Creases and caustics: non-smooth structures on black hole horizons

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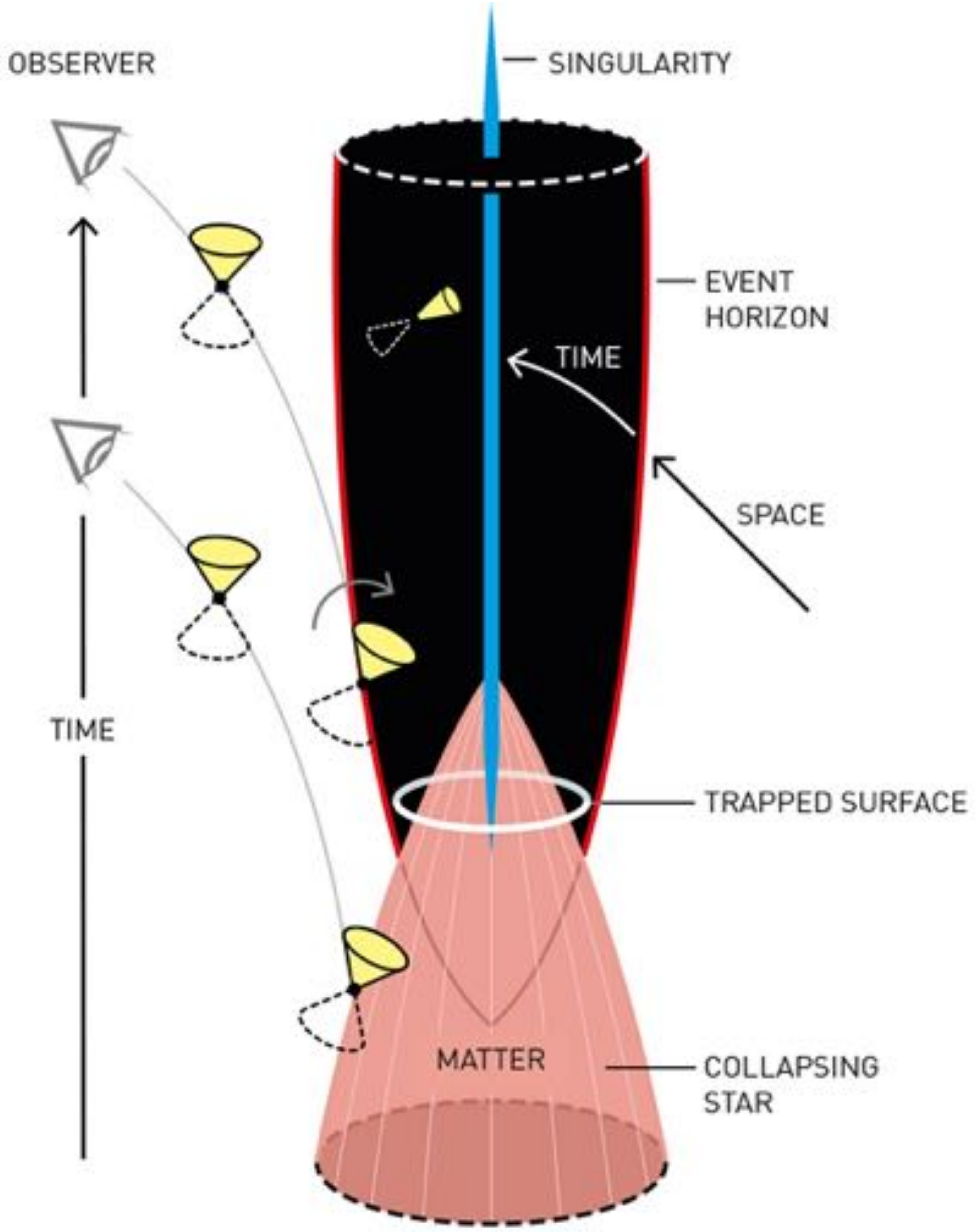
Black holes

- General relativity: gravity arises because spacetime is curved
- A black hole is a region of spacetime from which no signal can reach a "distant observer"
- The boundary of a black hole is its *event horizon*: a 3d surface in 4d spacetime



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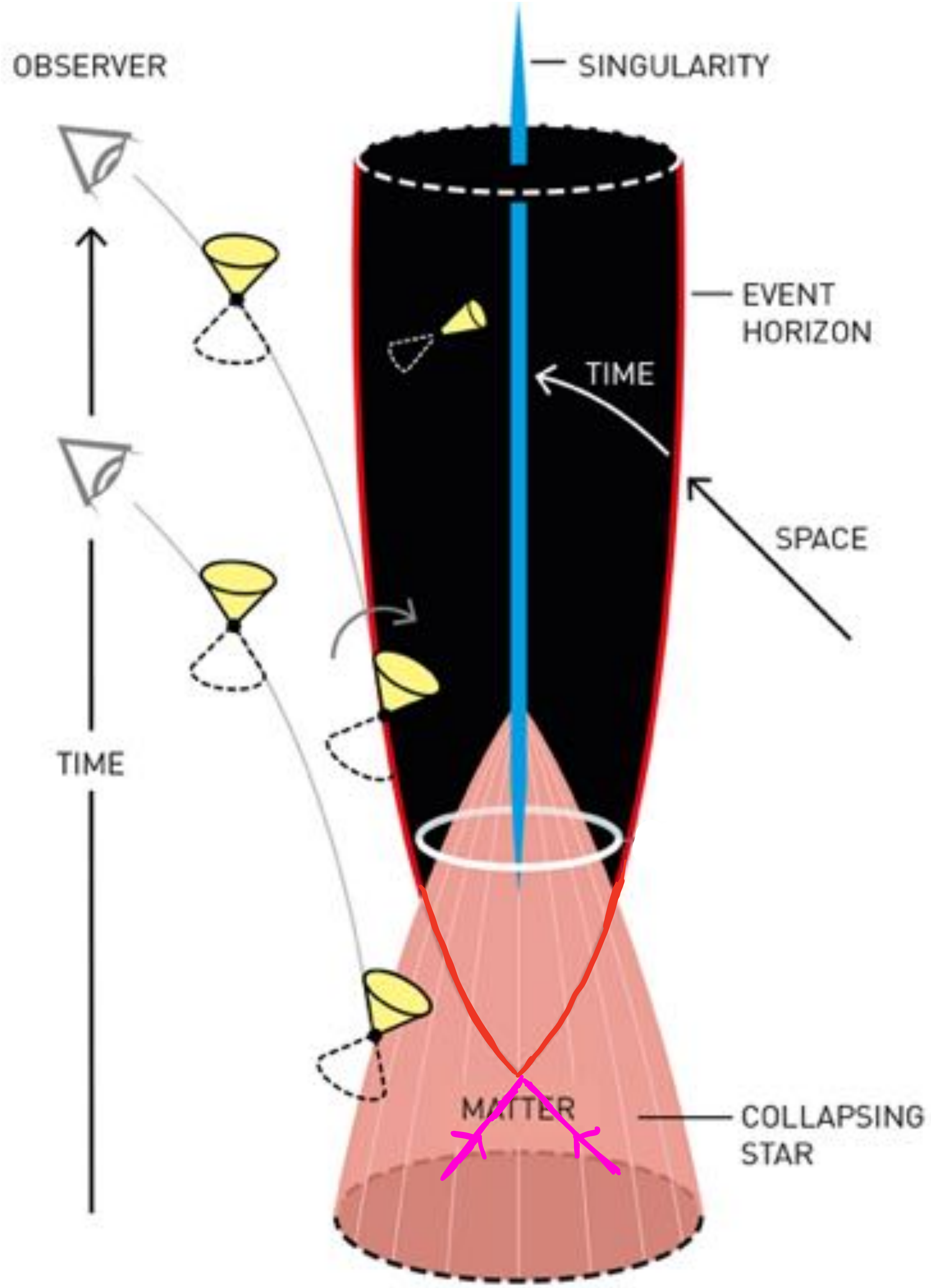
Event horizon

- Every point on an event horizon lies on a null geodesic (light ray) lying within the event horizon. These are called *generators* of the horizon.
- A generator cannot have a future endpoint: once in the horizon it cannot leave
- Generators can have *past* endpoints



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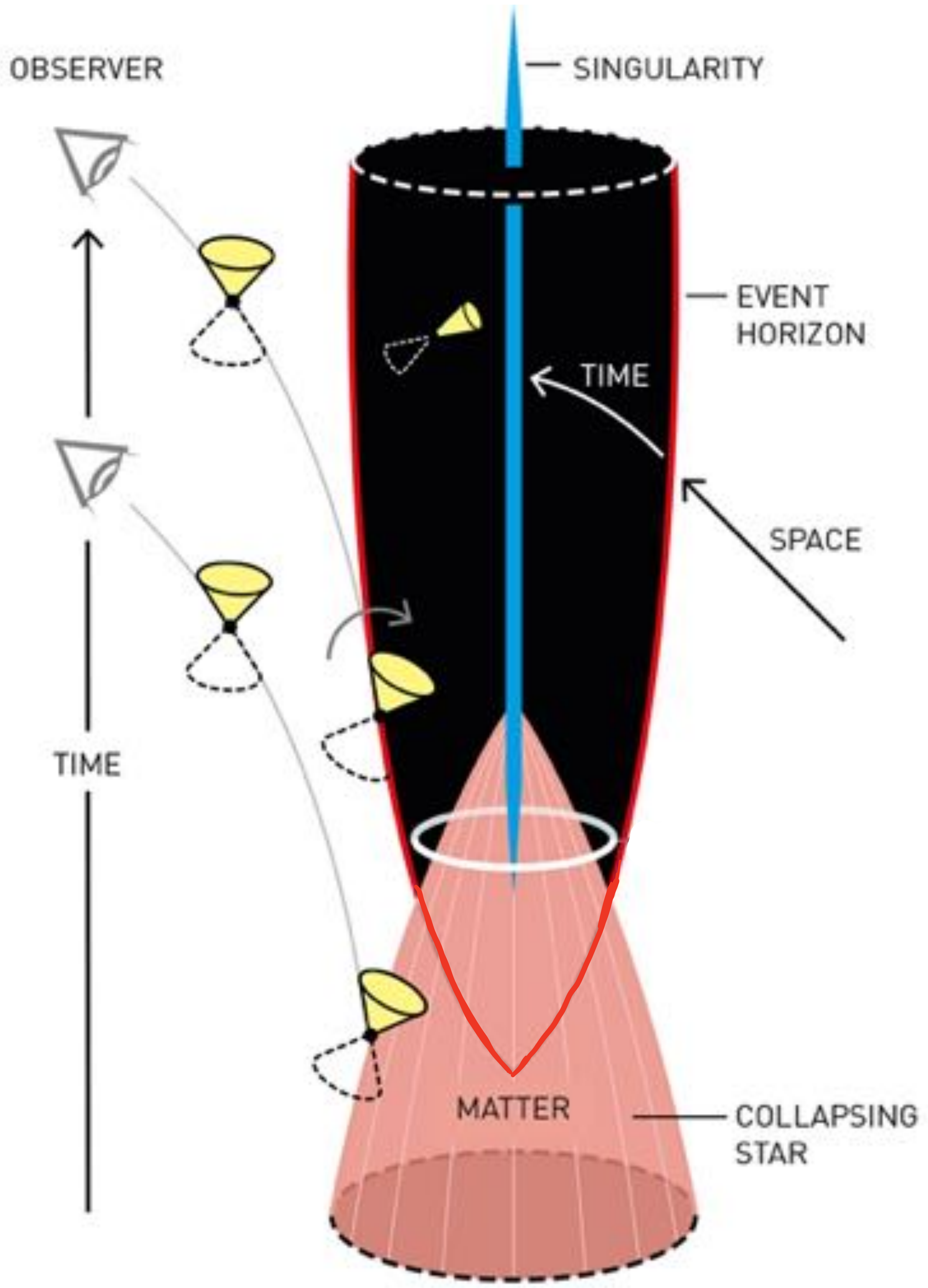
Properties of event horizons

- An event horizon is a continuous surface. But it is not smooth except in very special cases (We assume that *spacetime* is smooth)
- What is the nature of this non-smoothness?



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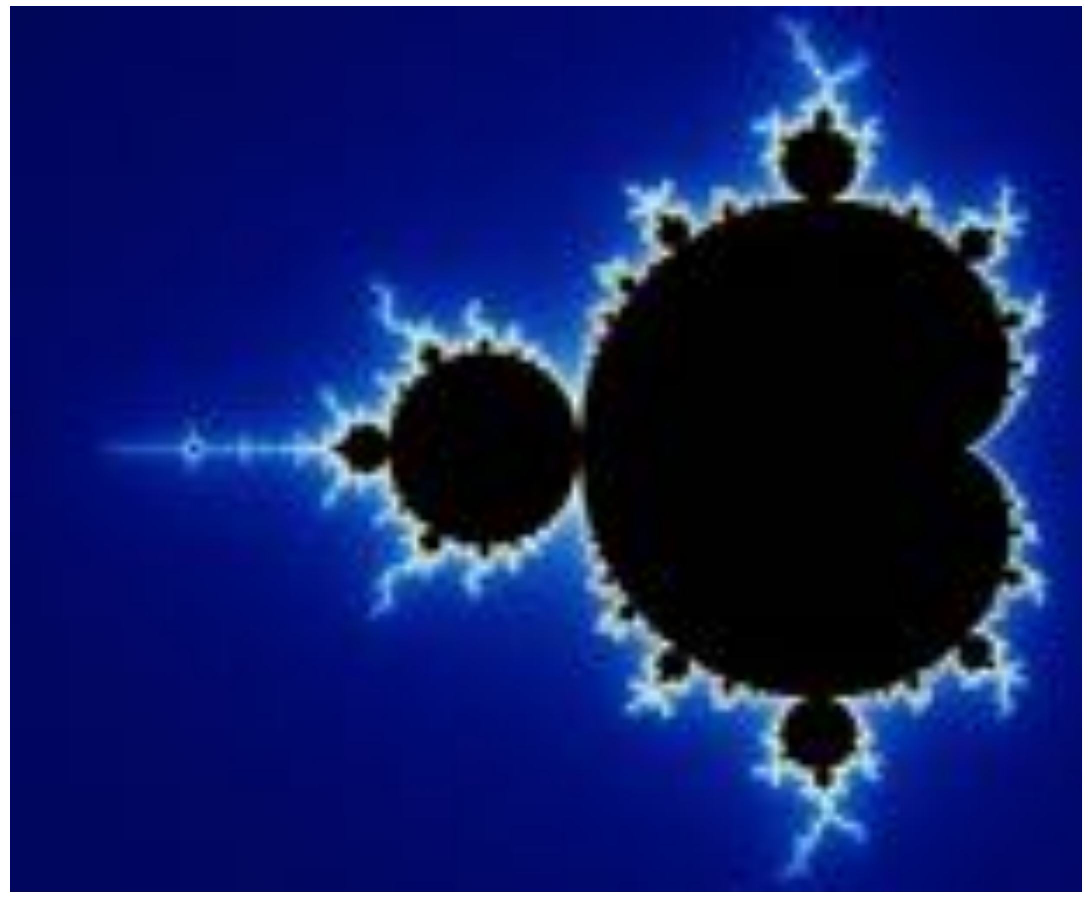
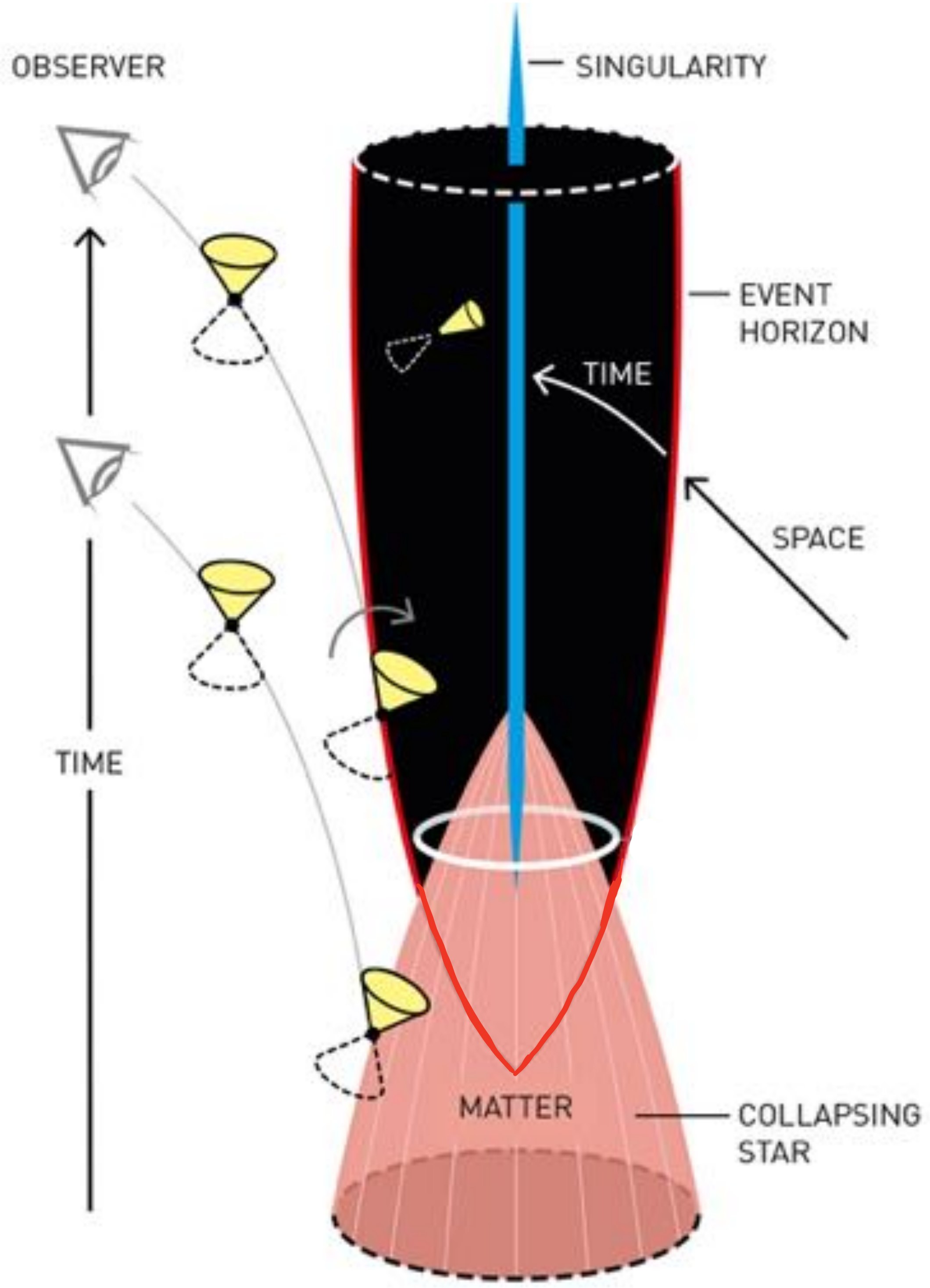
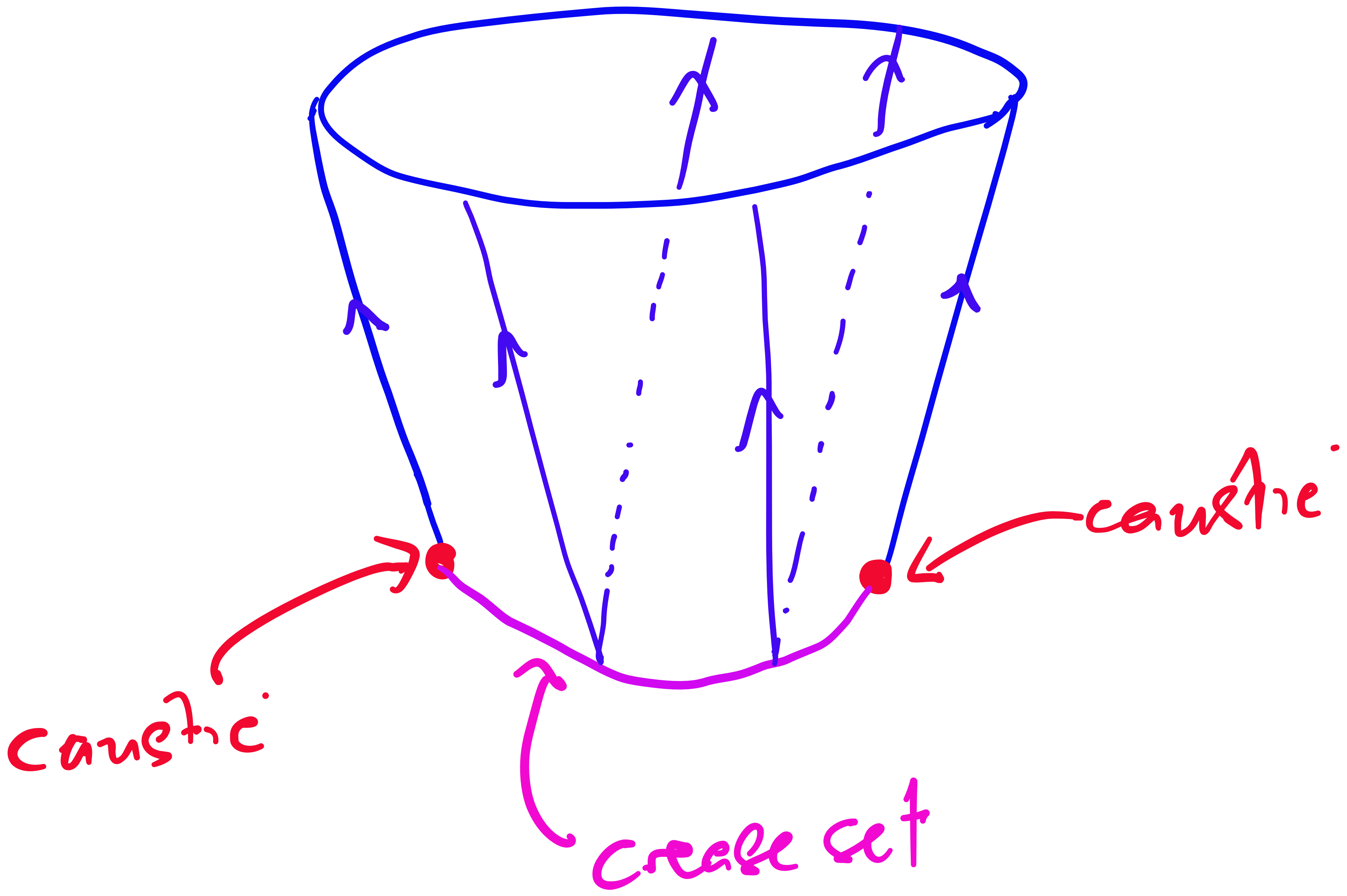
- There exist examples of spacetimes for which event horizon is non-differentiable on a dense set Chrusciel & Galloway 1996
- Theorem (Beem & Krolak 1997)
    - Event horizon is differentiable at p if and only if p lies on exactly one generator
    - A point lying on more than one generator is an endpoint (converse untrue)
    - So points where horizon is non-differentiable are endpoints



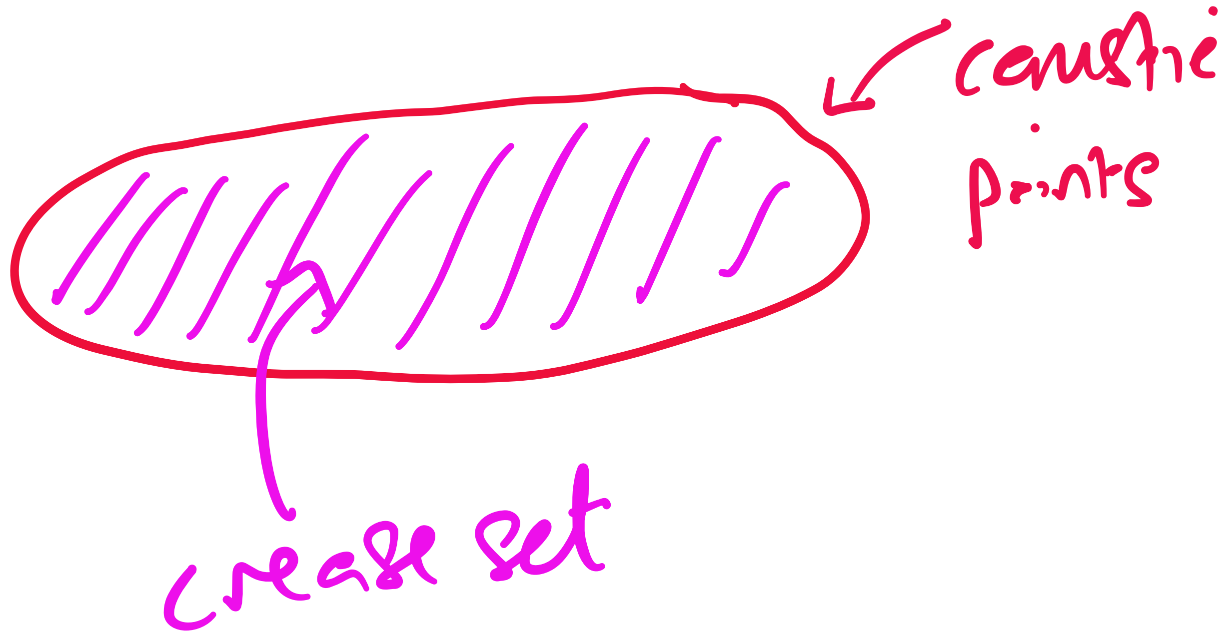


Examples

- In explicit examples of gravitational collapse or black hole mergers, set of endpoints consists of a 2d spacelike *crease set* where ≥ 2 generators enter horizon, together with its boundary, which is a line of caustic points (where “infinitesimally nearby generators intersect”) (Hughes et al 94, Shapiro et al 95, Lehner et al 99, Husa & Winicour ’99, Hamerly & Chen 10, Cohen et al 11, Emparan & Martinez 16, Bohn et al 16, Emparan et al 17)

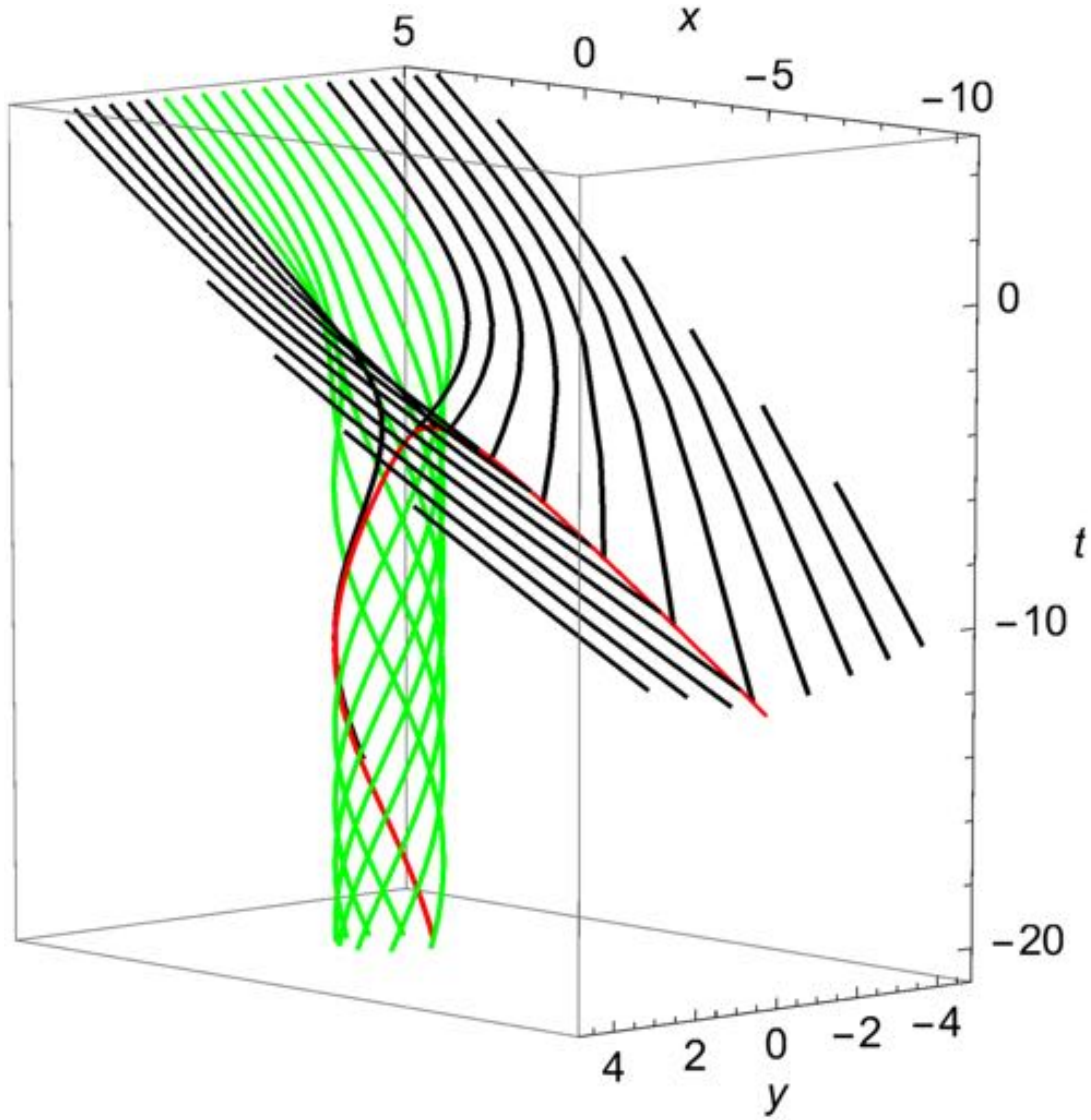


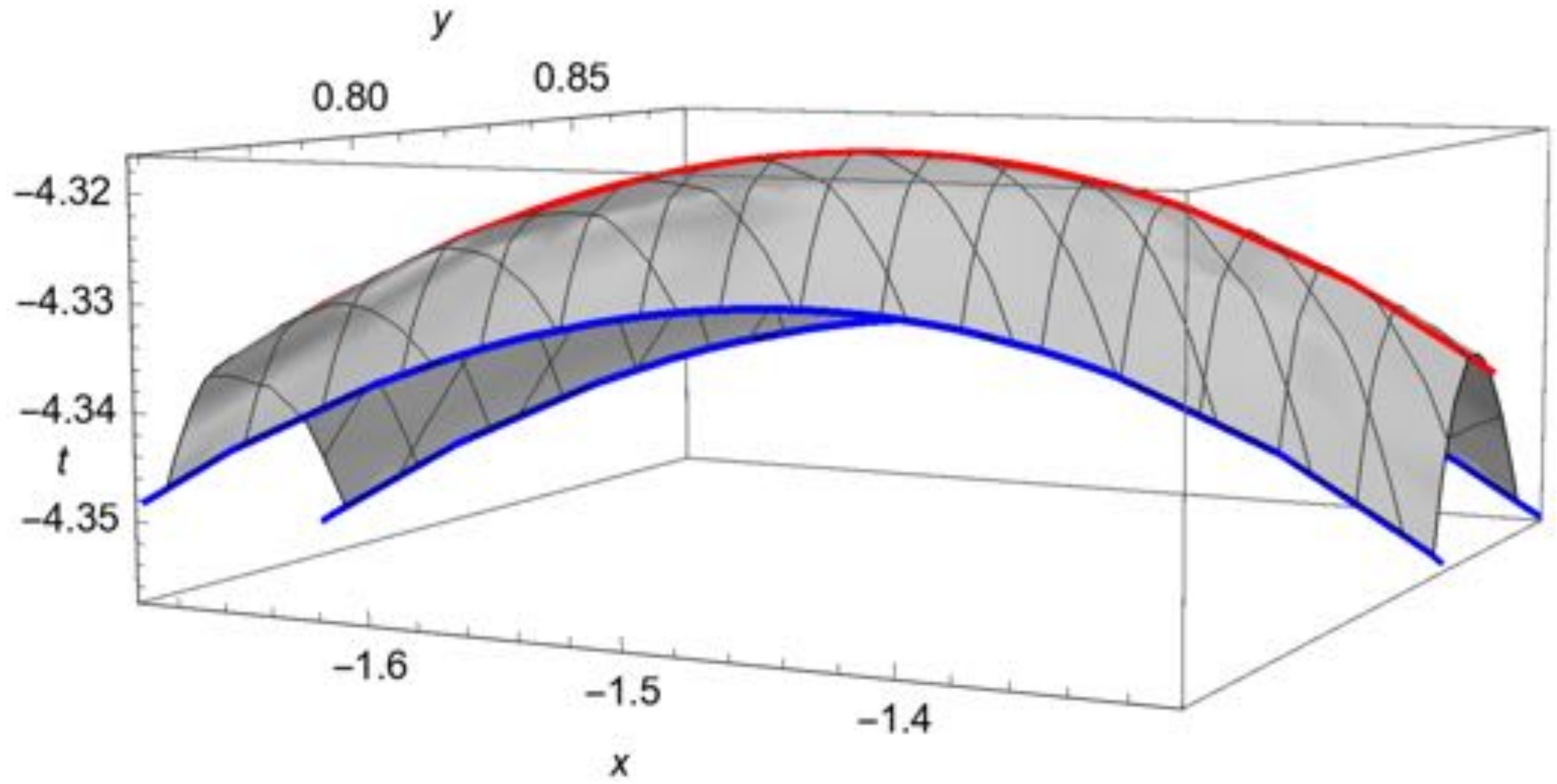
Event horizon in 3d gravitational collapse spacetime



Endpoint set in 4d gravitational collapse spacetime

Event horizon of non-axisymmetric extreme mass ratio merger (Emparan et al 17)





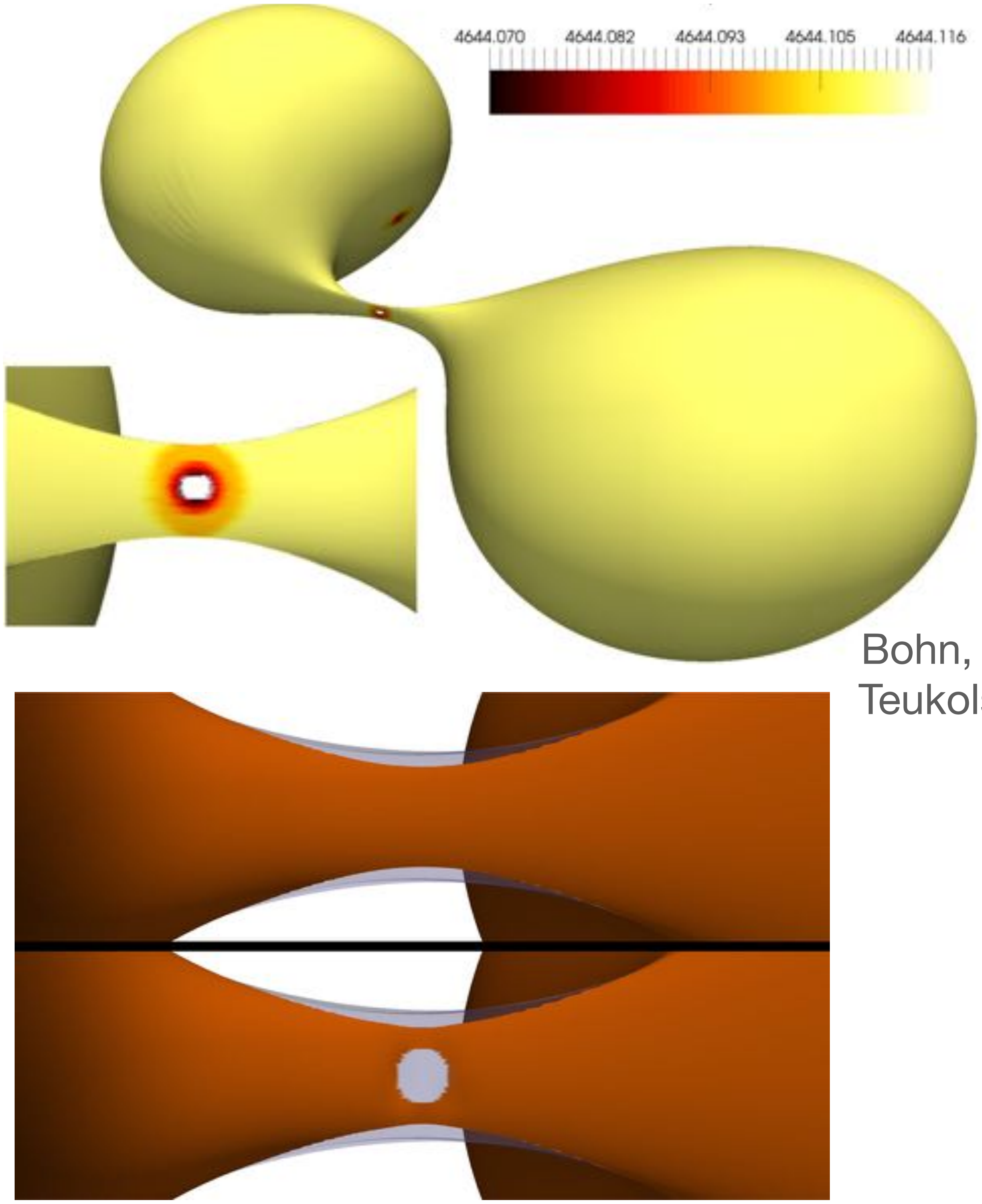
Creases

- Choose a time function, i.e., a foliation of spacetime into "constant time" hypersurfaces labelled by t
- Intersection with the event horizon is "the horizon at time t"
- In examples, if hypersurface intersects crease set then the horizon at time t will exhibits "creases": sharp edges, rounding off at caustic points

Husa & Winicour 99

Toroidal horizons

- In examples of gravitational collapse or black hole mergers, for some choices of time function, there is a brief period when horizon has toroidal topology (Hughes et al 94, Siino 97, Lehner et al 98, Husa & Winicour 99, Cohen et al 11, Bohn et al 16)
- The “hole in the torus” collapses superluminally
- Creases are present both around the ring of the torus and along the “edges of the bridge”



Bohn, Kidder & Teukolsky 2016

Our work

HSR & Maxime Gadioux 2023

- What explains the simple structure of the set of horizon endpoints in these examples?

- What other structures are possible?

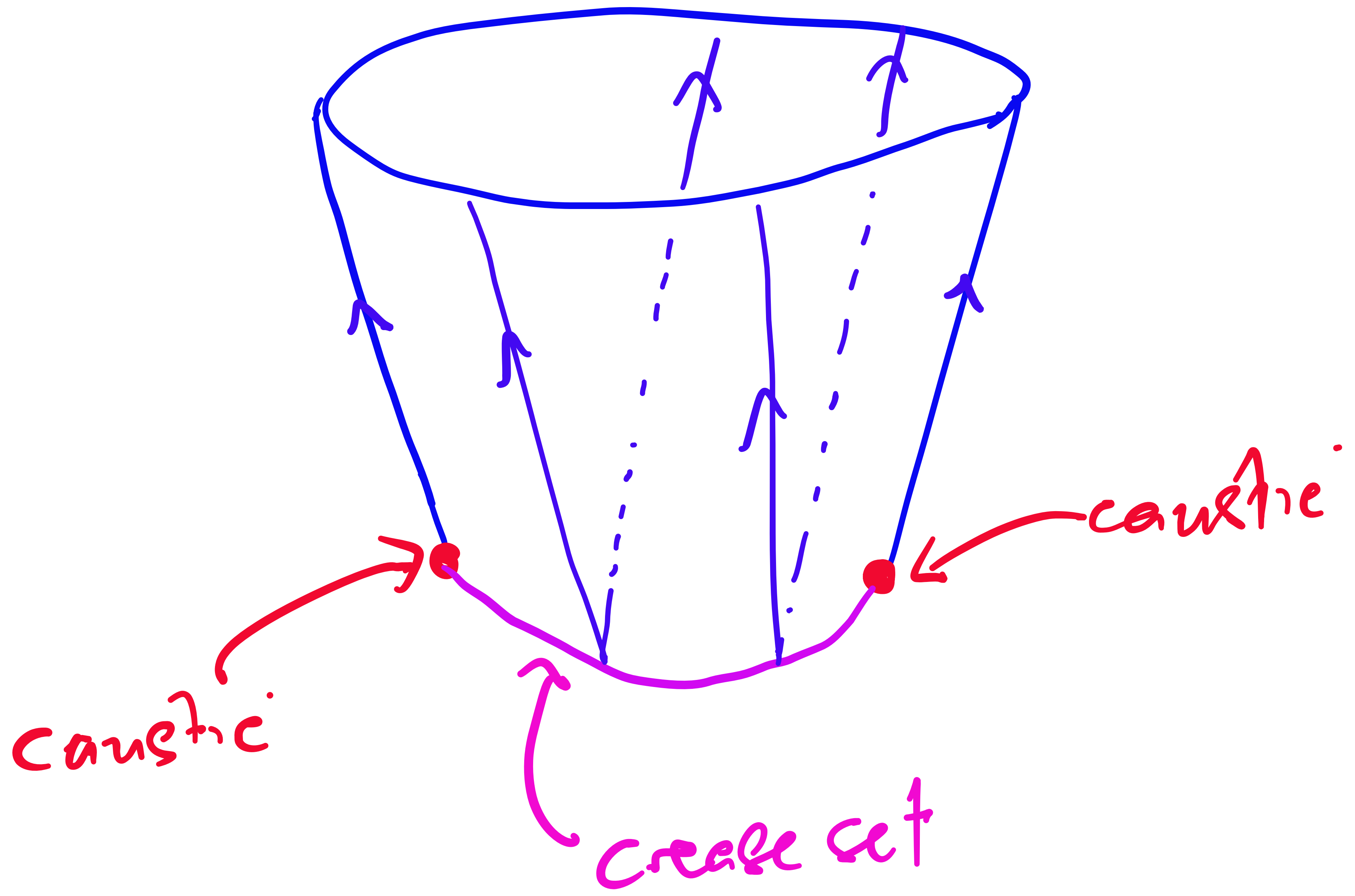
- Key assumption: event horizon is smooth *at late time* - there exists a smooth “constant time” cross-section of the event horizon

- Generalising some results from Riemannian geometry (Itoh & Tanaka 1998) we showed that

- Non-caustic points lying on exactly 2 generators form a 2-dimensional spacelike surface: the *crease submanifold*

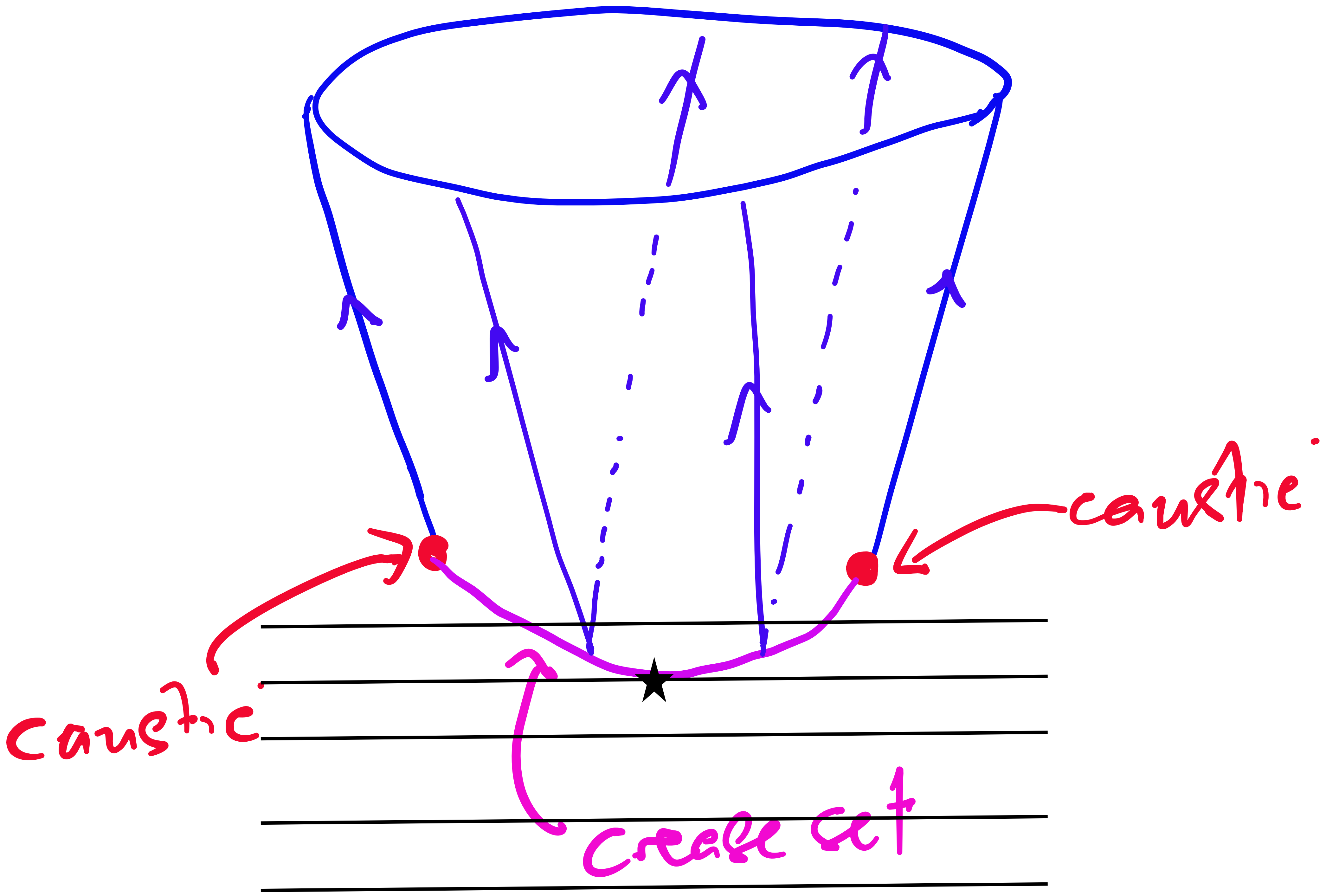
- All other endpoints form a set of (Hausdorff) dimension at most 1

- This explains the structure of the endpoint set seen in the examples



Perestroikas

- Choose a time function, i.e., a foliation of spacetime into “constant time” hypersurfaces labelled by t
- A *crease perestroika* occurs when a surface of constant t is tangent to the crease submanifold
- We classified perestroikas using local inertial coordinates at the point of tangency, adapted to surface of constant t
- 3 distinct cases. Shift t so perestroika occurs at t=0



Flying saucer

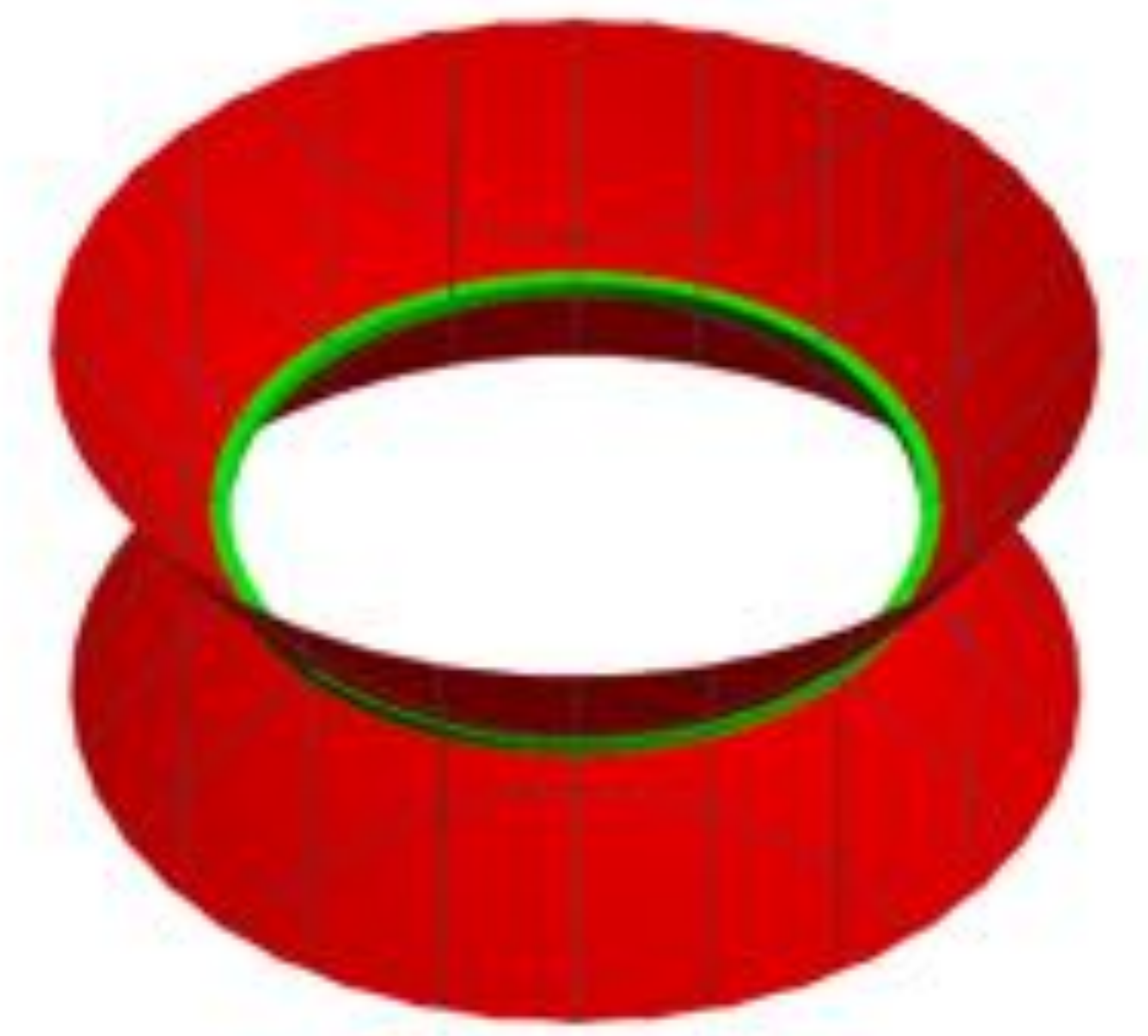
- This perestroika describes the nucleation of an event horizon in generic gravitational collapse
- Length of elliptical crease and angle at crease scale as $\sqrt{t}$, area scales as t

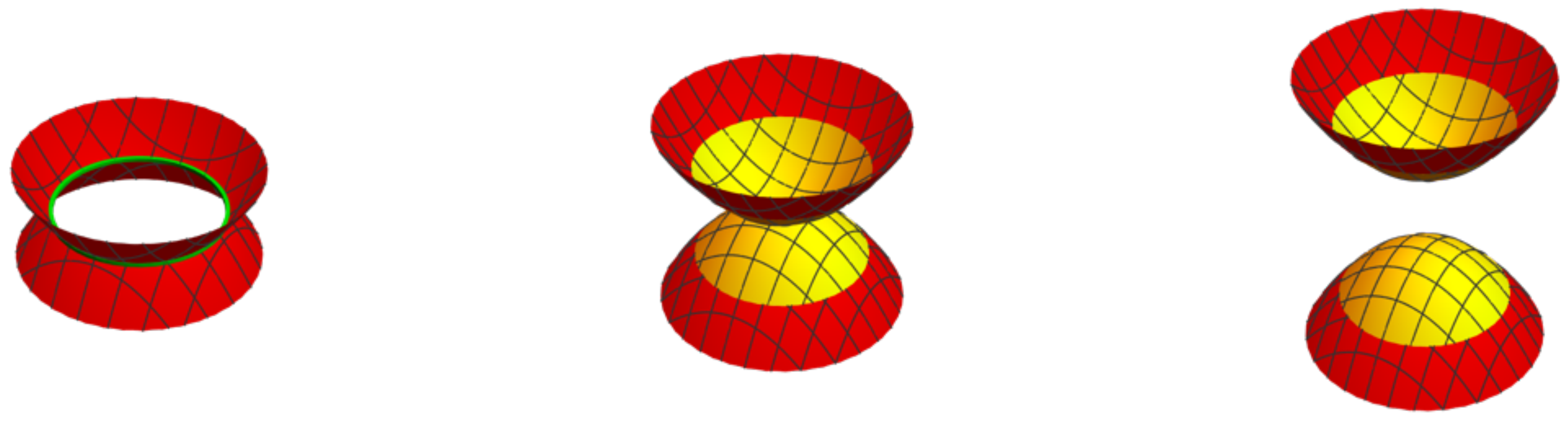
Collapse of hole in horizon

- Horizon can exhibit a short-lived phase of toroidal topology

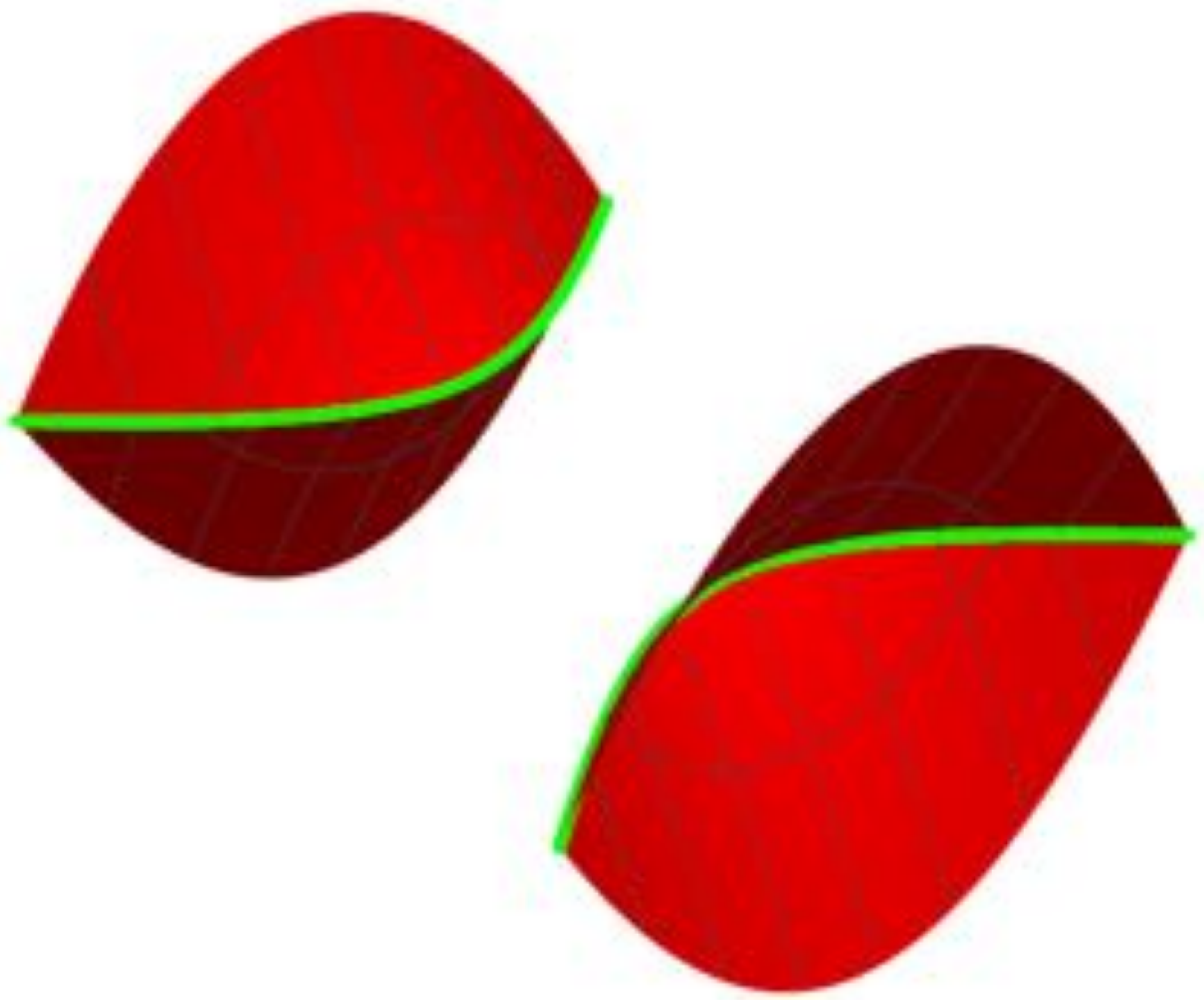
- The “hole in the torus” collapses superluminally. This is described by a perestroika

- Length of crease and angle at crease scale as $\sqrt{-t}$

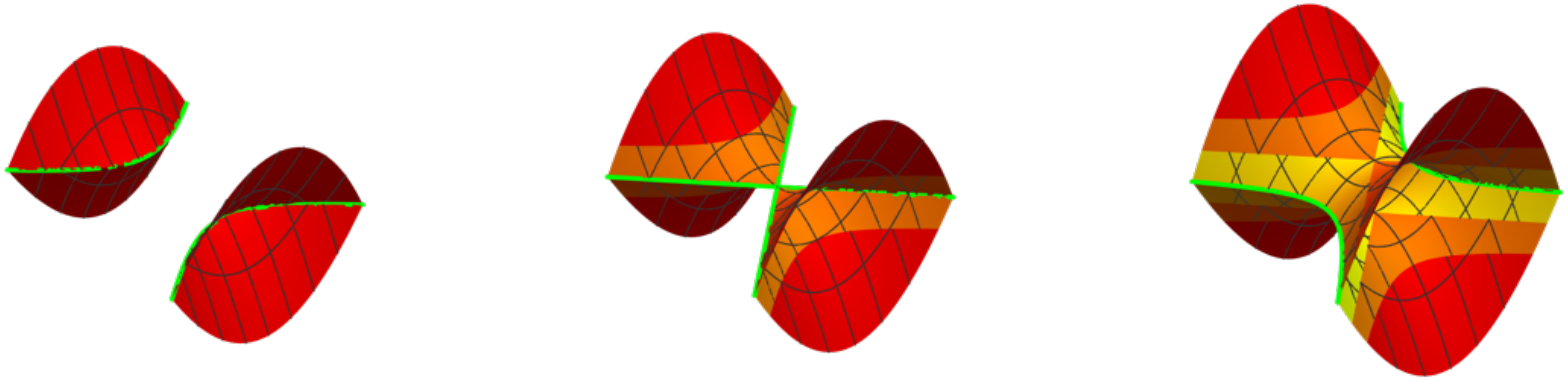




Black hole merger



- This perestroika describes the merger of two (locally) disconnected sections of horizon e.g. two merging black holes
- Angle at crease and width of bridge scale as $\sqrt{|t|}$, height of bridge scales as t



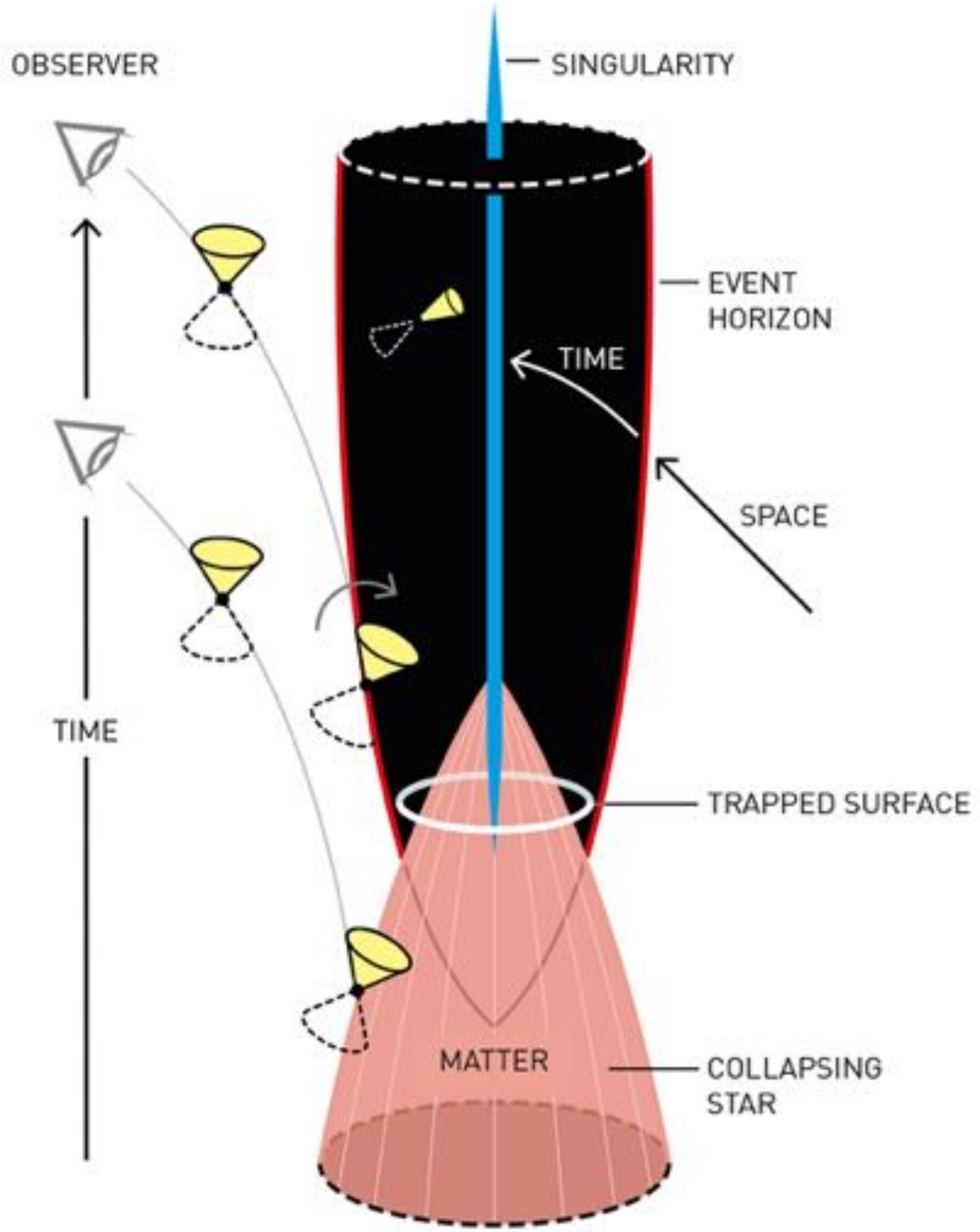
Crease contribution to black hole entropy

- Bekenstein, Hawking: a black hole has an entropy $S = A/4G\hbar$
- Old idea: some/all of this entropy arises from entanglement entropy of quantum fields across the black hole horizon (Bombelli et al 86, Srednicki 93, Susskind & Uglum 94)
- Entanglement entropy exhibits novel features in presence of a crease (Casini & Huerta 06, Hirata & Takayanagi 06, Klebanov et al 12, Myers & Singh 12)
- Suggests that a crease contributes to black hole entropy as $\frac{1}{\sqrt{G\hbar}} \int F(\Omega) dl$

  where Ω is angle at crease with $F < 0$ and $F \propto 1/\Omega$ as $\Omega \to 0$. Subleading compared to Bekenstein-Hawking.
- "Collapse of hole in the horizon" perestroika: this term is discontinuous but second law of thermodynamics is satisfied.

Stability and catastrophes

- Which features of the event horizon are stable under small perturbations?
- e.g. spherically symmetric gravitational collapse: single caustic point, unstable
- Siino & Koike 04: catastrophe theory classification of endpoints of horizon generators assuming a particular notion of stability
- Caustic points "of type A_3"



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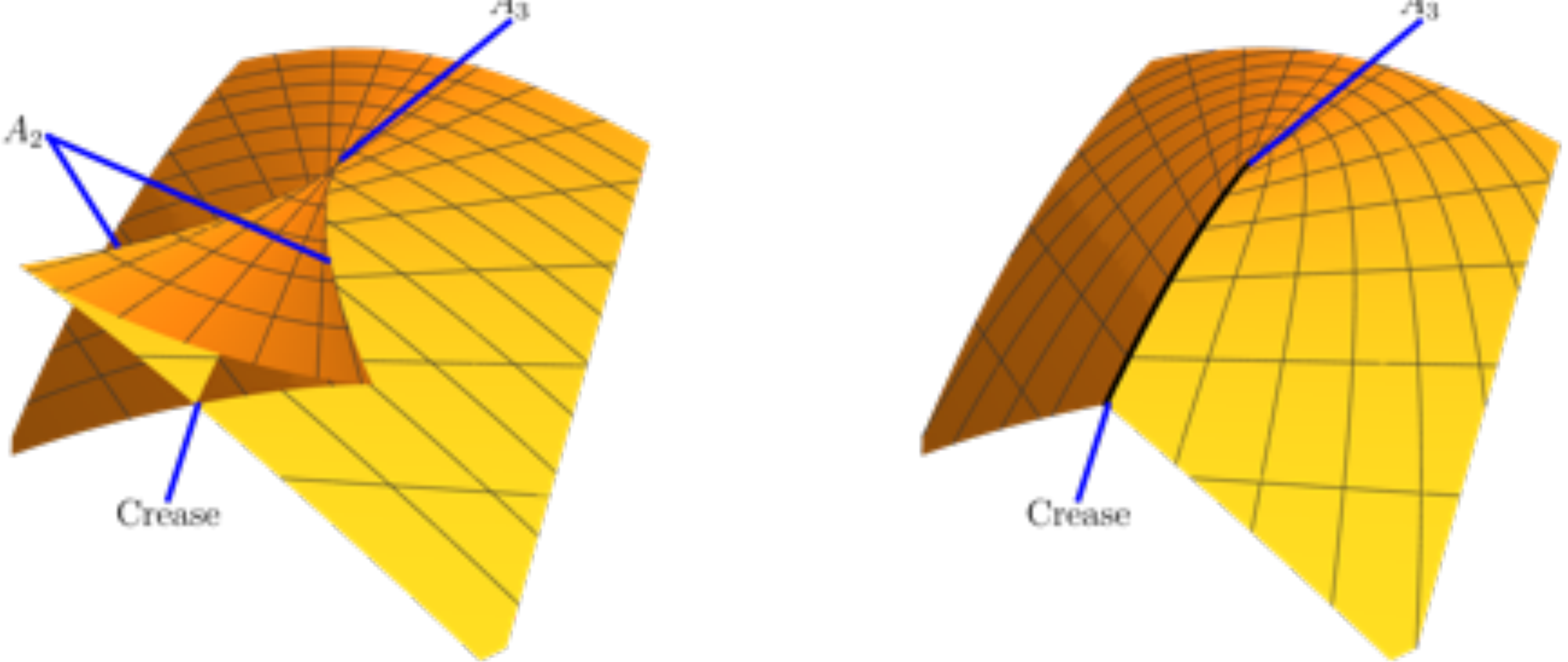
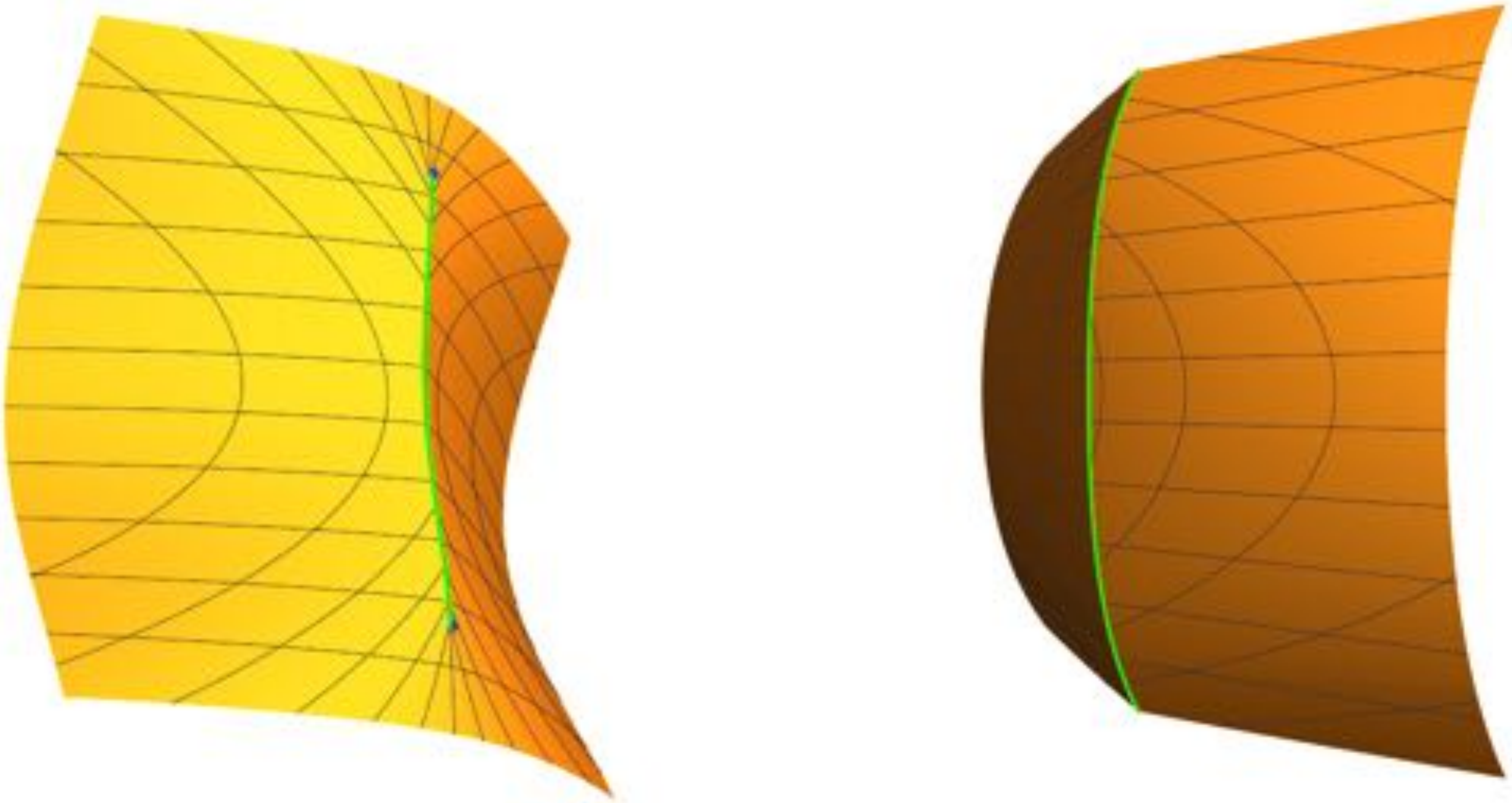
Generic caustics: A_3

- A_3 caustic points form spacelike lines in spacetime
- Constant time cross-section of horizon generically has isolated A_3 caustic points
- A single generator enters the horizon at an A_3 caustic



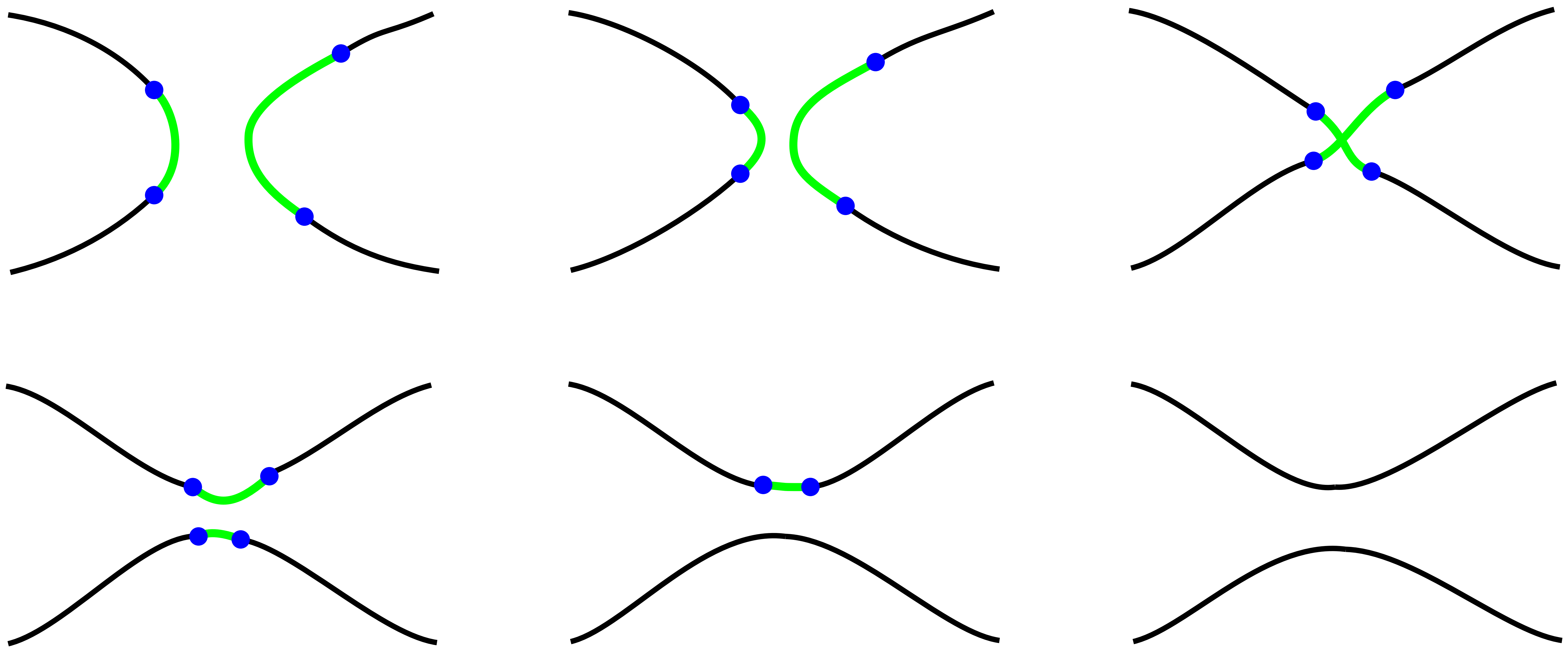
Caustic perestroikas

- Occur when constant time slice is tangent to A_3 line



Elements of a black hole merger

- A black hole merger can be decomposed into a sequence of crease and caustic perestroikas
- The instant of merger is, generically, always a crease perestroika



Summary

- An event horizon exhibits non-smooth features where new generators enter the horizon: creases, caustics
- We've determined the general structure of this endpoint set for a horizon that is smooth at late time
- We've classified perestroikas involving these structures, which play an important role in dynamical processes involving black holes
- We've argued that creases contribute to black hole entropy
- Other topics in our paper: corners, Gauss-Bonnet term in entropy, Bousso entropy bound, open questions concerning classification of caustics in curved spacetime