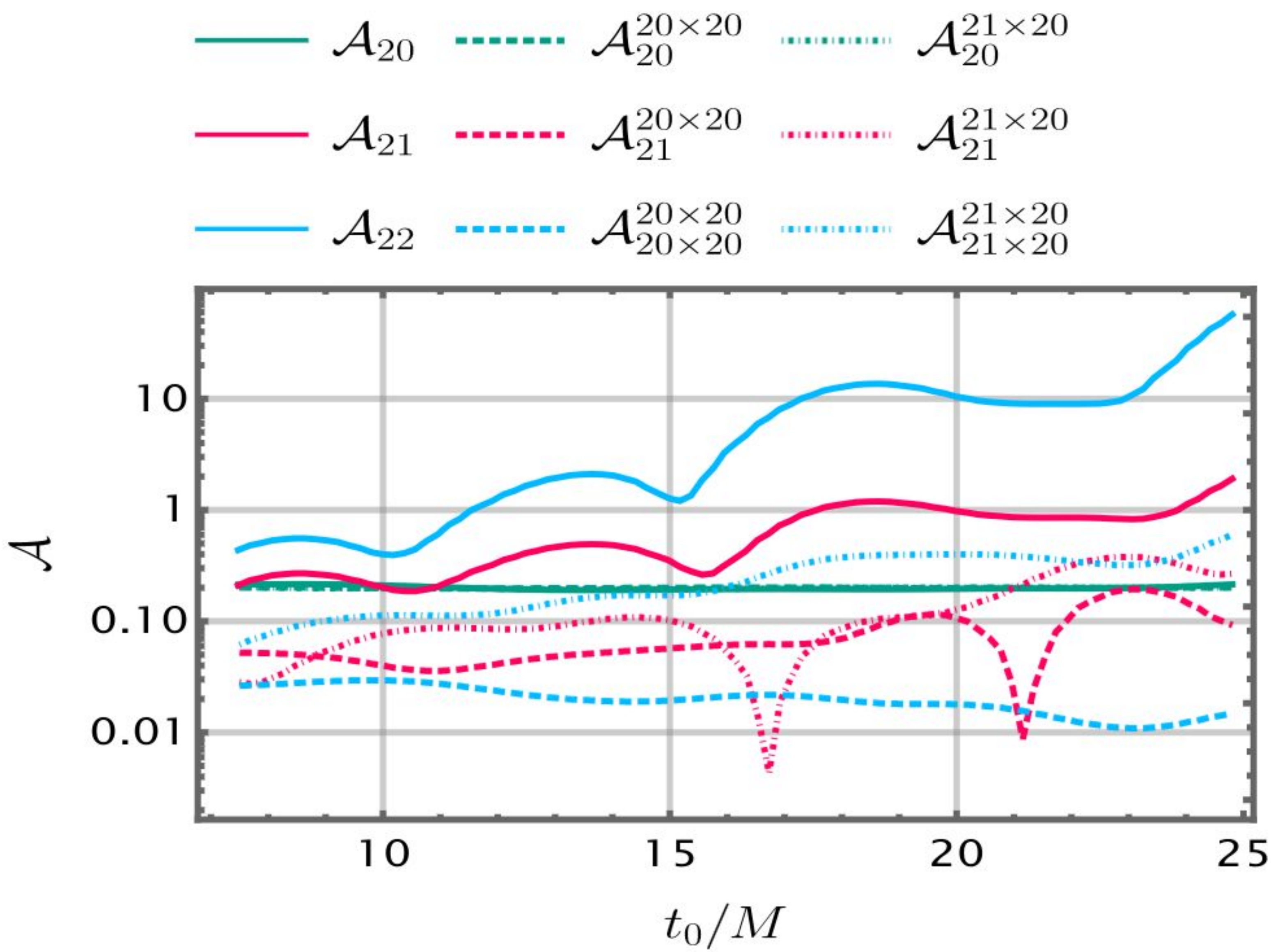
Mismatch S7 after fixing ω_{200} and ω_{201}



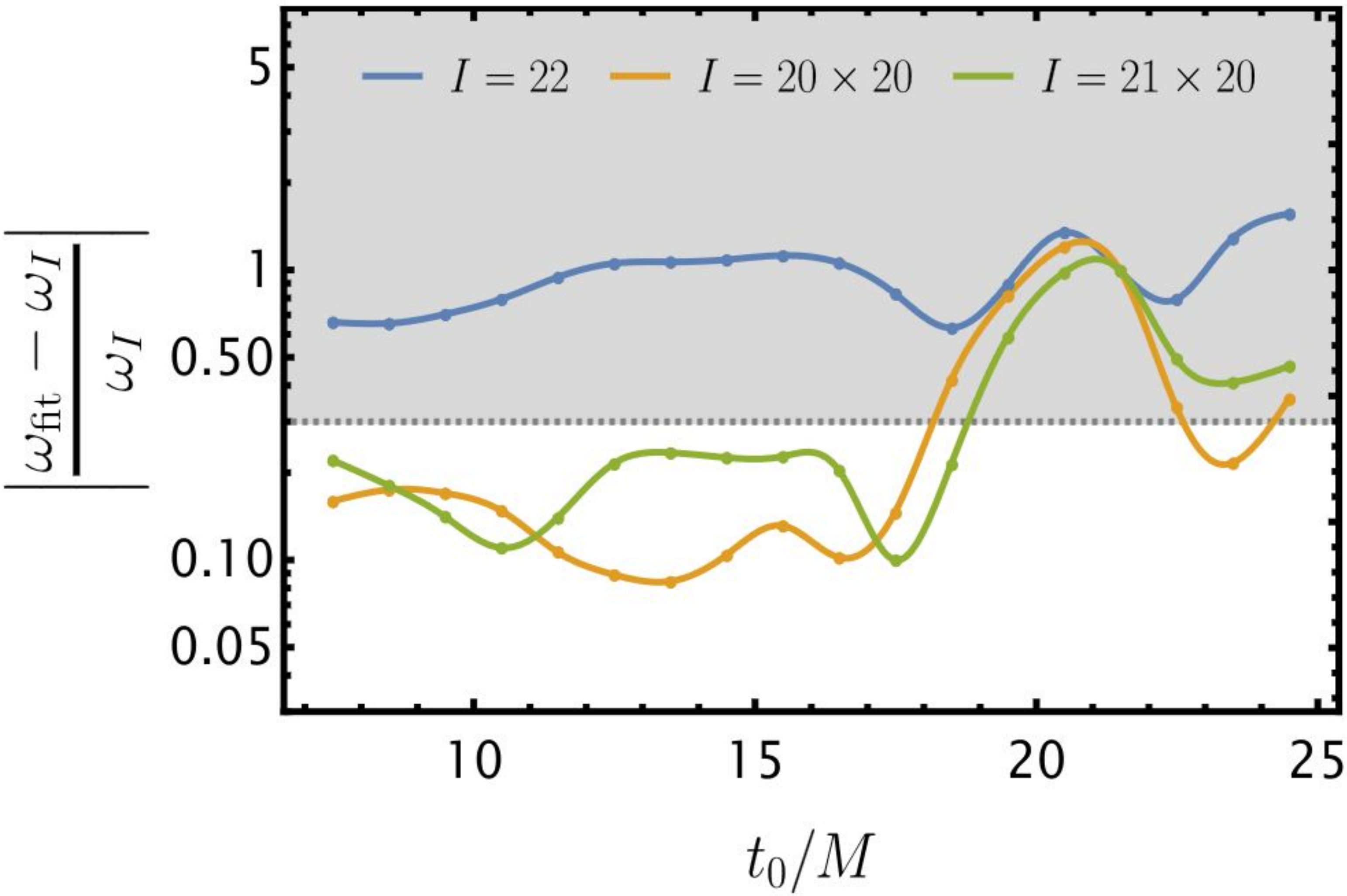
Mismatch S7 after fixing ω_{200} and ω_{201}



Stability amplitude

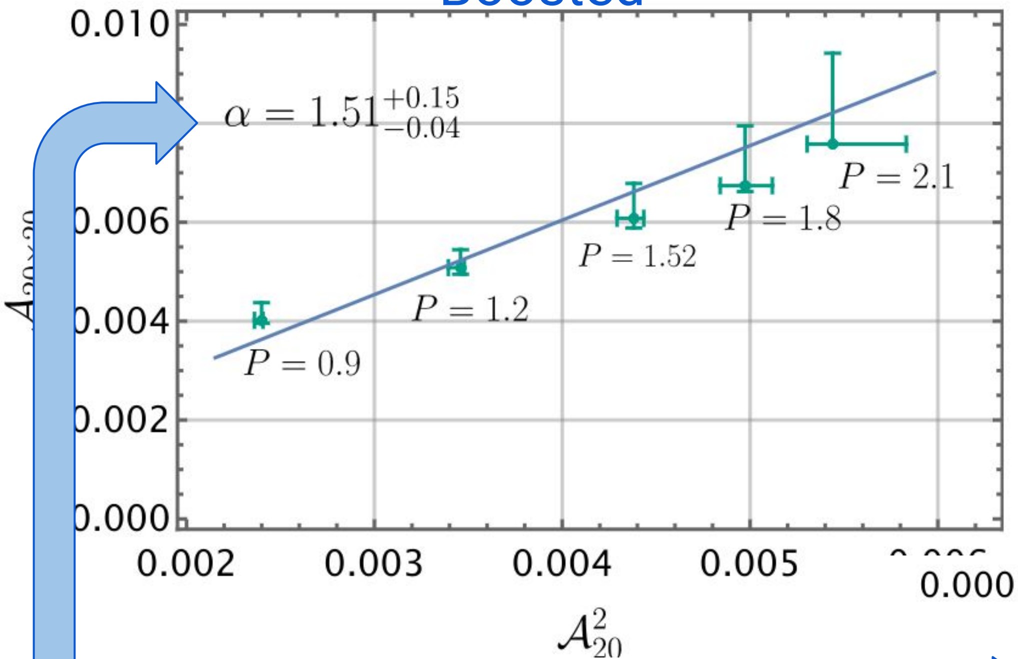


Relative variation of the optimal frequency

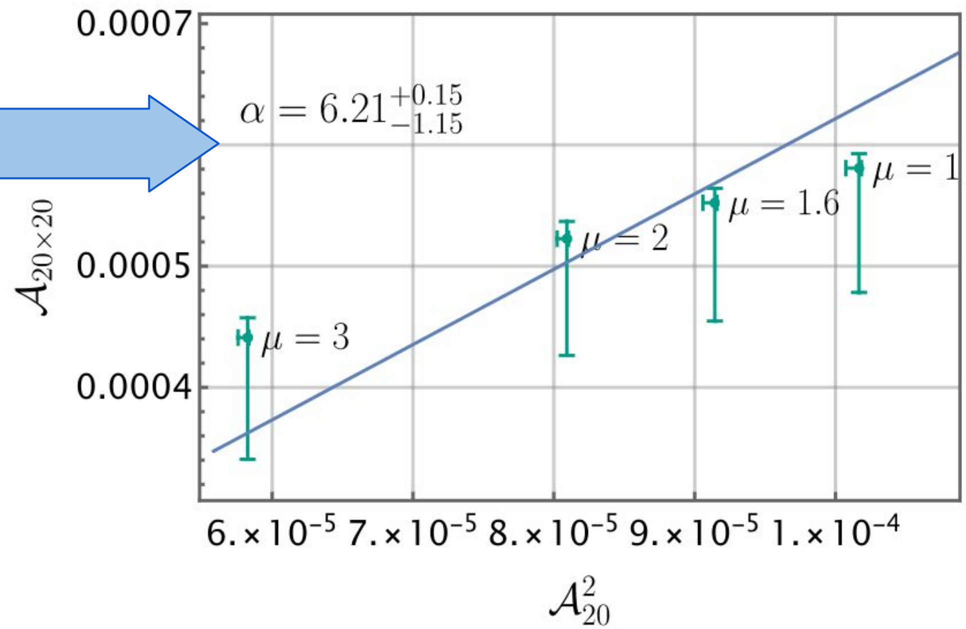


Amplitude relation

Boosted

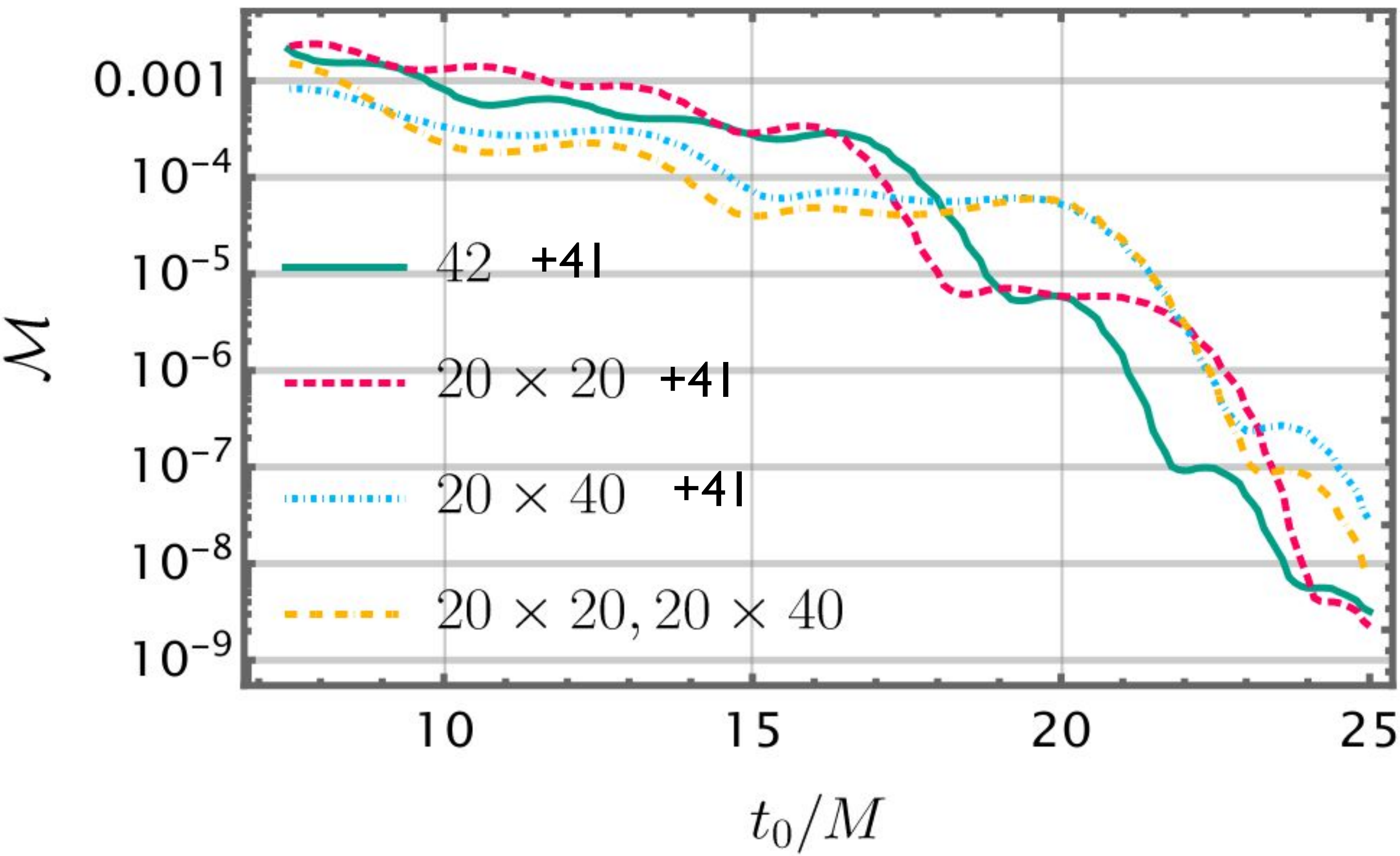


Unboosted



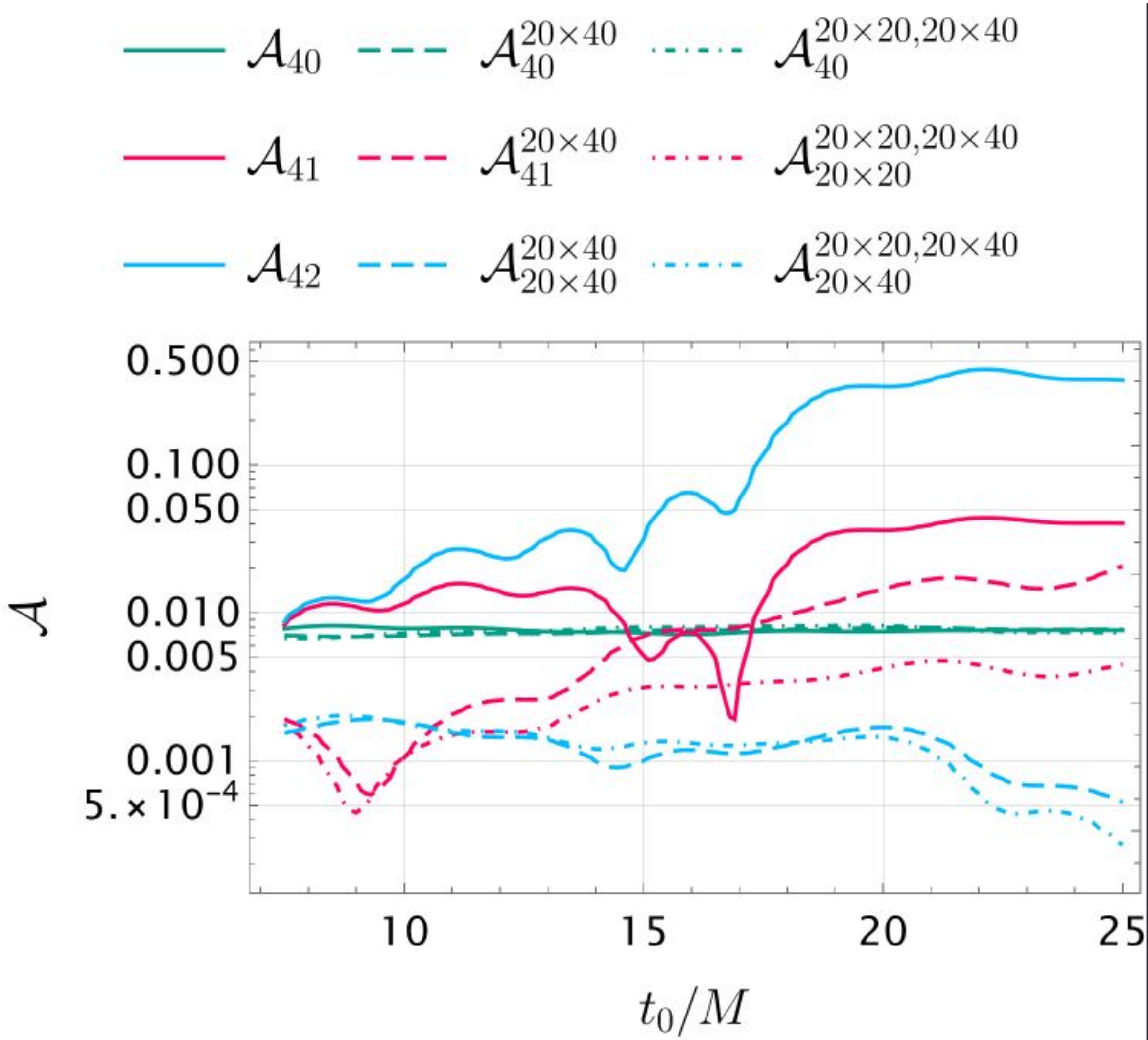
Puzzle: Why are these slopes different?

l=4 mode

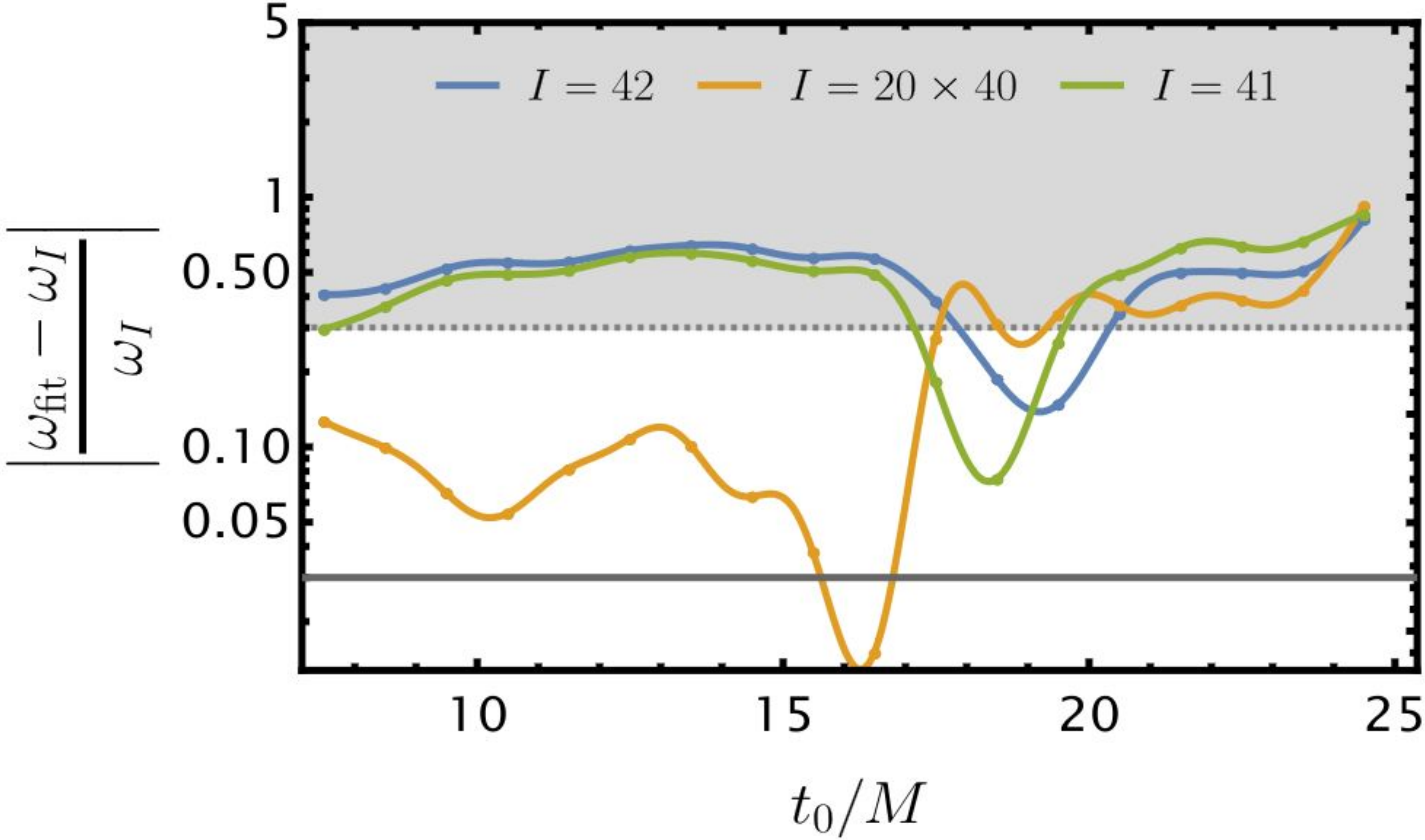


0.001
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$\mathcal{M}$
42 +4l
20×20 +4l
20×40 +4l
$20 \times 20, 20 \times 40$
10
15
20
25
t_0/M

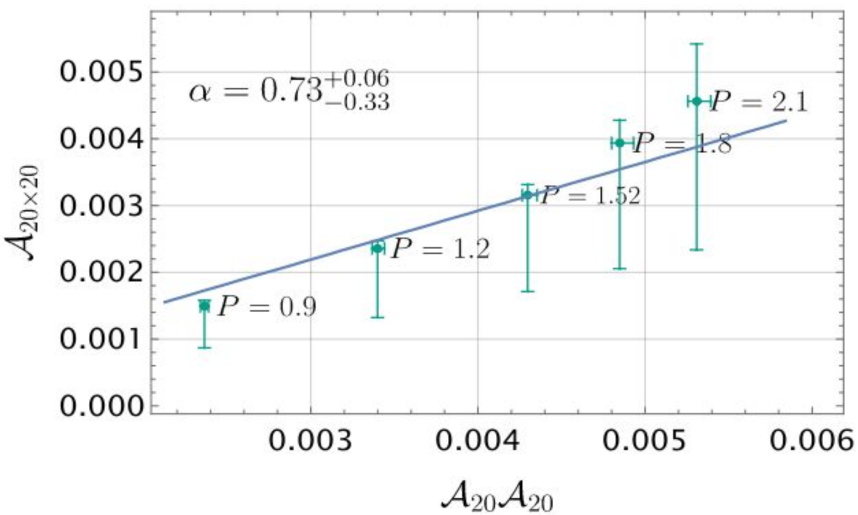
Stability amplitudes

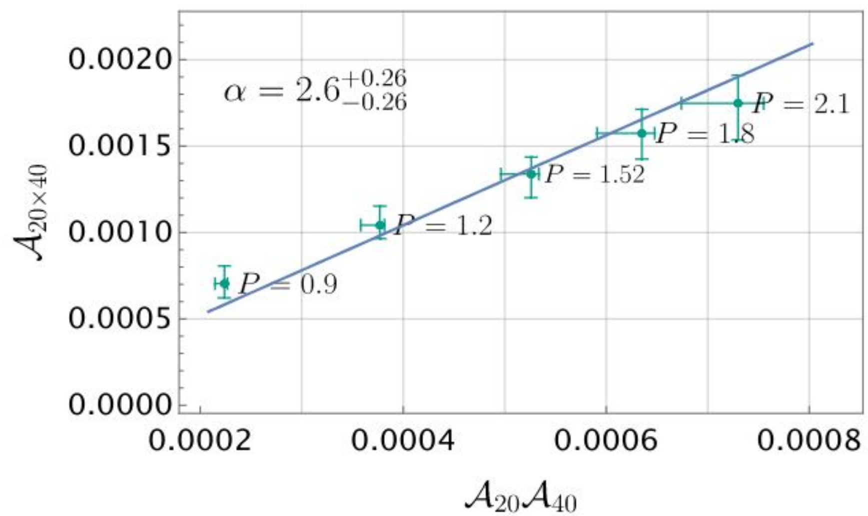


Optimal frequency fixing ω_{40} and ω_{20x20}



Amplitude relation





Data prefers model with fundamental tone + 2 quadratic modes!

Other l-modes

<table>
<tr><th>Mode</th><th>$\omega_{ln\times l'n'}$</th><th>Boosted (α)</th><th>Unboosted (α)</th></tr>
<tr><td>$l=2$</td><td>$\omega_{20\times 20}$</td><td>$1.51^{+0.15}_{-0.04}$</td><td>$6.21^{+0.15}_{-1.15}$</td></tr>
<tr><td rowspan="2">$l=4$</td><td>$\omega_{20\times 20}$</td><td>$0.73^{+0.06}_{-0.33}$</td><td>-</td></tr>
<tr><td>$\omega_{20\times 40}$</td><td>$2.6^{+0.26}_{-0.26}$</td><td>-</td></tr>
<tr><td rowspan="4">$l=6$ *</td><td>$\omega_{20\times 40}$</td><td>$1.78^{0.53}_{-0.74}$</td><td>-</td></tr>
<tr><td>$\omega_{20\times 60}$</td><td>$2.52^{+1.29}_{-0.59}$</td><td>-</td></tr>
<tr><td>$\omega_{20\times 40}$</td><td>$1.78^{0.44}_{-0.65}$</td><td>-</td></tr>
<tr><td>$\omega_{40\times 40}$</td><td>$2.82^{+1.5}_{-0.62}$</td><td>-</td></tr>
</table>

Connection horizon and infinity

- For l=4, same quadratic modes found at infinity
- For l=6, also $\omega_{200 \times 400}$ found at infinity

[Cheung et al, 2022 + private correspondence]

Conclusion

★ Quadratic QNMs fit the shear (and multipole) data at the horizon better than models with overtones
  - lower mismatch
  - more stable amplitudes wrt changes in starting time
  - closer to the optimal frequency
  - amplitude relation is satisfied

★ Some of the same (quadratic) modes found at horizon and infinity

★ Puzzling: why is the amplitude relation for boosted and unboosted simulations different?

Open questions

(1) All results based on fitting observations, are there better ways to do this?

(2) Why are the slopes for boosted/unboosted simulations different?

(3) Is there a well-motivated choice of slicing/time?

(4) Can we link observations at infinity more directly to horizon properties?


ONE DOES NOT SIMPLY
SAY THANK YOU WITHOUT A
MEME