Non-linear behavior (of black hole horizons) from different angles

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Outline: *multiple angles*

"Using a term like nonlinear science is like referring to the bulk of zoology as the study of non-elephant animals." [S. Ulam]

- Full GR numerical simulations
- Perturbative analysis
- Analogies

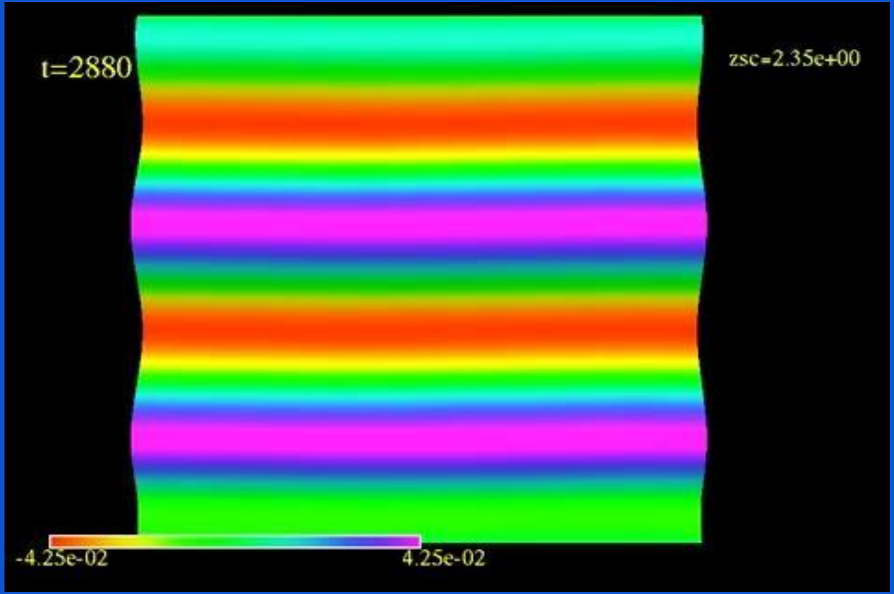
– singling out particular scenarios that can shed light on underlying phenomenology

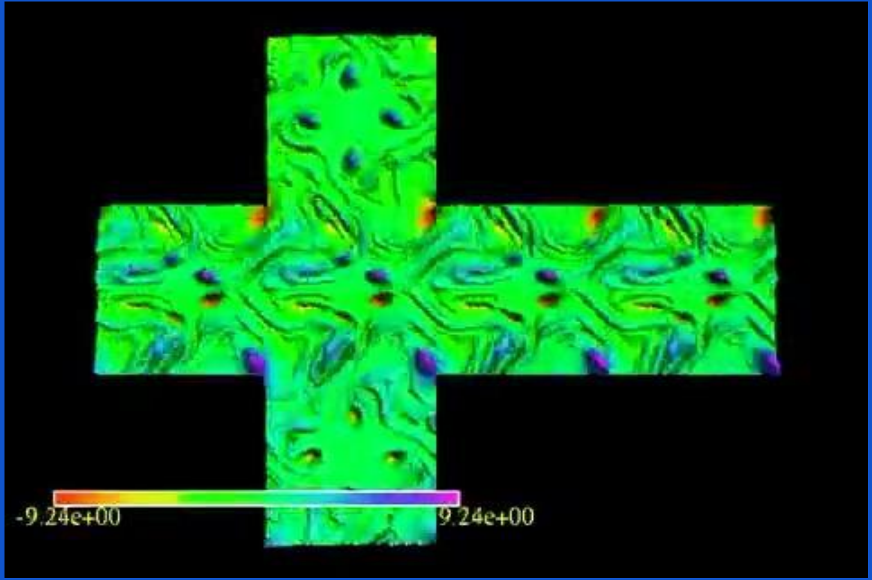
– Both within AF and asymptotically AdS boundary conditions

(from the point of view of ‘non-linear couplings’, cosmological constant does not change the picture in a crucial way)

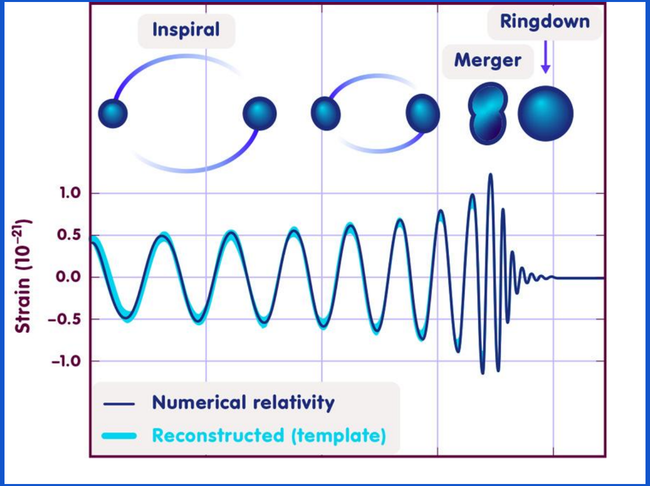
how perturbed' ?

What analogy & why?

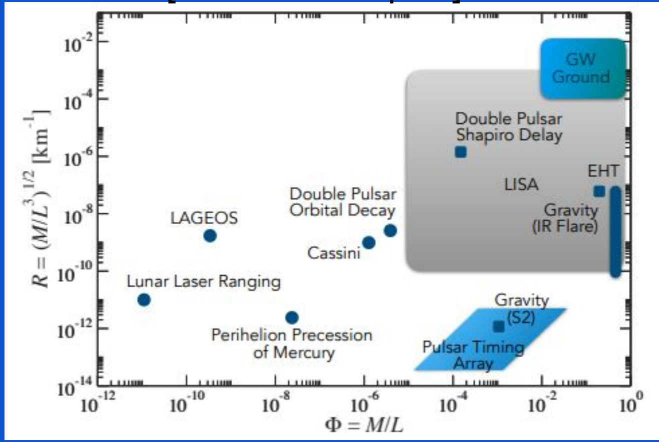


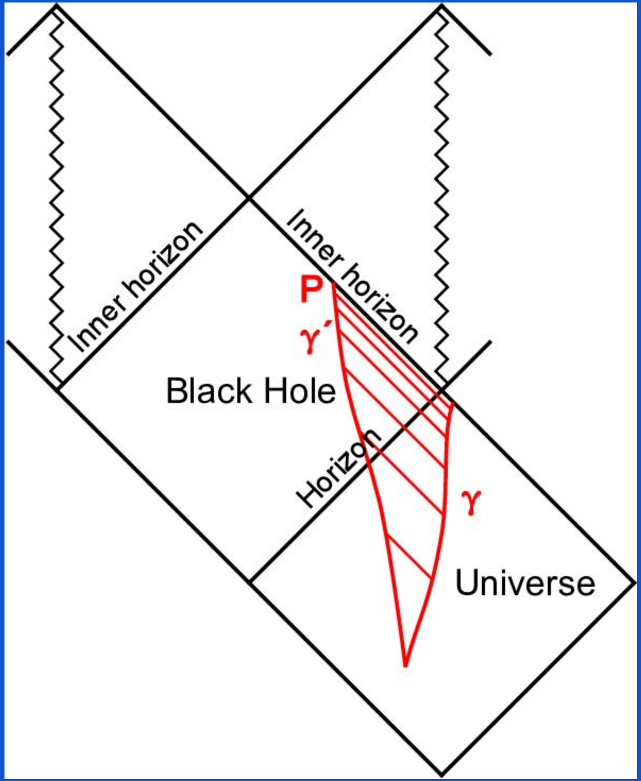


What target?

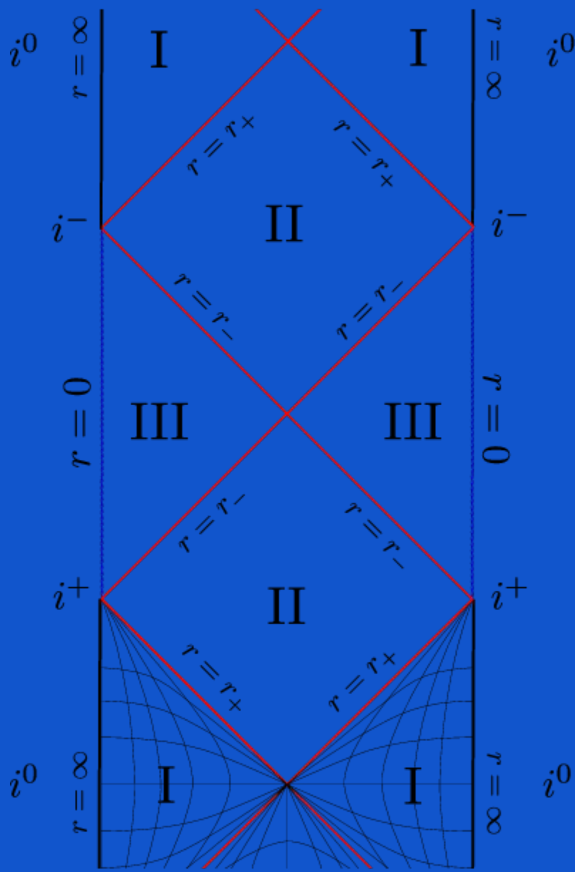


[LSC & 3G report]





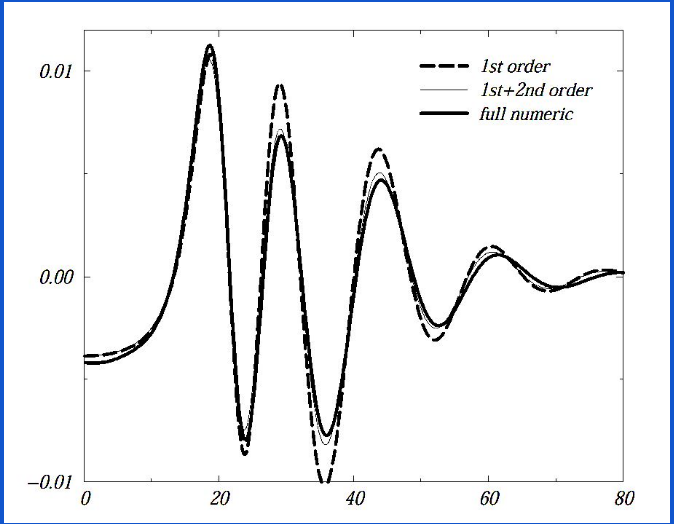
[Romero]



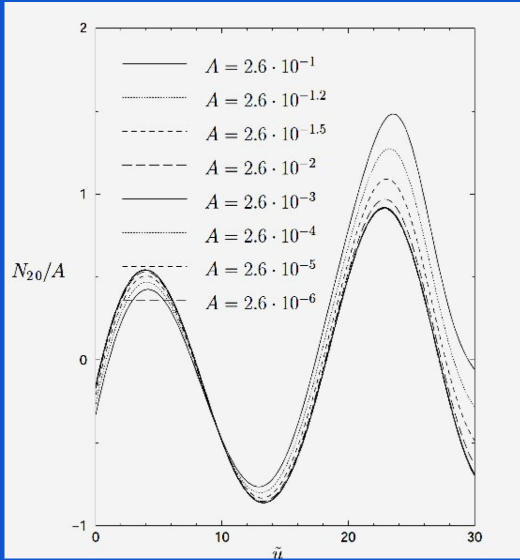
[Novaes]

Some ‘oldies’

- Close-limit approximation: in head-on collisions: [Nicasio+ PRD ‘99] -> 1st+2nd order perts discernibly better at approximating post-merger
- Scattering of BHs: amplitude changes in impinging mode, resulting in a non-linear behavior off scattered radiation [Zlochower+ PRD ‘03]



[pert]

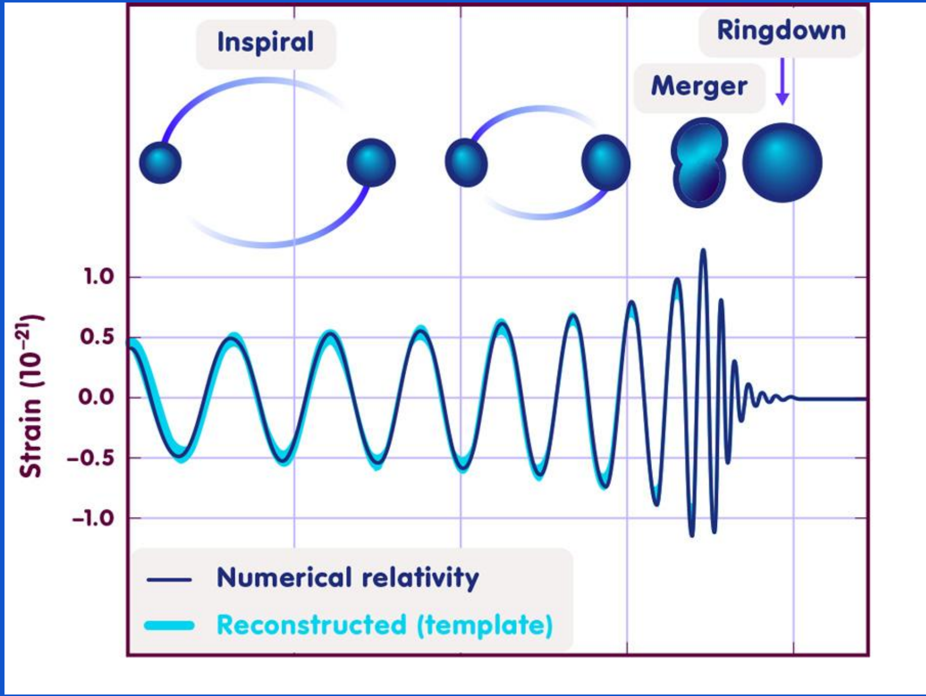


[full GR]

Binary black hole collisions: *Linear from almost the peak?*

- *reductio ad absurdum*. If so, one can estimate the flux of energy through horizon → {M,a} change by a few %, hardly an 'unchanging background', what is its impact? timescales?

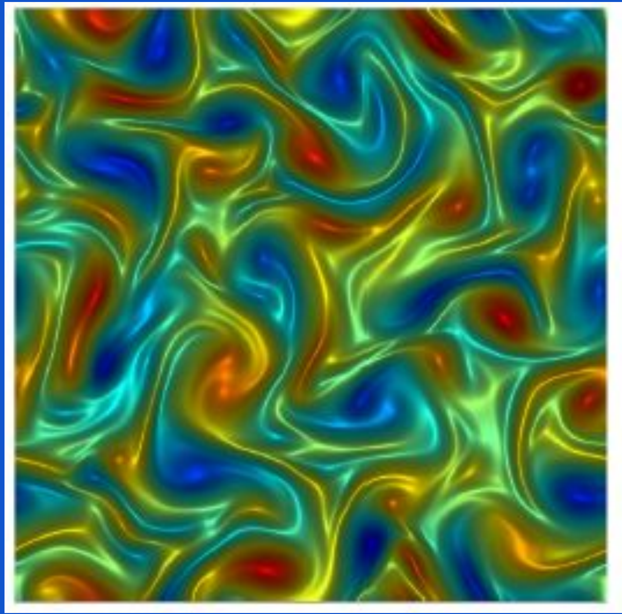
- What other phenomena might also indicate non-linear behavior?



[Also, talks/flash talks in this meeting!]

Perturbed BHs in AdS

- Perturbed boosted black brane in 3+1 dimensions, with a long-wavelength perturbation
- ‘Pure’ mode, develops a complex structure, which extends throughout the spacetime, displaying ‘vortices’ of gravitational waves (a-la ‘geons’) from horizon to boundary of AdS
- Dynamically, energy flows to longer wavelengths
- *[Chesler,Adams,Liu PRL’14] What drives such behavior?*



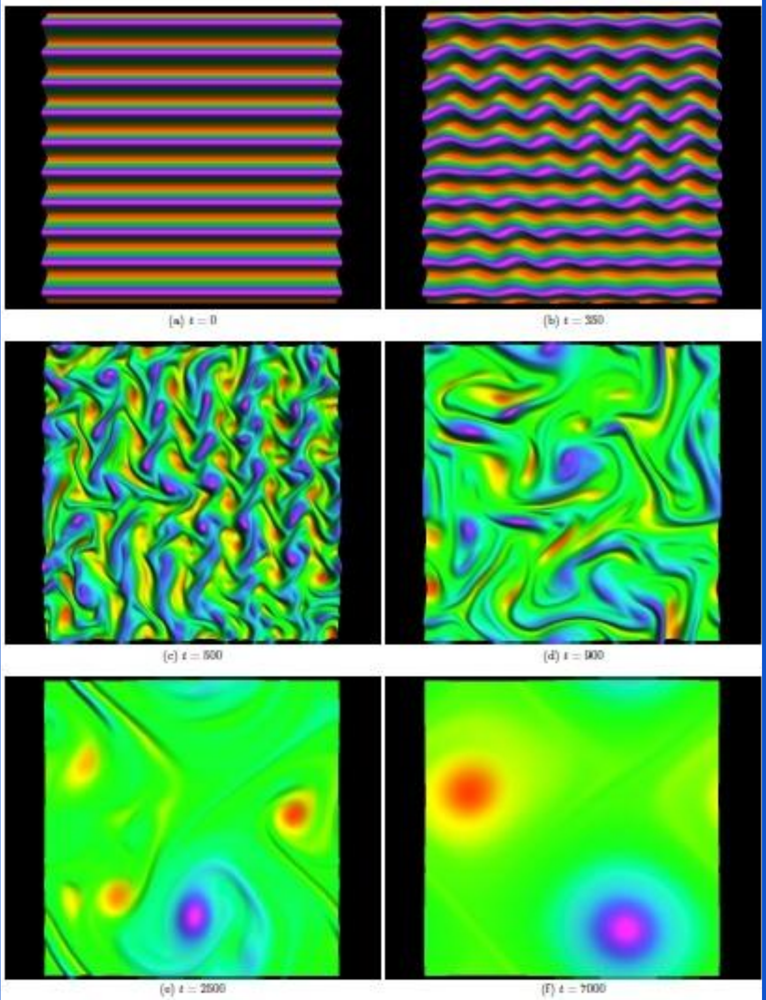
[Chesler,Adams,Liu PRL’14]

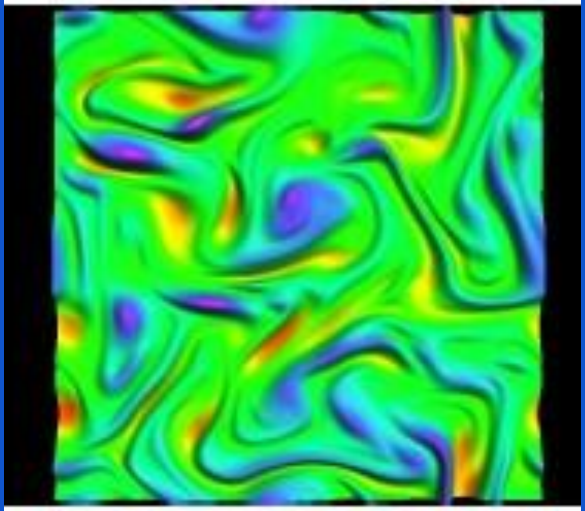
- *AdS/CFT <->gravity/fluid correspondence* *[dictionary!]*

[Bhattacharya,Hubeny,Minwalla,Rangamani; VanRaamsdonk; Baier,Romatschke,Son,Starinets,Stephanov]

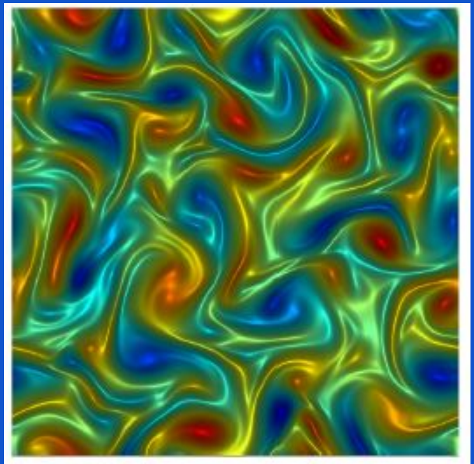
$$ds^2_{[0]} = -2u_\mu dx^\mu dr + r^2 \left(\eta_{\mu\nu} + \frac{1}{(br)^d} u_\mu u_\nu \right) dx^\mu dx^\nu .$$

- $T_{ab} = T_{ab} = \frac{\rho}{d-1}(du_a u_b + \eta_{ab}) + \Pi_{ab}$
- Subject to :
  - $u_a u^a = -1$; $T^a{}_a = 0$; $\Pi_{ab} = -2\eta\sigma_{ab} + \cdots$
  - $\nabla_a T^{ab} = 0$.
- Do these eqns/eos give rise to turbulence?
  - Non-relativistic limit → Navier-Stokes eqn. why wouldn't they?
  - If so, NS eqns have indirect cascade for 2+1 dimensions. Why? There exists a conserved quantity: *enstrophy. Does it exist for these eqns/eos?* Yes! [Carrasco+ '12]





[Carrasco,Green,LL PRX’13]



From the fluid’s perspective

- Non-linearities naturally expected, the regime corresponds to Re>>1
- Energy flows primarily to longer wavelengths due to a topological ‘constraint’, –*enstrophy conservation*– which restricts energy flow to shorter ones.

- ‘Clean’ duality in AdS, can we draw similar observations in asymptotically flat spacetimes? Can we unearth a phenomena describing it from GR?
  - Ultimately what mediated this non-linear behavior?
    - AdS ‘trapping energy’ -> slowly decaying QNMs & turbulence
    - Or `more slowly’ decaying QNMs -> time for non-linearities to ``do something interesting’’?

Parametric instabilities in BHs (AF)

- In AF spacetimes, claims of fluid-gravity as well.
  - NS from membrane paradigm on a timelike surface (80's)
  - Raychaudhuri/Damour equations on horizon (?)

However the first is potentially delicate, intuition but not a rigorous message.

Let's try something else, taking a page from what we learnt from fluids.

- First, recall the behavior of parametric oscillators:
  - $q_{,tt} + \omega^2 (1 + f(t)) q + \gamma q_{,t} = 0$
  - Soln is generically bounded in time *except* when f(t) oscillates approximately with $\omega' \sim 2\omega$. [e.g. $f(t) = f_o \cos(\omega' t)$] . If so, an unbounded solution is triggered behaving as $e^{\alpha t}$ with $\alpha = (f_o^2 \omega^2/16 - (\omega'-\omega)^2)^{1/2} - \gamma$
  - (referred to as *parametric instability* in classical mechanics and optics)

- As a simplification: consider a BH perturbed by single mode h_1 and take only an additional scalar perturbation over the resulting spacetime. One obtains:

$$[\, \mathrm{Box}_{kerr} + O(h_1)\ ]\, \Phi = 0.$$

- With the solution having the form: $e^{t(\alpha - \omega_\imath)}$ with

$$\alpha = \pm\sqrt{|H h_0(t)/Qm'|^2 - (\omega'_R - \omega_R/2)^2}\,,$$

- So exponentially growing solution if:

$$h_0(t)/(m'\omega'_I) - |Q/H|\sqrt{(\omega'_R - \omega_R/2)^2/\omega'^2_I + 1} > 0\,.$$

[Yang,Zimmerman,LL ‘15]

- if Φ has l, m/2 -> a parametric instability can turn on; an 'inverse cascade'.
- Further, one can find 'critical values' for growth onset.

(l,m)	$l'=1$	$l'=2$	$l'=3$	$l'=4$	$l'=5$	$l'=6$	$l'=7$	$l'=8$
$(2,2)$	0.287	0.163	0.130	0.122	0.117	0.115	0.113	0.111
$(4,2)$	43.2	62.1	92.7	123	118	118	117	117
$(4,4)$	–	3.62	0.00676	0.0114	0.0108	0.0104	0.0101	0.0100

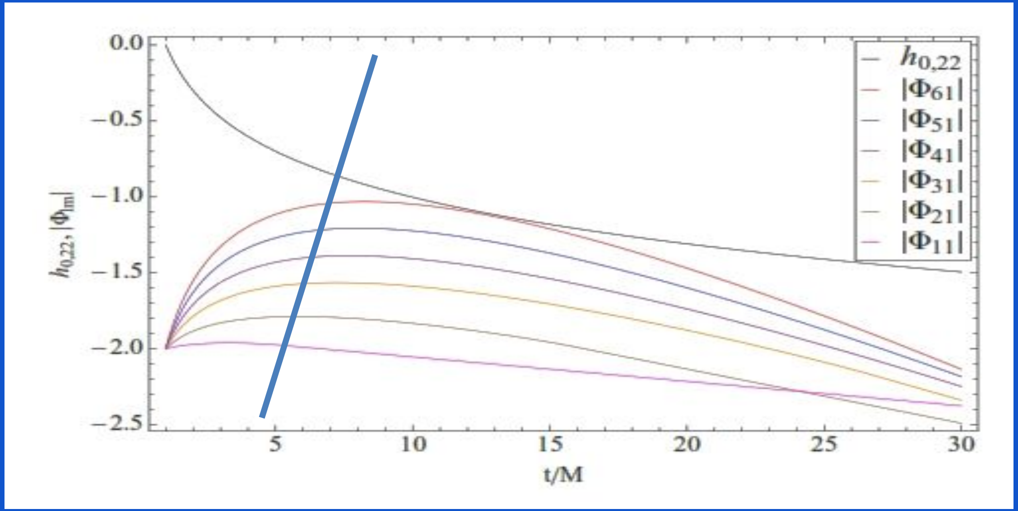
- One can also define a 'separatrix' value as:

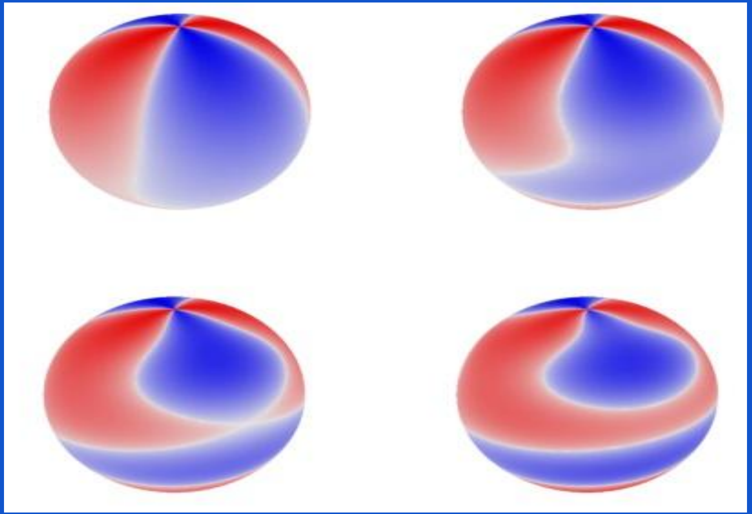
$$Re_g = h_o/(m\,\omega_\nu)$$

- identify $\lambda <-> 1/m$; v <-> h_o ; ν/ρ <-> ω_ν

$$\rightarrow\ Re_g = Re$$

Critical ``Reynolds'' number & instability consequence





a = 0.998, perturbation ~ 0.02%, initial mode l=2,m2

Could 'potentially' have observational consequences

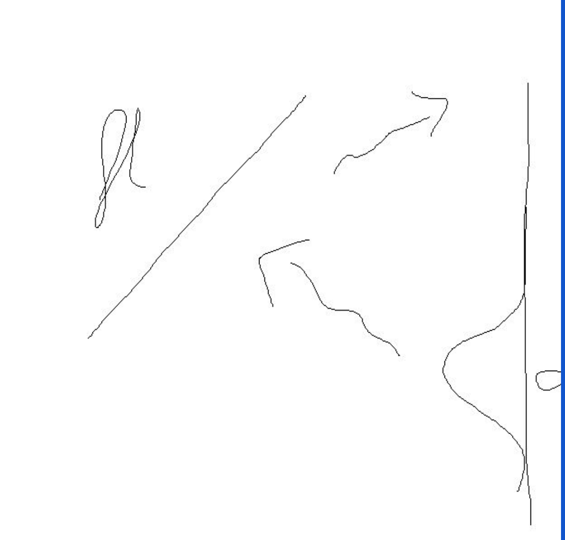
Mode excitation and bh/spacetime response

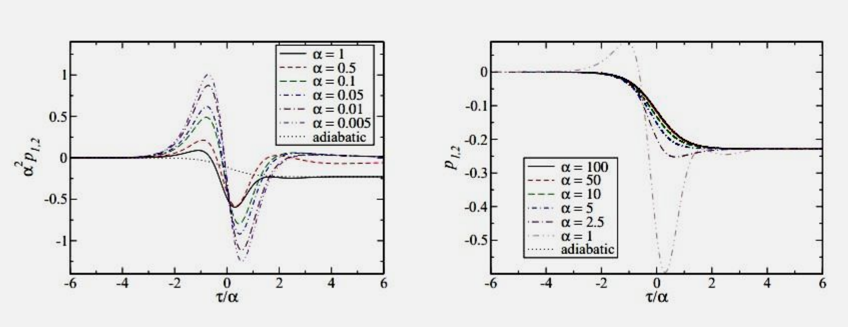
– Timescales: black hole response to ‘arbitrary slow or fast’ perturbations? [Buchel+ ‘12]

– For arbitrary low, → adiabatic regime, pure mode stays pure

– For arbitrary fast: ‘quench’ → universal response

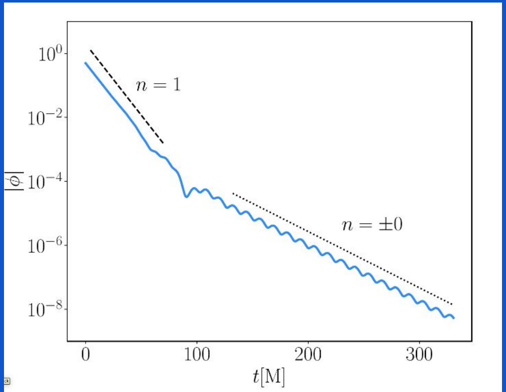
- But in between?



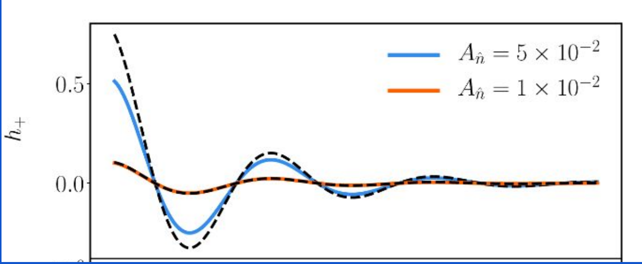


Non-linearities through accretion onto BH: AIME

- Modes propagate on a changing background as BH accretes, and the time-scale of such change is not adiabatic [Sberna+ PRD 105 '22]
- *Secular effect*, mass/angular momentum changes -> a mode will be 'projected' onto new ones. ***AIME**: absorption induced mode excitation*



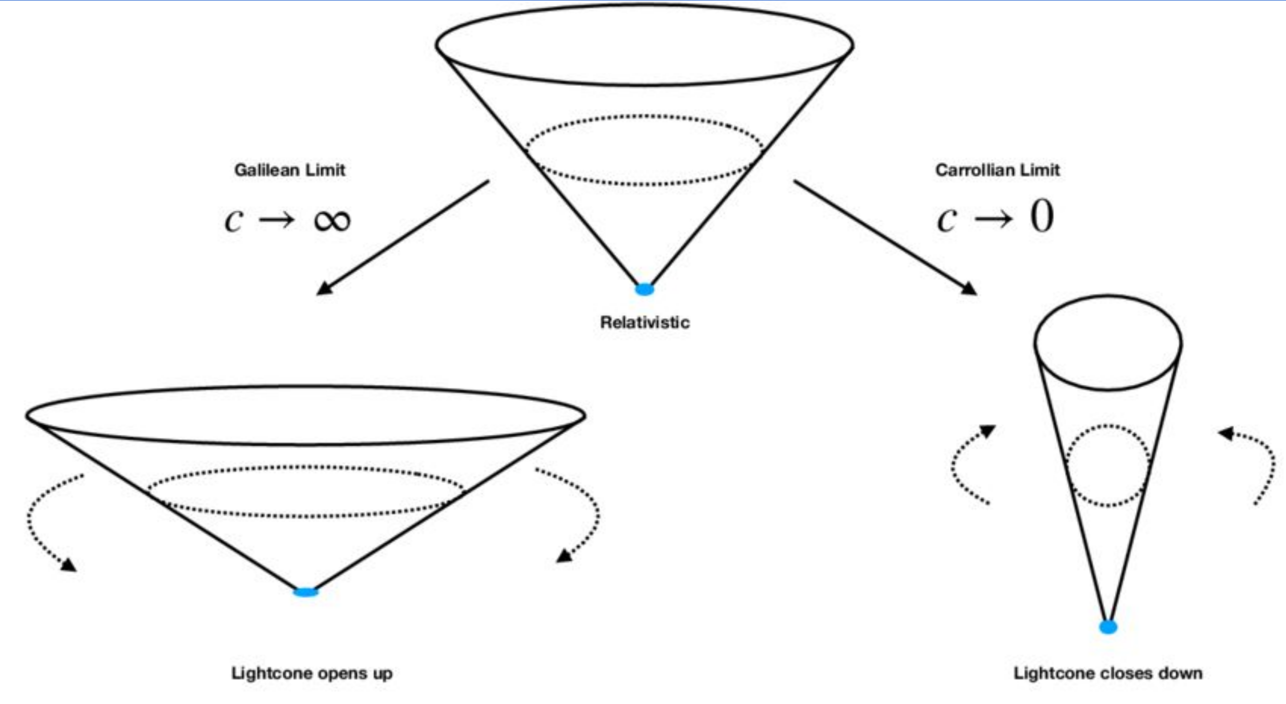
- CAV: sims and thorough comparison for a scalar field in asymptotic AdS spacetimes
- Mode influence assuming no time offset in comparison



- BUT: basic mechanism is clear, and will take place in AF.

[Sberna,Bosch-Gomez,Green,East,LL '22]

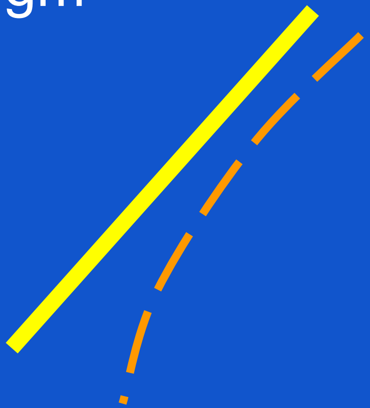
‘Revisiting’ some more…



[Bagchi]

And back to horizons, what happens 'there'

- Revisit 'membrane paradigm'



- (Horizon) null surface is a special surface. Brings a welcomed degree of robustness to MP → a unique surface
  - Raychaudhuri + Damour equations also describe ' hydrodynamics… but with Carrollian symmetry

[Donnay,Ciambelli,C.Marteau,Petkou,Petropoulos, Siampos,Freidel,Jai-akson,Jafari,Speranza,Adami,Grumiller,Zwickel…]

Null surfaces in GR & Carrollian hydrodynamics

$$ds^2 = -Vdv^2 + 2dvd\rho + 2\Upsilon_A dvdx^A + \gamma_{AB}dx^A dx^B, \quad (1)$$

where v is the advanced null time, ρ parametrizes the distance to the surface, and x^A are the angular coordinates on the sphere, $A = 2, 3$. We expand the metric in powers of the distance to the surface

$$V = 2\kappa\rho + O(\rho^2), \quad \Upsilon_A = U_A\rho + O(\rho^2), \quad \gamma_{AB} = \Omega_{AB} - 2\lambda_{AB}\rho + O(\rho^2). \quad (2)$$

→

$$\dot{\theta^{(l)}} - \kappa\theta^{(l)} + \frac{1}{2}\theta^{(l)2} + N_{AB}^{(l)}N^{AB\,(l)} = 0,$$
$$\dot{\mathcal{H}}_A + \theta^{(l)}\mathcal{H}_A - \nabla_A\kappa - \frac{1}{2}\nabla_A\theta^{(l)} + \nabla^B N_{AB}^{(l)} = 0,$$

↕

$$\dot{e} + \theta^{(l)}e + \alpha(\hat{\nabla}_A + 2\varphi_A)\mathcal{J}^A + \left(N_{AB}^{(l)} + \frac{1}{2}\theta\Omega_{AB}\right)(\Pi^{AB} + p\Omega^{AB}) = 0,$$
$$\dot{\pi}_B + \theta^{(l)}\pi_B + \alpha\left(\hat{\nabla}_A + \varphi_A\right)(\Pi^A_B + p\delta^A_B) + \alpha\varphi_B e - \varpi^{AB}\mathcal{J}_A = 0.$$

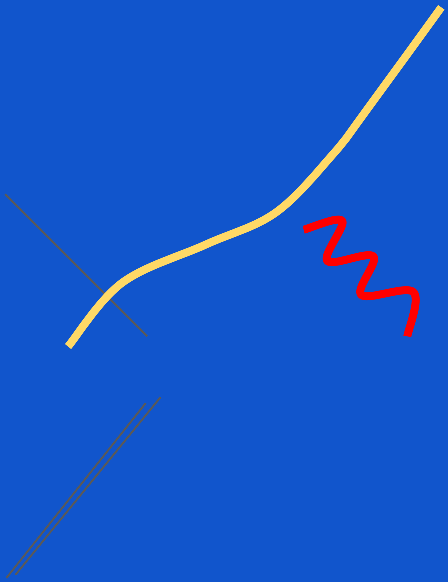
$$e = \theta^{(l)}, \qquad p = -\kappa,$$
$$\Pi_{AB} = 2\eta N_{AB}^{(l)} + \zeta\theta^{(l)}\Omega_{AB}, \qquad \eta = \frac{1}{2}, \qquad \zeta = -\frac{1}{2},$$
$$\pi_A = -\frac{1}{\kappa}\left(\nabla_A\kappa - \mathcal{H}^B\dot{\Omega}_{AB} - \frac{1}{\kappa}\mathcal{H}_A\dot{\kappa}\right), \quad \Sigma^{AB} = \mathcal{J}^A = 0.$$

← Under these identifications

$$\dot{\theta^{(l)}} - \kappa\theta^{(l)} + \frac{1}{2}\theta^{(l)2} + N^{(l)}_{AB}N^{AB\,(l)} = 0,$$
$$\dot{\mathcal{H}}_A + \theta^{(l)}\mathcal{H}_A - \nabla_A\kappa - \frac{1}{2}\nabla_A\theta^{(l)} + \nabla^B N^{(l)}_{AB} = 0,$$

- But what are N, k? both *gauge* and physics in them. Consider perturbing Schwarzschild (vacuum case)

$$\dot{\theta}^{(n)} + \kappa\theta^{(n)} + \theta^{(l)}\theta^{(n)} - (\nabla_A\mathcal{H}^A + \mathcal{H}_A\mathcal{H}^A) + \frac{R}{2} = 0,$$
$$2\dot{N}^{(n)}_{AB} - 4N^{C\,(n)}_{(A}N^{(l)}_{B)C} + \theta^{(n)}N^{(l)}_{AB} + (2\kappa - \theta^{(l)})N^{(n)}_{AB} +$$
$$+R^{\text{T-F}}_{AB} - 2(\mathcal{H}_A\mathcal{H}_B)^{\text{T-F}} - 2(\nabla_{(A}\mathcal{H}_{B)})^{\text{T-F}} = 0,$$



$$g^{\text{Sch}}_{ab} = -\frac{\rho}{4m}dv^2 + 2dvd\rho + (4m^2 + 4m\rho)q_{AB}dx^A dx^B, \tag{19}$$

$$\kappa = \frac{1}{4m}(1 + \epsilon k + \epsilon^2 K)$$
$$U_A = -2(\epsilon h_A + \epsilon^2 H_A),$$
$$\Omega_{AB} = 4m^2(q_{AB} + \epsilon c_{AB} + \epsilon^2 C_{AB}),$$
$$\lambda_{AB} = 2m(-q_{AB} + \epsilon s_{AB} + \epsilon^2 S_{AB}).$$

[Redondo-Yuste,LL '22]

Involve thing rest of EEs, which link horizon to bulk behavior.
To 2nd order…

- {k,N} functions of (H,C,S,θ) , and evoln eqn $C_{,t}$=F(H,C,S,θ)

→ replacing:

$$
\begin{aligned}
-4m\dot{H}_A = {} & \nabla^2 H_A + \nabla_B \nabla_A H^B \\
& + m^{-1}\left(H_A \nabla^B H_B + H_B \nabla^B H_A\right) + \\
& + c^{BC}(\nabla_C \nabla_A H_B + \nabla_C \nabla_B H_A \\
& - \nabla_A \nabla_B H_C) \\
& - \nabla_A C^{BC} \nabla_B H_C + \nabla^B C_{AB} \nabla^C H_C .
\end{aligned}
$$

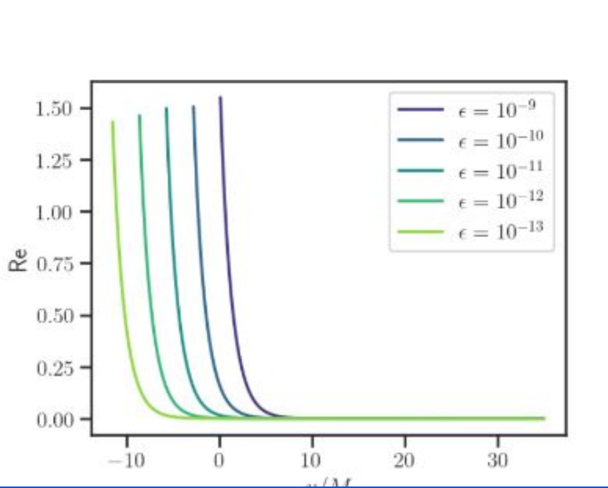
Which now bears some resemblance to Navier Stokes eqns

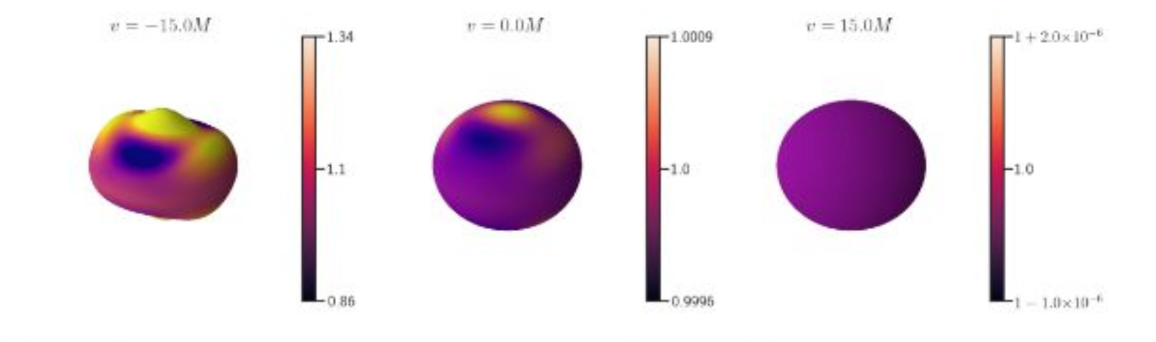
$$U_{,t} \simeq U \partial U + \eta / \rho \, \partial^2 U$$

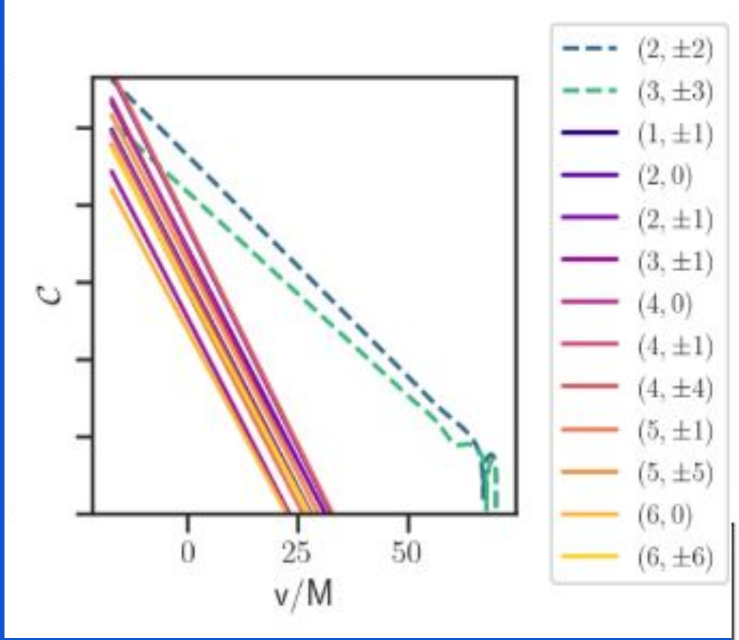
and…being very 'liberal' with dimensions,

- Re → H (L /mass +c)/(1+c) [recall, in NS, Re -> λu/(η/ρ)]

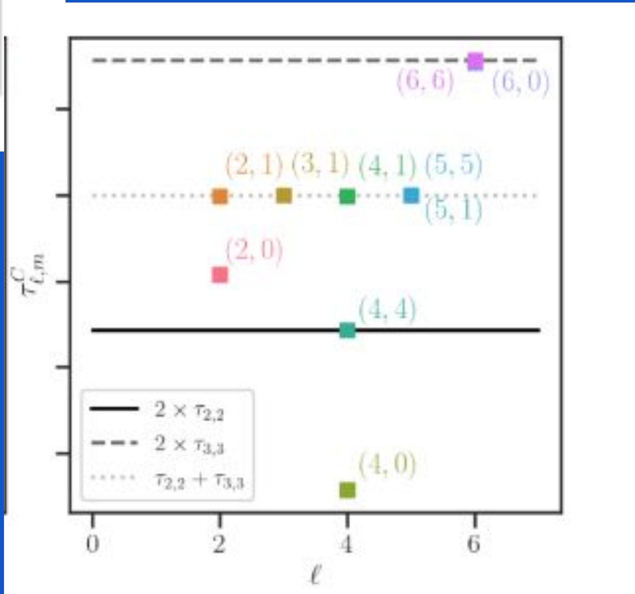
Does it make sense?







$\mathcal{C}$
0
25
50
v/M
(2, ±2)
(3, ±3)
(1, ±1)
(2, 0)
(2, ±1)
(3, ±1)
(4, 0)
(4, ±1)
(4, ±4)
(5, ±1)
(5, ±5)
(6, 0)
(6, ±6)



$\tau^{C}_{\ell,m}$
(6, 6)
(6, 0)
(2, 1) (3, 1) (4, 1) (5, 5)
(5, 1)
(2, 0)
(4, 4)
(4, 0)
$2 \times \tau_{2,2}$
$2 \times \tau_{3,3}$
$\tau_{2,2} + \tau_{3,3}$
0
2
4
6
ℓ

SUMMARY:

- Several identified interactions/mechanisms argue for non-linear behavior with discernible consequences throughout the spacetime

- Relevant time scales ~ decay time of (leading) mode

- Connection with hydrodynamics, interesting analogy for unearthing particular phenomena, and hinting of even deeper ones
    - In particular, ‘inverse’ cascade underlying simplicity of waveforms?