Quantum effects inside black hole spacetimes

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Outline

1. Cauchy horizon

2. Classical perturbations of CH

3. Semiclassical gravity

4. Semiclassical effects on CH of Kerr(-de Sitter)

5. Conclusion

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Kerr Black Holes

Astrophysical BHs are believed to be described by the Kerr metric:

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt^2 - a\sin^2\theta d\varphi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left(\left(r^2+a^2\right)d\varphi - adt\right)^2$$

$$\Delta = (r - r_+)(r - r_-)$$

$$\Sigma \equiv r^2 + a^2\cos^2\theta$$

Event horizon: $r_\pm = M \pm \sqrt{M^2 - a^2}$

Cauchy

mass

angular momentum per unit mass

Maximally-rotating (extremal) Kerr is for $a = M$

Horizon at $r = r_\pm$ has angular veloc. $\Omega_\pm \equiv \frac{a}{r_\pm^2 + a^2}$

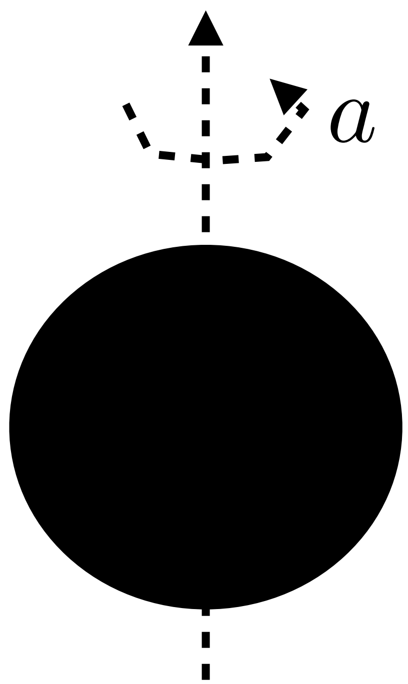
and surface gravity $\kappa_\pm = \frac{r_+ - r_-}{2\left(r_\pm^2 + a^2\right)}$

It has a curvature singularity at $r = 0$ and $\theta = \pi/2$

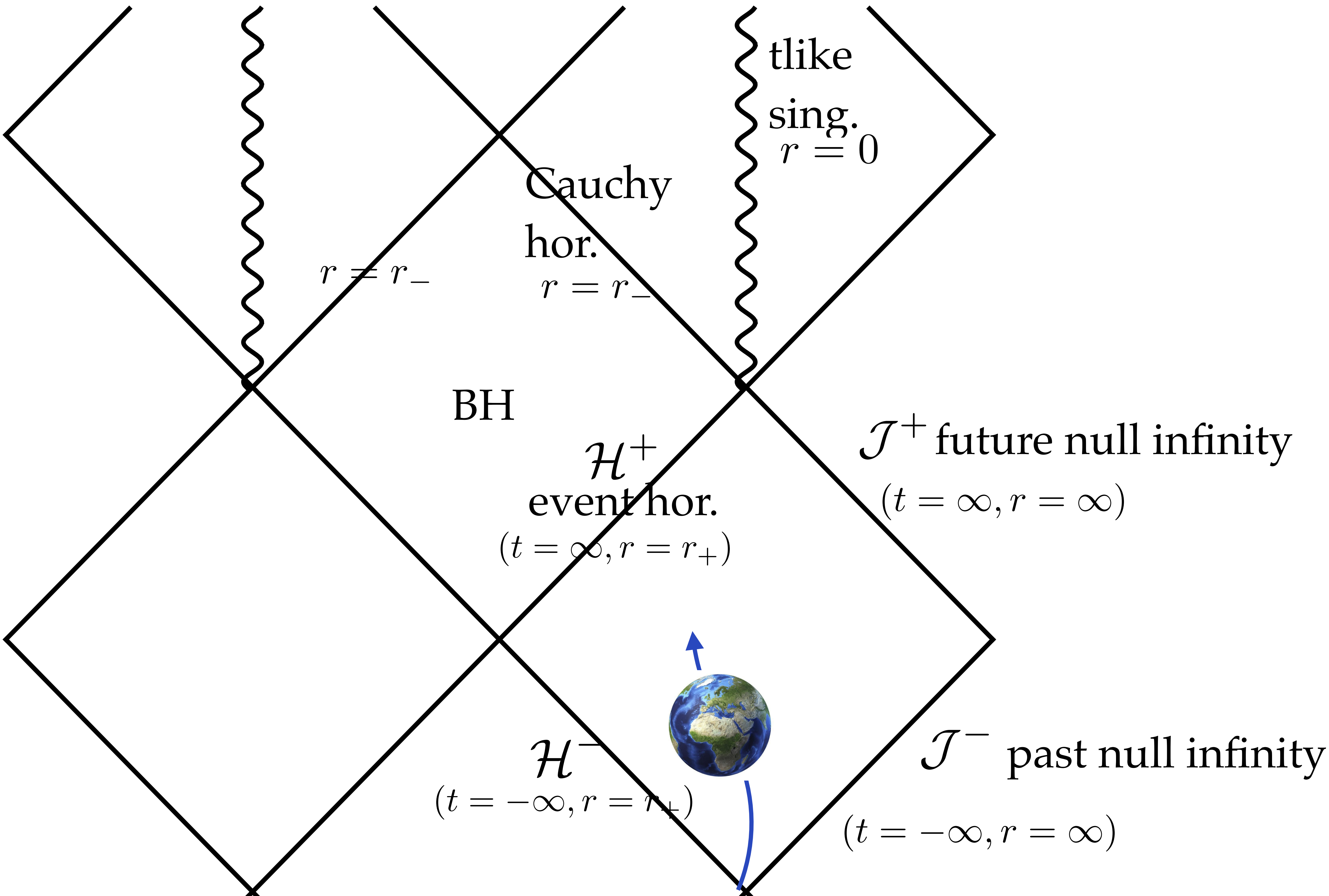
It has two (Killing) symmetries:

stationarity (∂_t) and axi-symmetry (∂_φ)

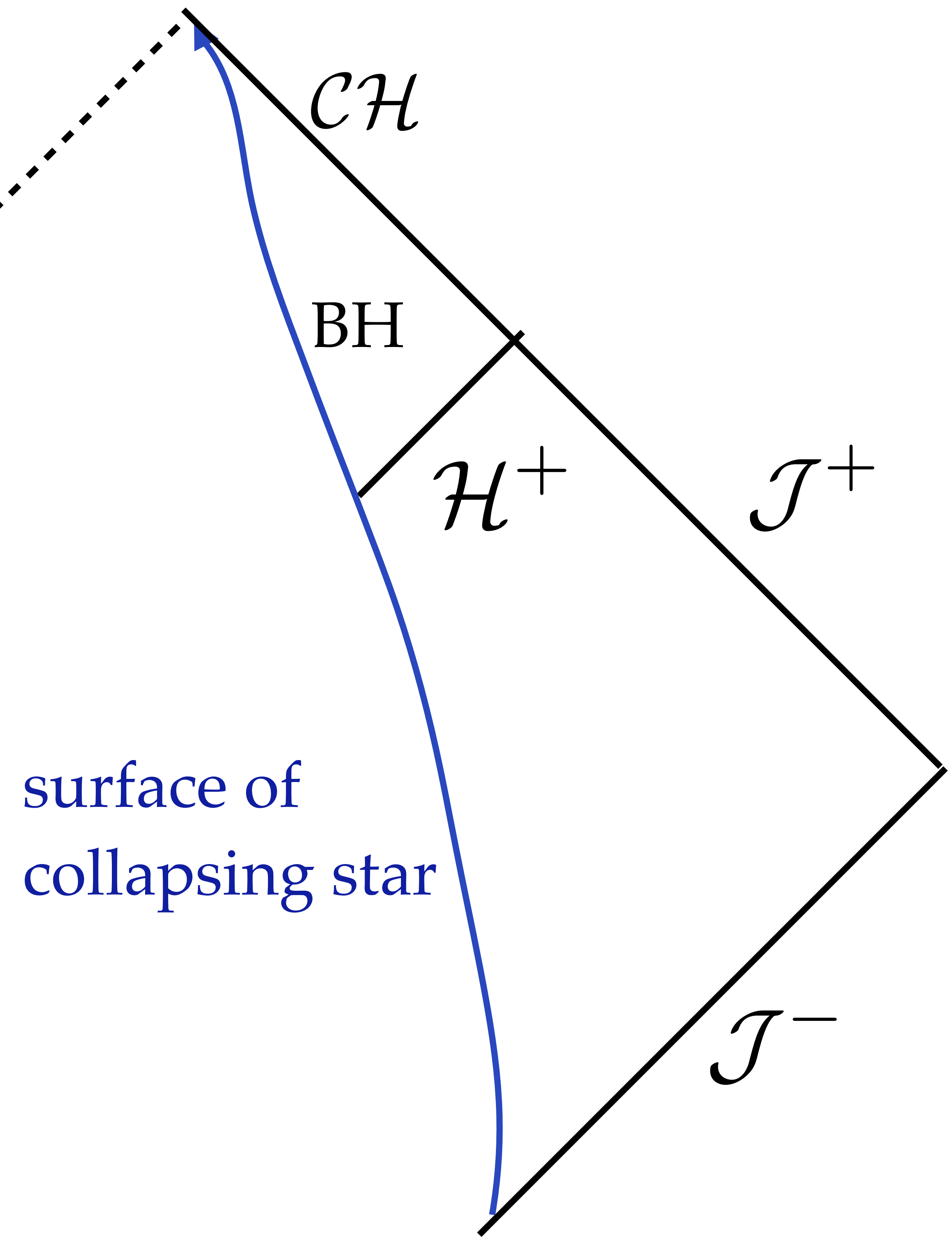
It represents a *rotating* (astrophysical) BH



Penrose diagram

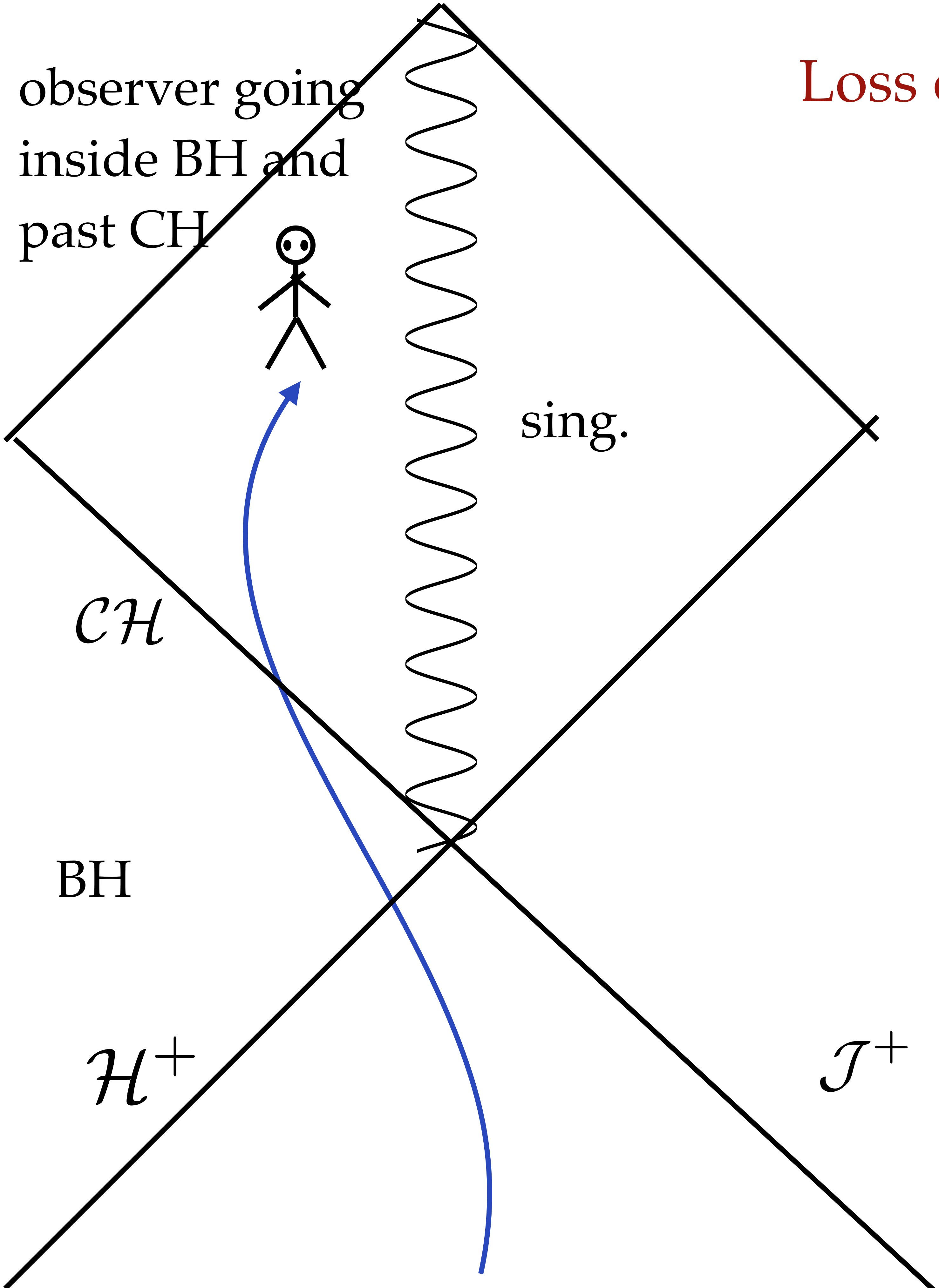


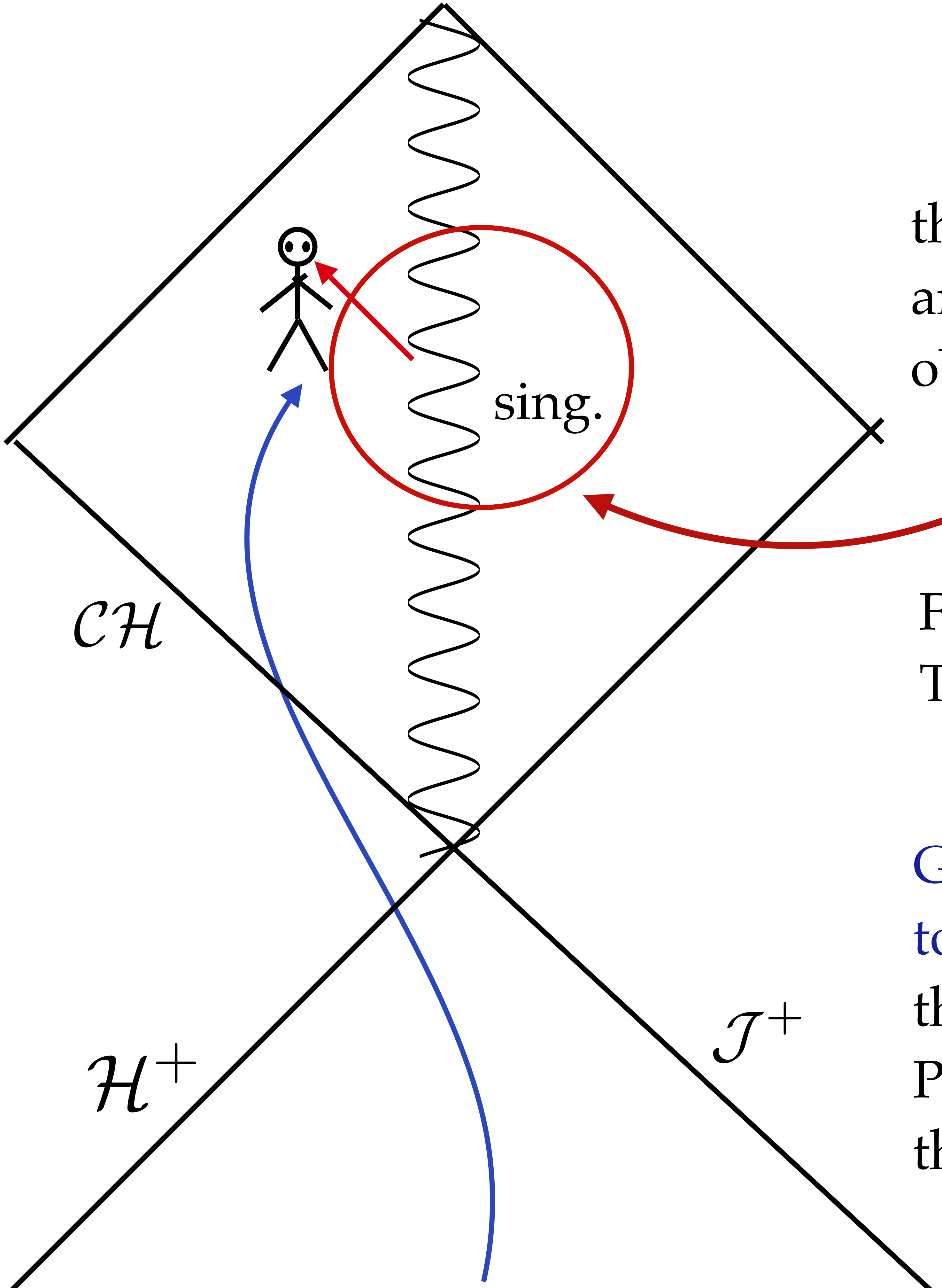
Penrose diagram: collapse to Kerr



In gravitational collapse, the CH on the left might not be there but, in principle, part of the CH on the *right* is there, so that's the physically meaningful one

Loss of predicability inside the BH





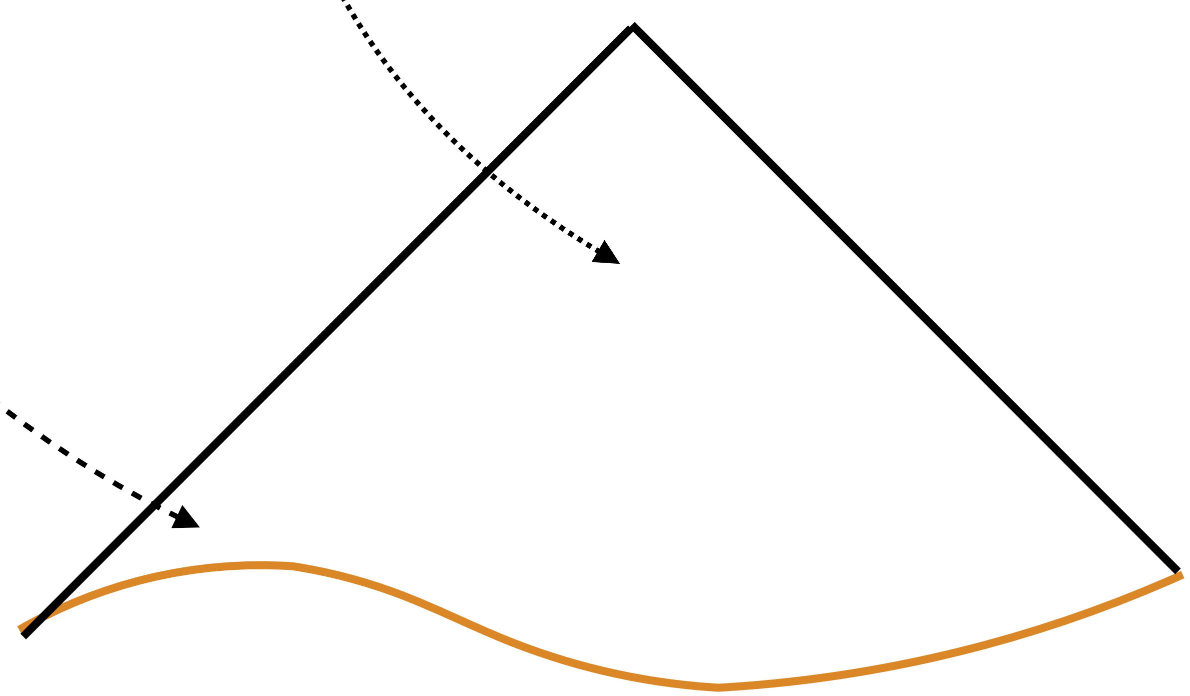
this is a *timelike* singularity, and so it's *visible* to an observer going into the BH

Furthermore, there exist Closed Timelike Curves near the sing.

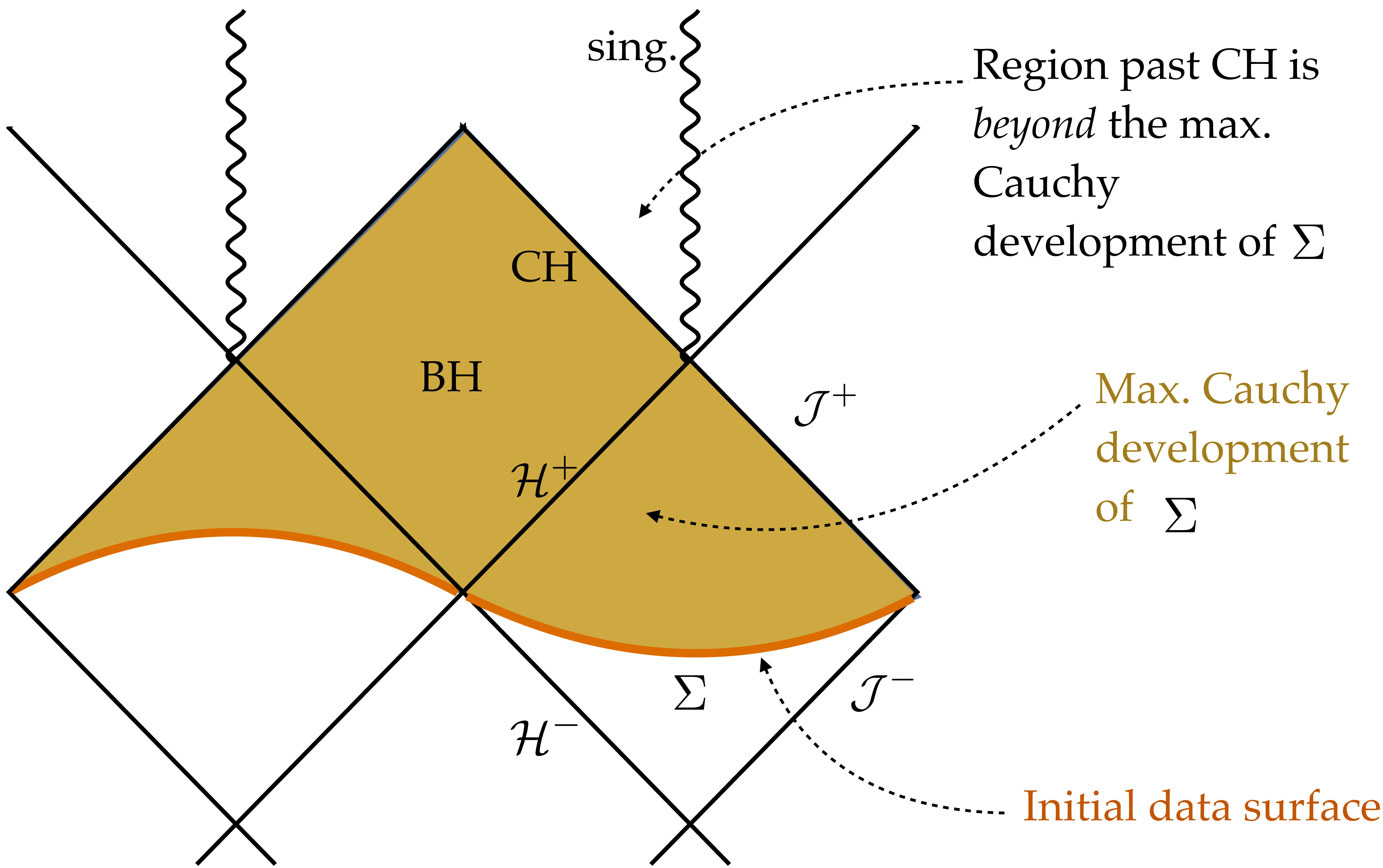
General Relativity seems to cease to be a *predictable* th. inside BHs: the Cauchy ("initial") Value Problem is not well posed beyond the CH

Strong Cosmic Censorship hypothesis

Strong Cosmic Censorship (SCC) Hypothesis (Penrose'72), essentially:
the maximal Cauchy development via Einstein equations of generic initial data is inextendible (the degree of *irregularity* of field depends on version)



Applying SCC to Kerr means that initial data for the metric field should be inextendible past the CH

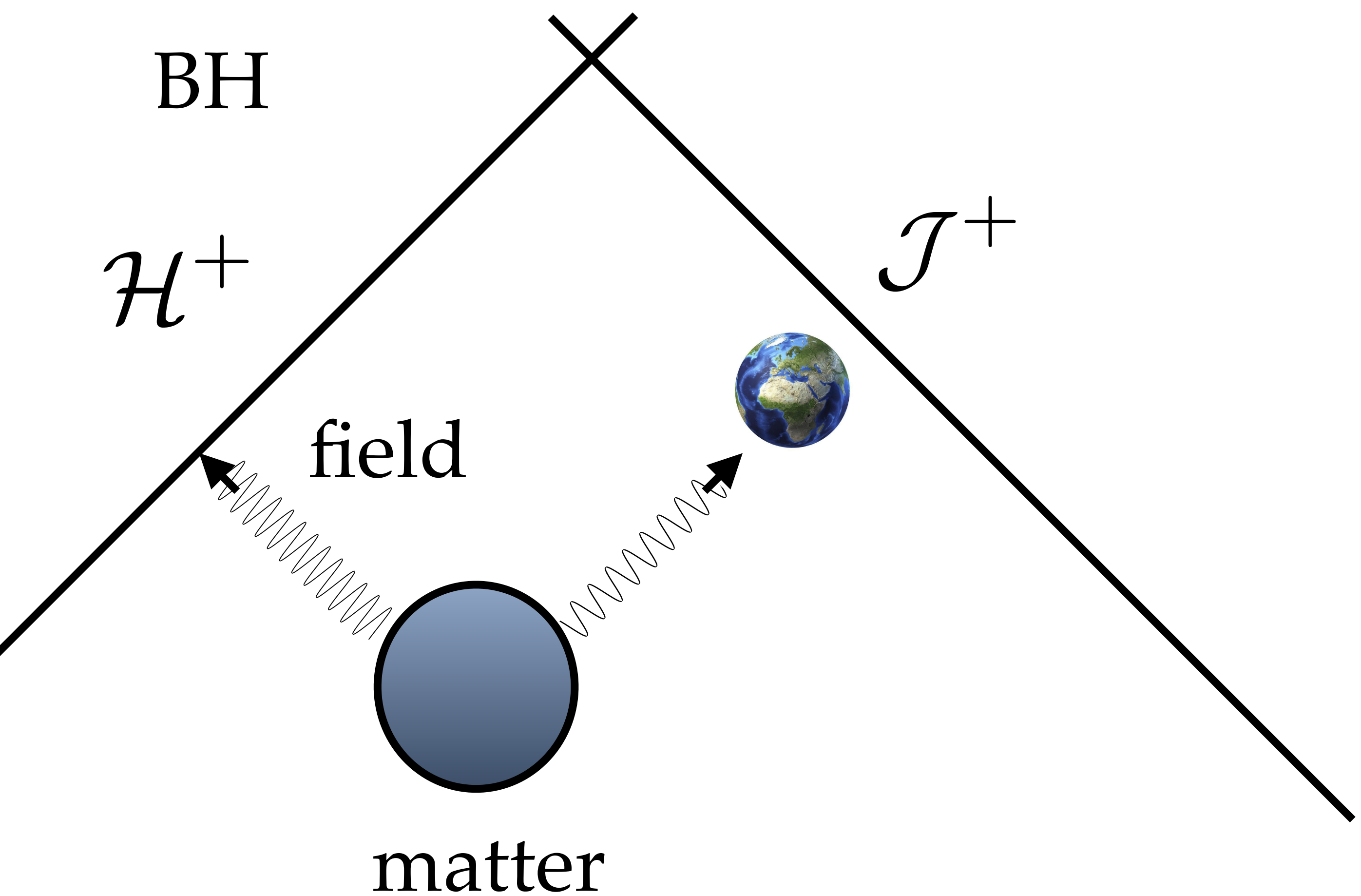


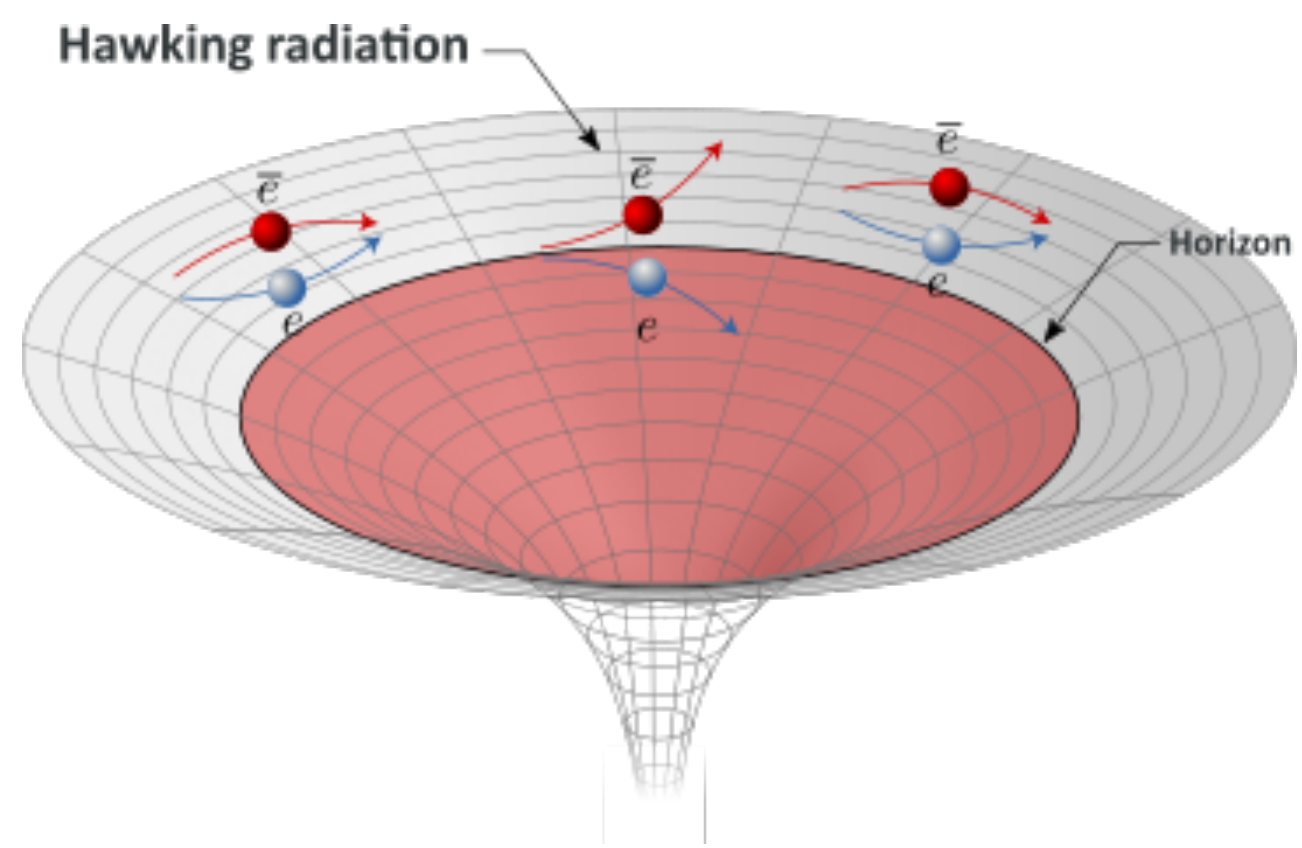
BH Perturbations - saviours of SCC?

BHs in Nature are not *exactly* Kerr but they are ‘perturbed’ by classical or quantum fields (scalar, fermion, electromagnetic, gravitational…) due to:

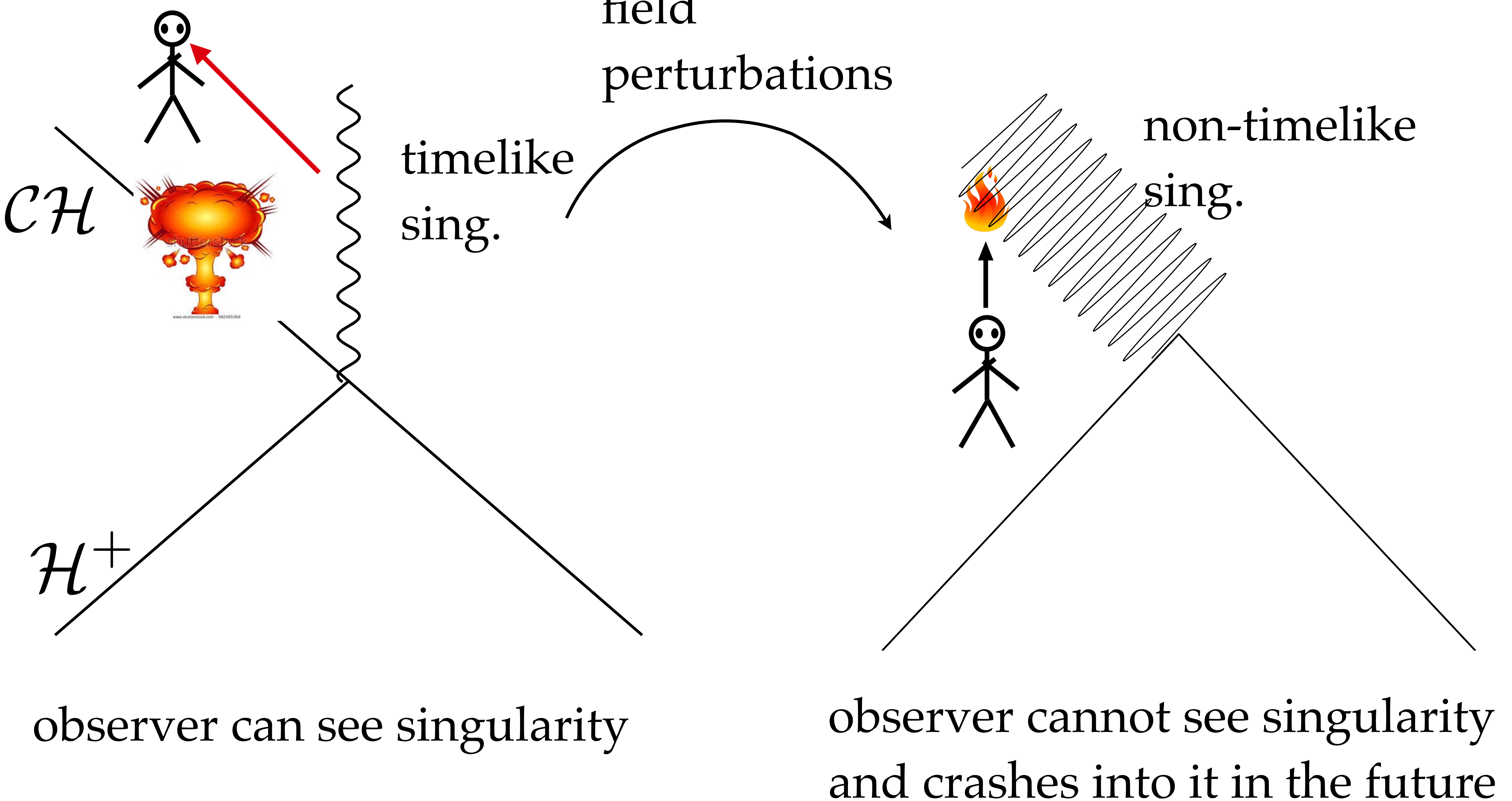
Possible neighbouring classical matter (eg, accretion disk, neutron star, etc) or another BH

Quantum vacuum (Hawking’75) - *always* present!





SCC could be upheld for BHs if their CH is "destroyed" by perturbations from classical and/or quantum fields



N.B.: a similar CH exists for electrically charged BHs, whether non-rotating (Reissner-Nordstrom) or rotating (Kerr-Newman), whether with cosmological const. $\Lambda = 0$, $\Lambda > 0$ (asympt. de Sitter) or $\Lambda < 0$ (asympt. anti-de Sitter)

But SCC is a hypothesis - it needs to be verified!

Questions:

Is GR a predictable th. inside astrophysical BHs? if the CH becomes irregular, how irregular does it become? does it become a null sing. or a slike sing.? what would happen to an observer trying to cross that region? what are stronger, the classical effects or the quantum effects?…

1. Cauchy horizon

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Wave Equation

We consider *linear* field perturbations of a *fixed* BH *background* g (ie, we do not consider the backreaction of the field on the BH)

E.g., scalar field perturbations ϕ of a BH satisfy a wave eq.

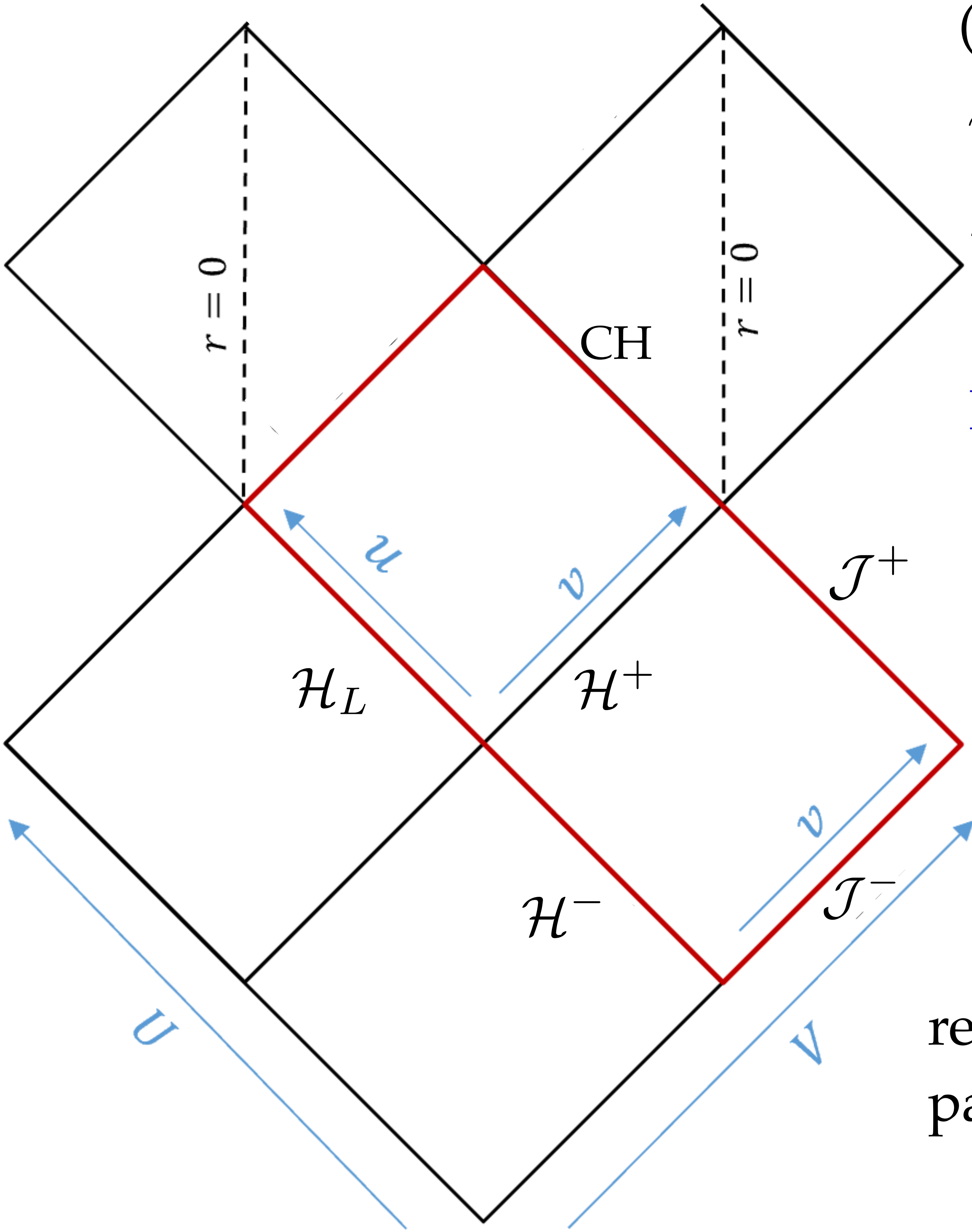
$$\Box\phi(x) \equiv g_{\mu\nu}\nabla^{\mu}\nabla^{\nu}\phi(x) = T(x)$$

spacetime point

source of field

Linear perturbations by other fields (fermions, emag, linear grav) of Kerr(-Newman)(-A)(dS) BHs obey a similar wave-like eq.

"Time" coordinates



Eddington-Finkelstein coords.:

(range over $\mathbb{R}$)

$$u \equiv t - r_*$$

$$v \equiv t + r_* \quad \text{affine par. along } \mathcal{J}^-$$

Kruskal coords.:

$$V(v) \equiv \frac{1}{\kappa_-} \exp(-\kappa_- v) \text{ regular}$$

on CH (V=0)

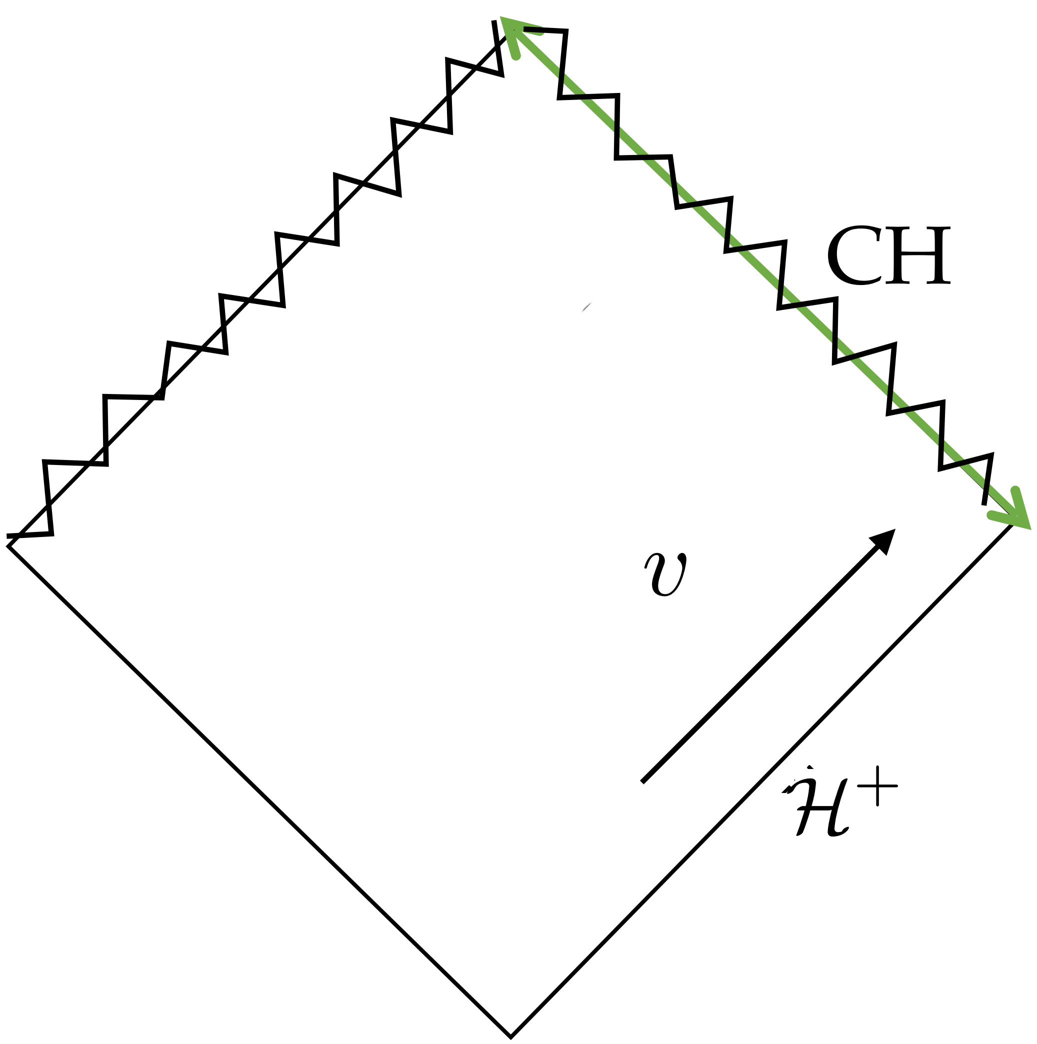
$$U(u) \equiv -\frac{1}{\kappa_+} \exp(-\kappa_+ u)$$

regular on $\mathcal{H}^+$ (U=0) and affine par. along H_L and $\mathcal{H}^-$

Classical (ir)regularity of CH of asymptotically-*flat* BHs

CH & region of unpredictability of Kerr [Ori'92, Dafermos&Luk'17] and of Reissner-Nordstrom [Poisson & Israel'90] are "somewhat destroyed" by the perturbation:

SCC holds in the sense that the field is not C^1 but is C^0



a *null* curvature sing. forms

it's a "*weak*" sing.

$$\sim \frac{\partial^2 g}{\partial V^2} \sim v^{-n} e^{2\kappa_- v} \quad \text{for} \quad n > 0$$

$v \to \infty$

depends on field type

Classical (ir)regularity of CH of asymptotically-*dS* BHs

Kerr-dS: the CH becomes irregular (field is not C^1) -> region of unpredictability "disappears" -> preservation of SCC [Dias et al.'18]

😀

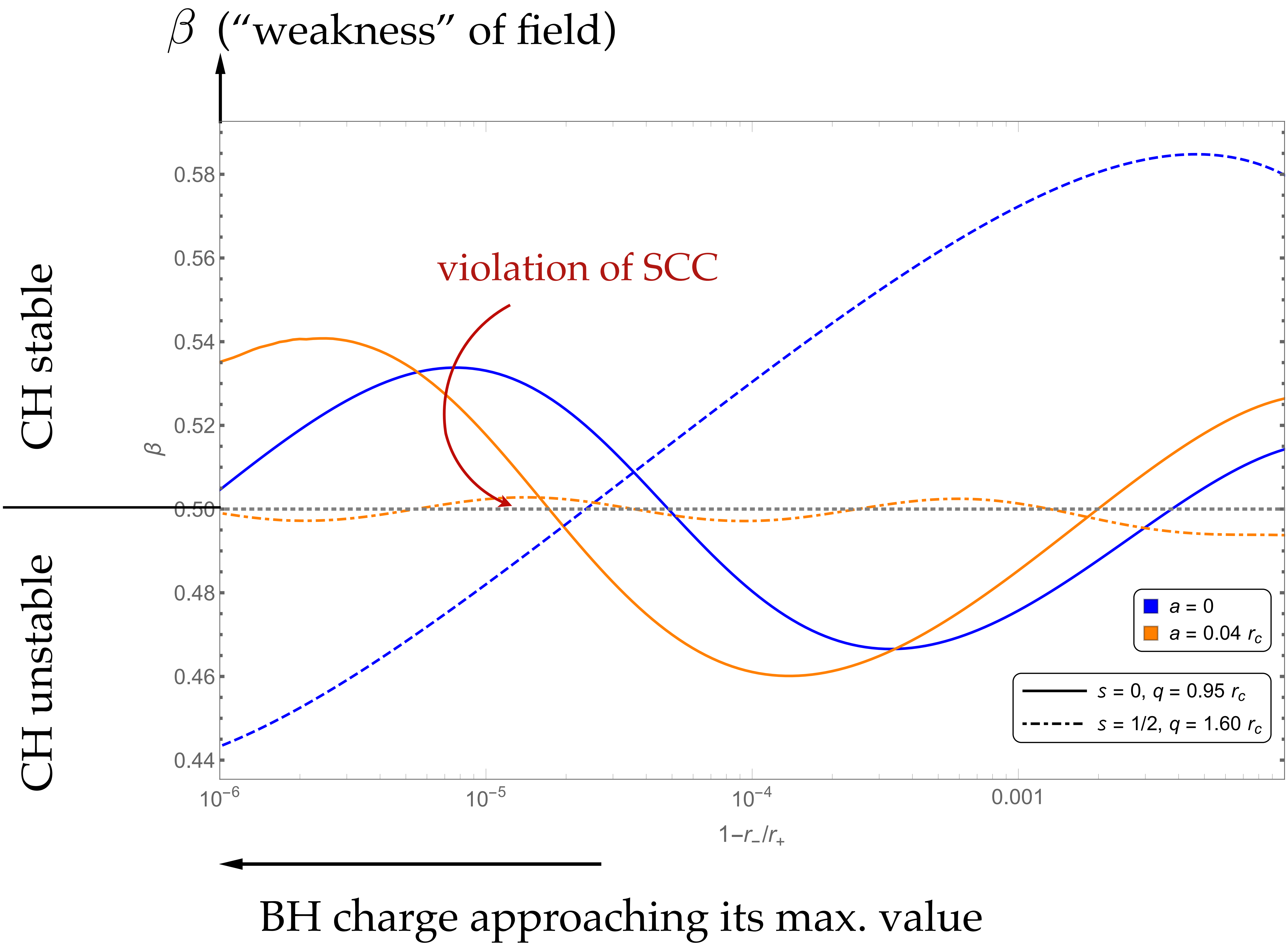
However…

Reissner-Nordstrom-dS: the CH remains regular ($T_{VV} \in L^1_{loc}$) -> region of unpredictability remains -> violation of SCC [Cardoso et al.'18]

😱

Is there still violation if one includes BH *rotation* (ie, Kerr-Newman-dS)?

Also violation of SCC in Kerr-Newman-dS [Casals & Marinho'22]



Questions

What happens when including quantum-backreaction effects:

Is SCC upheld in asymptotically-dS BHs?

What is the irregularity of the CH of Kerr(-dS) due to quantum effects?

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Semiclassical Einstein Eqs.

There exists no theory of Quantum Gravity that is fully satisfactory yet

In its absence, we use semiclassical theory of Quantum Gravity:

$$R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) + \Lambda\, g_{\mu\nu} = 8\pi G \left\langle \hat{T}_{\mu\nu}(g) \right\rangle^{\Psi}_{ren}$$

Renormalized expectation value of the stress-energy tensor (RSET) of the matter field in a state $|\psi\rangle$

Matter fields are quantized but gravitational field is kept classical

This theory is valid in the limit that the length and time scales of the physical processes $\gg$ Planck length and time

$$\left(G\hbar/c^3\right)^{1/2} \sim 10^{-33} cm \quad \text{and} \quad \left(G\hbar/c^5\right)^{1/2} \sim 10^{-44} s$$

It's very hard to solve the above semiclassical eqs. *self-consistently* for the metric g

In practise, one solves the semiclasssical eqs. *perturbatively*:

- first solve Einstein eqs. for a classical *background* ('vacuum') metric g

$$R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) + \Lambda\, g_{\mu\nu} = 0$$

- next place a quantum matter field in some state $|\Psi\rangle$ on background g and find its RSET $\left\langle \hat{T}_{\mu\nu}(g) \right\rangle_{ren}^{\Psi}$

- finally, place this RSET on the rhs of Einstein eqs. and try to solve for the quantum-backreacted metric $g^{(c)}$:

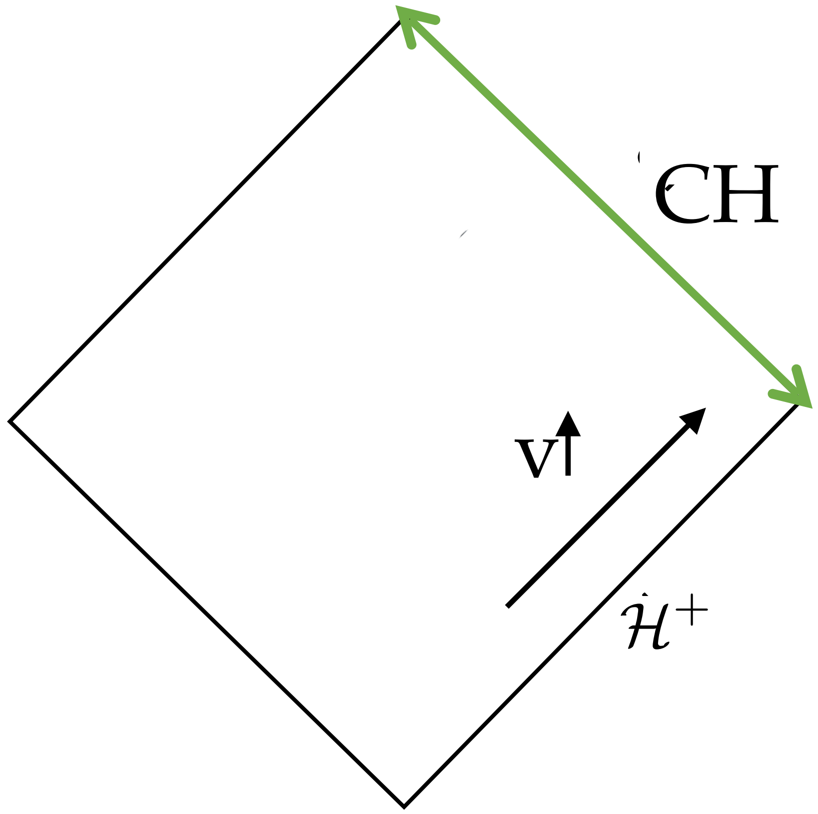
$$R_{\mu\nu}\left(\underset{\substack{\uparrow \\ \textit{quantum-corrected}\ \text{metric}}}{g^{(c)}}\right) - \frac{1}{2} g^{(c)}_{\mu\nu} R\left(g^{(c)}\right) + \Lambda\, g^{(c)}_{\mu\nu} = 8\pi G \left\langle \hat{T}_{\mu\nu}\left(\underset{\substack{\uparrow \\ \textit{background}\ \text{metric}}}{g}\right) \right\rangle_{ren}^{\Psi}$$

Already $\left\langle \hat{T}_{\mu\nu}(g) \right\rangle_{ren}^{\Psi}$ can give us some properties of $g^{(c)}$ even if the latter is not obtained (eg, irregularity in RSET -> curvature sing.?)

Literature results on quantum effects on CH: 4D (spherical)

In 3+1-D spherical symmetry, RSET diverges at the CH

$$\left\langle \hat{T}_{VV} \right\rangle_{\text{ren}} \underset{v \to \infty}{\sim} e^{2\kappa_- v}$$



(*stronger* than the classical $v^{-n} e^{2\kappa_- v}$)

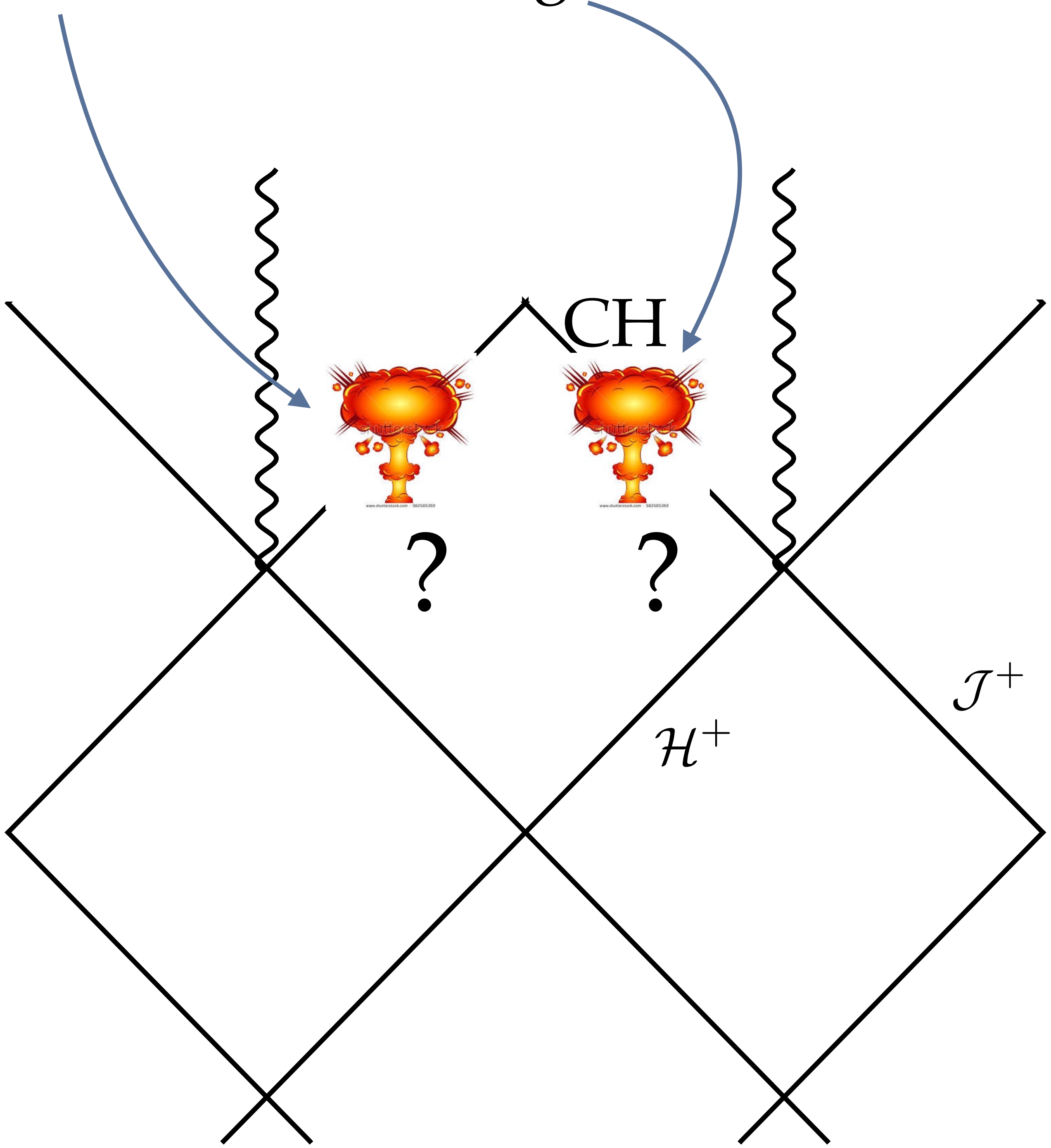
when background g is:

- Reissner-Nordstrom [Zilberman, Levi & Ori'19]

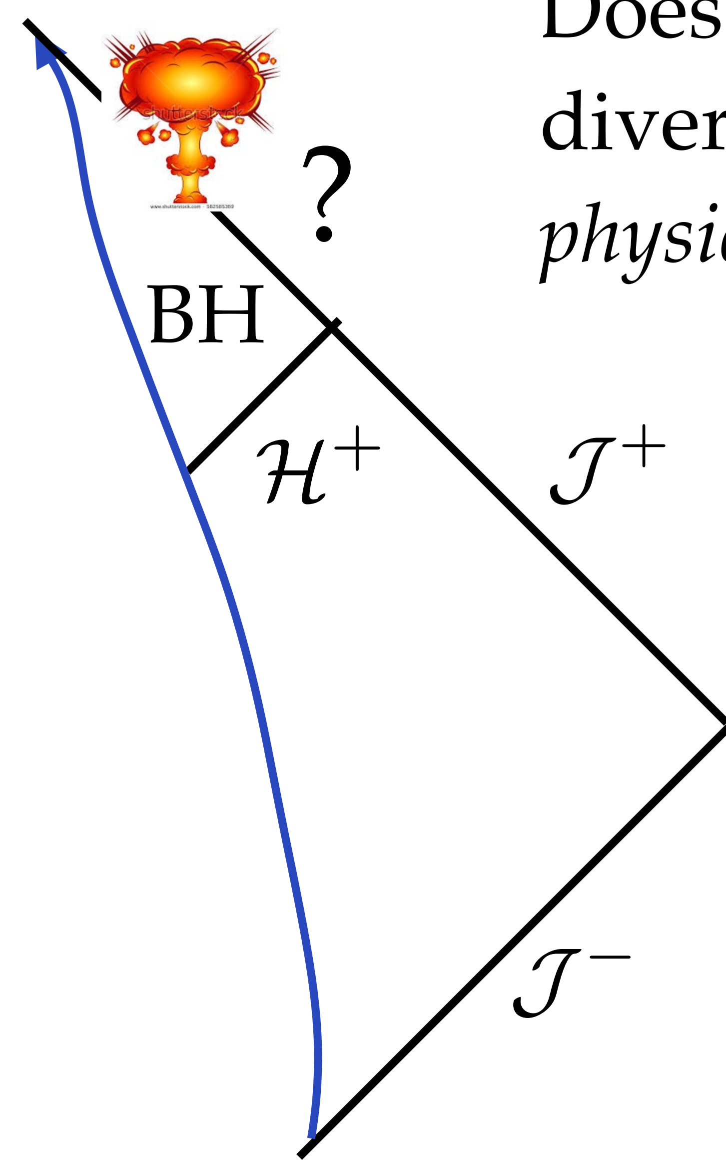
- Reissner-Nordstrom-de Sitter [Hollands,Wald,Zahn'20] -> quantum physics also acts as a strong Cosmic Censor!

Literature results on quantum effects on CH: 4D (Kerr…)

When g is Kerr, there're *indications* that RSET diverges on the left and/or the right CH [Hiscock'80 and Ottewill & Winstanley'00]



Remember that the *right* CH is the physical one:



Does the RSET diverge on the *physical* CH?

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Scalar field in Kerr

Massless scalar field ϕ perturbations of the background Kerr metric g

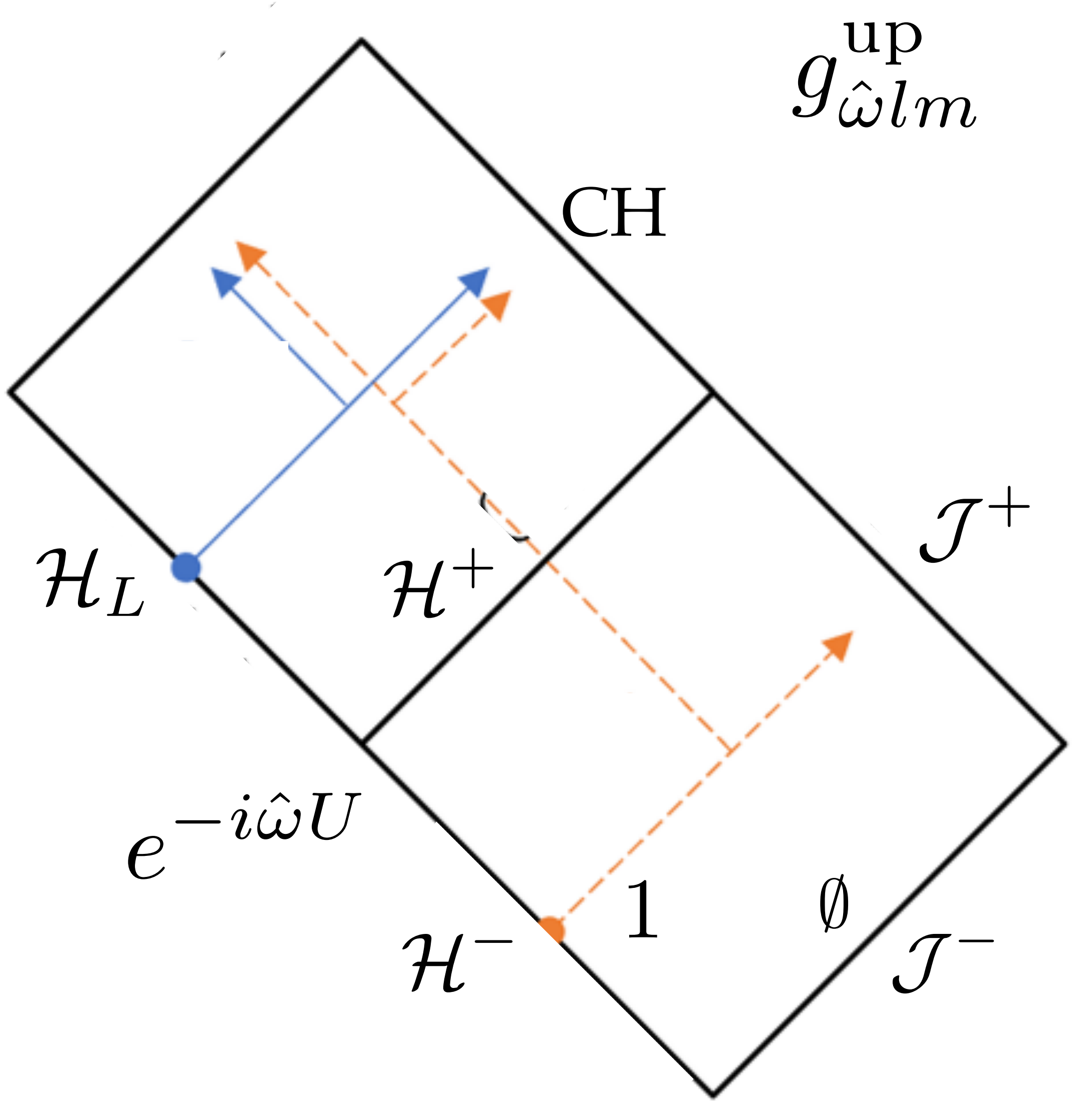
satisfy a wave eq.

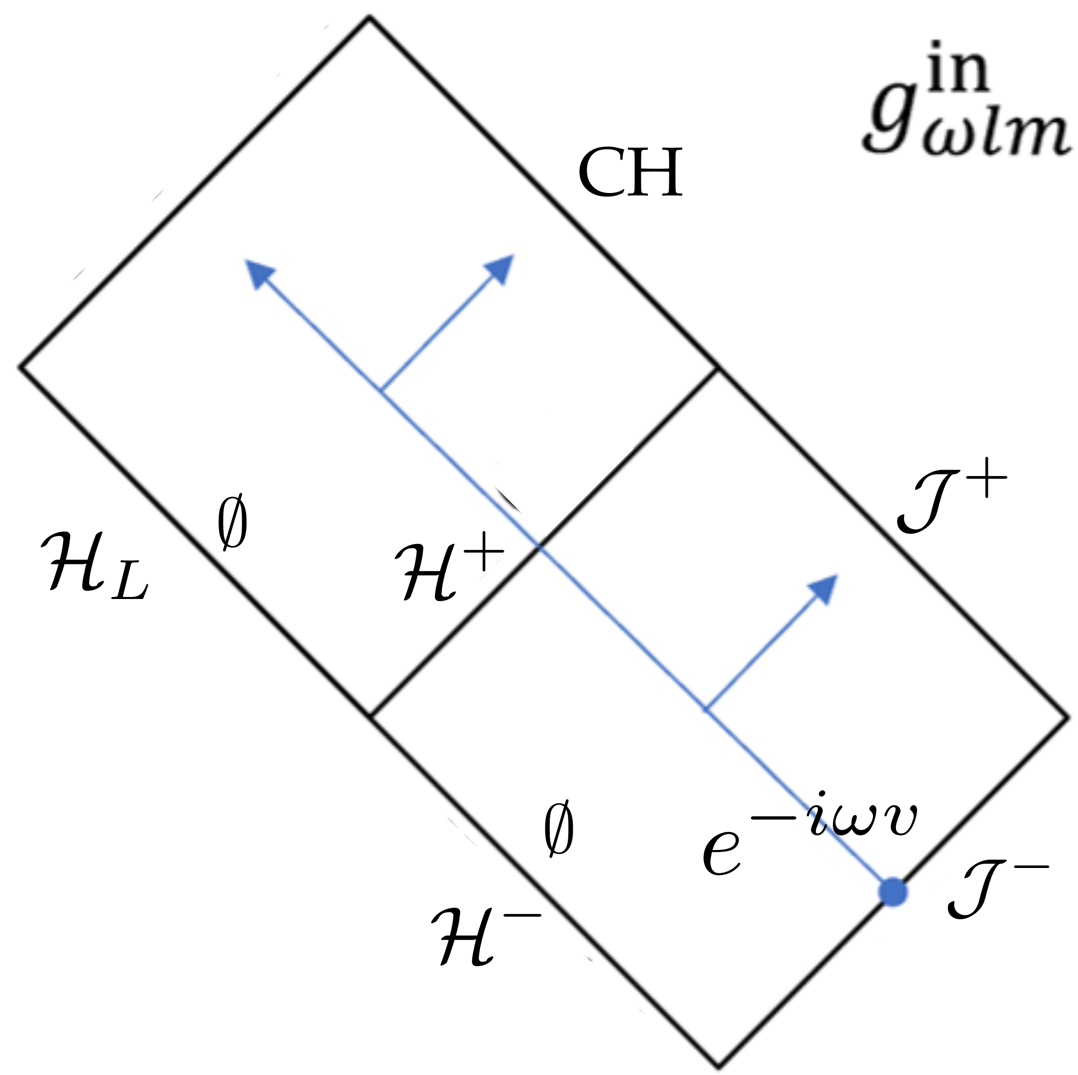
$$\Box\phi(x) = g_{\mu\nu}\nabla^{\mu}\nabla^{\nu}\phi(x) = 0$$

(N.B.: similar wave eq. for fields of other spins and/or other BHs)

We need to quantize the field ($\phi \to \hat{\phi}$) and so choose a quantum state for the field. The state is defined in terms of *positive* frequency modes wrt some "time" coordinate

Unruh Modes





The Unruh modes are positive frequency wrt U on $\mathcal{H}_L \cup \mathcal{H}^-$ and wrt v on $\mathcal{J}^-$, i.e., wrt the affine parameters in the corresponding hypersurfaces

Unruh state

The Unruh state is constructed to model the state of the quantum matter fields around an astrophysical BH, and so evaporating via emission of thermal Hawking radiation at a temperature $T_H = \kappa_+/(2\pi)$ outside BH

Construction: expand the scalar field in terms of the Unruh modes:

$$\hat{\phi}(x) = \sum_{l,m}\left(\sum_{\hat{\omega}>0} \hat{a}^{\mathrm{up}}_{\hat{\omega}lm}\, g^{\mathrm{up}}_{\hat{\omega}lm}(x) + \sum_{\omega>0} \hat{a}^{\mathrm{in}}_{\omega lm}\, g^{\mathrm{in}}_{\omega lm}(x) + \mathrm{h.c.}\right)$$

The Unruh state [Unruh'76] is the quantum state $|U\rangle$ which is annihilated by the corresponding coefficients:

$$\hat{a}^{\mathrm{up}}_{\hat{\omega}lm}\,|U\rangle = 0 = \hat{a}^{\mathrm{in}}_{\omega lm}\,|U\rangle$$

So, anti-commutator: $\left\langle\left\{\hat{\Phi}(x), \hat{\Phi}(x')\right\}\right\rangle^{U} =$

$$\hbar\sum_{l,m}\left(\int_0^\infty \mathrm{d}\hat{\omega}\,\left\{g^{\mathrm{up}}_{\hat{\omega}lm}(x), g^{\mathrm{up}*}_{\hat{\omega}lm}(x')\right\} + \int_0^\infty \mathrm{d}\omega\,\left\{g^{\mathrm{in}}_{\omega lm}(x), g^{\mathrm{in}*}_{\omega lm}(x')\right\}\right)$$

Expectation value of stress-energy tensor

Formal expression for the *bare* (unrenormalized) exp. val. of the stress-energy tensor for the quantum (minimally-coupled) scalar field $\hat{\phi}$:

$$\left\langle \hat{T}_{\alpha\beta} \right\rangle_{\rm ren} = \langle \overline{T}_{\alpha\beta} \rangle_{\rm ren} - \frac{1}{2} g_{\alpha\beta} \left\langle \overline{T}^{\mu}{}_{\mu} \right\rangle_{\rm ren}$$

$$\langle \overline{T}_{\alpha\beta} \rangle_{\rm ren} (x) \equiv \frac{1}{2} \lim_{x' \to x} \left(\left\langle \{ \hat{\phi}(x), \hat{\phi}(x') \} \right\rangle_{,\alpha\beta'} - (\text{renormalization terms}) \right)$$

Quantum backreaction on CH

In the case of spherical symmetry (Zilberman, Levi & Ori'19):

Metric ansatz: $$ds^2 = -e^{\sigma(u,v)}dudv + r^2(u,v)d\Omega^2$$

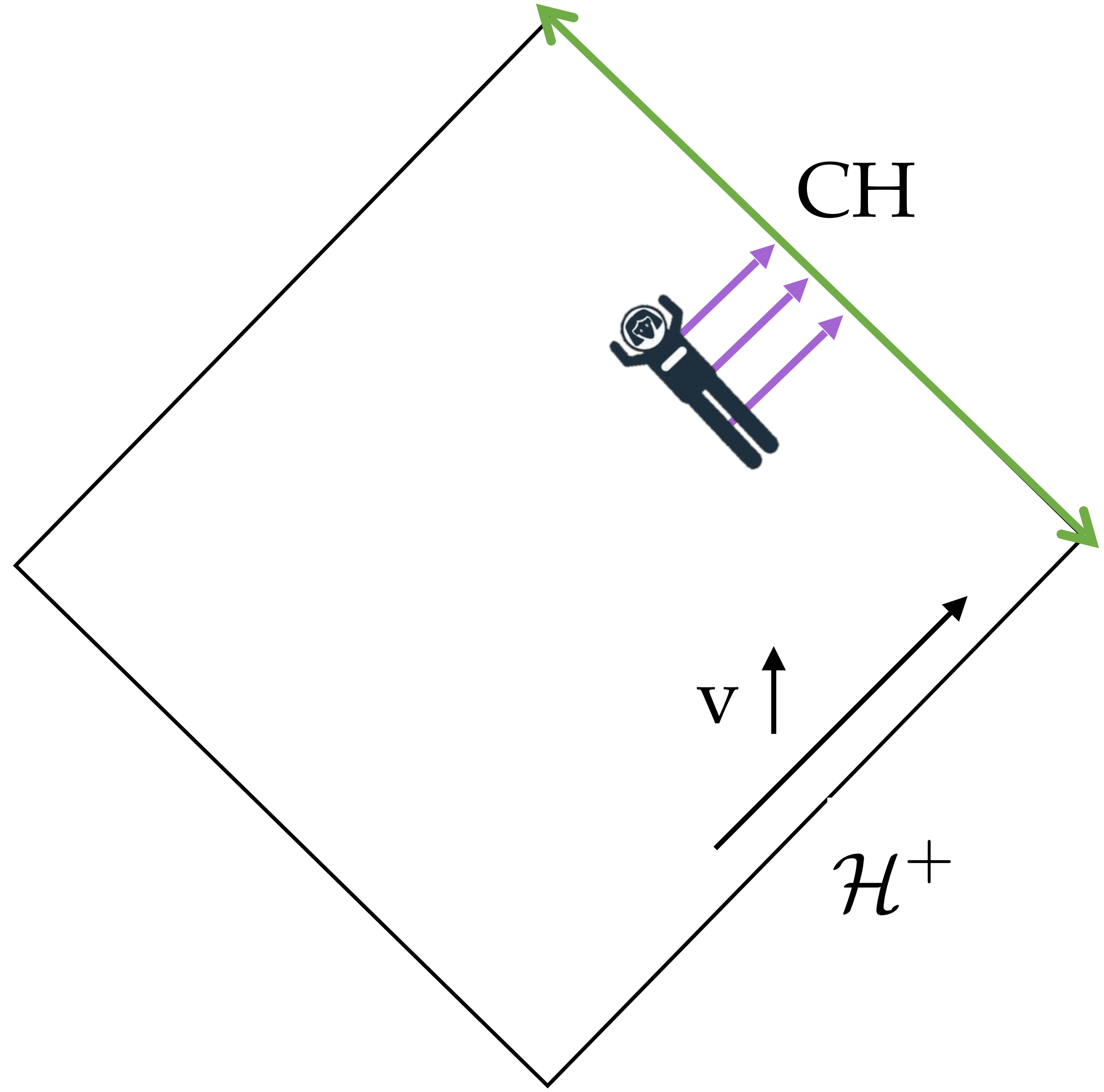
Semiclassical EFE: $$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = 8\pi \left\langle \hat{T}_{\mu\nu}(g) \right\rangle_{ren}^{U}$$

$$\rightarrow \quad r_{,yy} - r_{,y}\sigma_{,y} = -4\pi r \left\langle \hat{T}_{yy}(g) \right\rangle_{ren}^{U} \qquad y = u, v$$

Weak backreaction domain (not too close to evaporation timescale nor to CH):

$r, \sigma_y, \left\langle \hat{T}_{yy}(g) \right\rangle_{ren}^{U}$ approximated by their values in RN background on CH ($v \to \infty$)

$$\frac{\partial r}{\partial v} \propto - \left\langle \hat{T}^{-}_{vv} \right\rangle^{U}_{\mathrm{ren}}$$



Regularity of CH and tidal deformation:

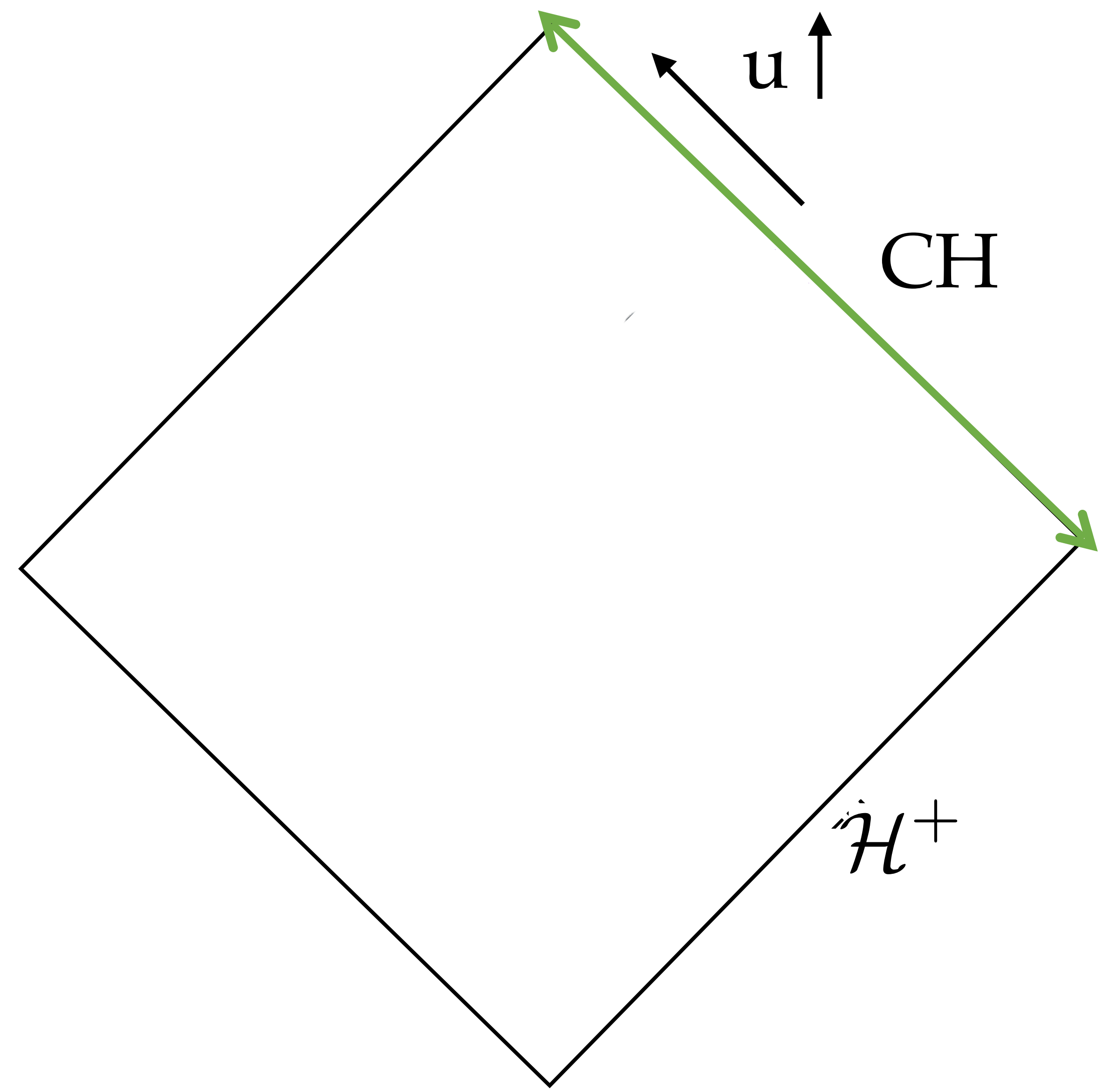
If $\left\langle \hat{T}^{-}_{vv} \right\rangle_{\mathrm{ren}} \neq 0$, then $\left\langle \hat{T}^{-}_{VV} \right\rangle^{U}_{\mathrm{ren}} \propto e^{2\kappa_{-} v} \left\langle \hat{T}^{-}_{vv} \right\rangle^{U}_{\mathrm{ren}}$ diverges as $v \to \infty$

-> curvature singularity & tidal deformation of observer crossing the CH:

$\left\langle \hat{T}^{-}_{vv} \right\rangle^{U}_{\mathrm{ren}} > 0 \longrightarrow$ abrupt contraction of observer

$\left\langle \hat{T}^{-}_{vv} \right\rangle^{U}_{\mathrm{ren}} < 0 \longrightarrow$ abrupt expansion of observer on CH

$$\frac{\partial r}{\partial u} \propto -\left\langle \hat{T}^{-}_{uu} \right\rangle^{U}_{\mathrm{ren}}$$



Deformation of CH:

$\left\langle \hat{T}^{-}_{uu} \right\rangle^{U}_{\mathrm{ren}} > 0$ $\longrightarrow$ contraction of CH

$\left\langle \hat{T}^{-}_{uu} \right\rangle^{U}_{\mathrm{ren}} < 0$ $\longrightarrow$ expansion of CH

Results for quantum effects inside a Kerr BH

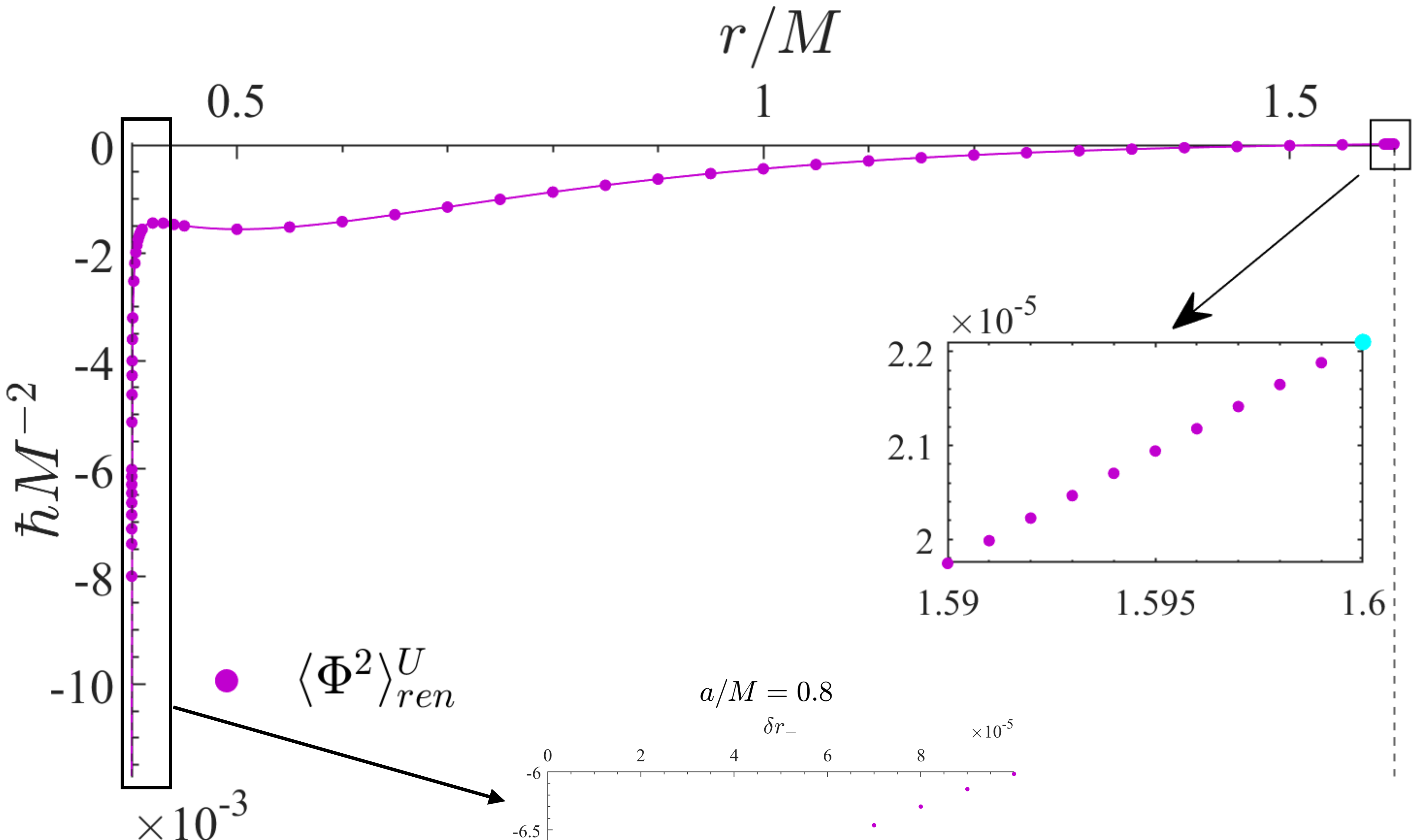
Together with *N. Zilberman, A. Ori & A. Ottewill* [PRL'22 + PRD'223

+ In preparation] we calculated $\left\langle \hat{\phi}^2 \right\rangle_{\mathrm{ren}}^{U}$ and the fluxes $\left\langle \hat{T}_{vv} \right\rangle_{\mathrm{ren}}^{U}$

and $\left\langle \hat{T}_{uu} \right\rangle_{\mathrm{ren}}^{U}$ on the CH $(r = r_{-})$ and everywhere inside

the EH $(r \in (r_{-}, r_{+}))$

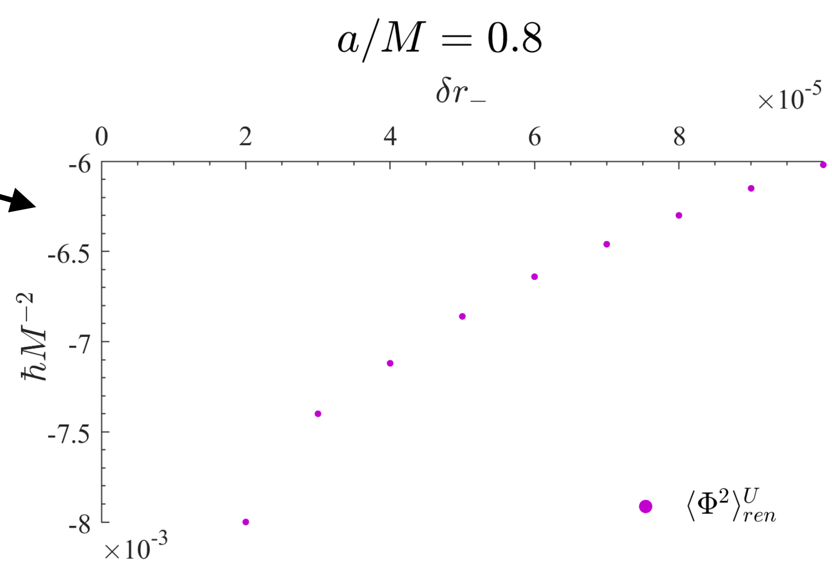
Results *off* the CH

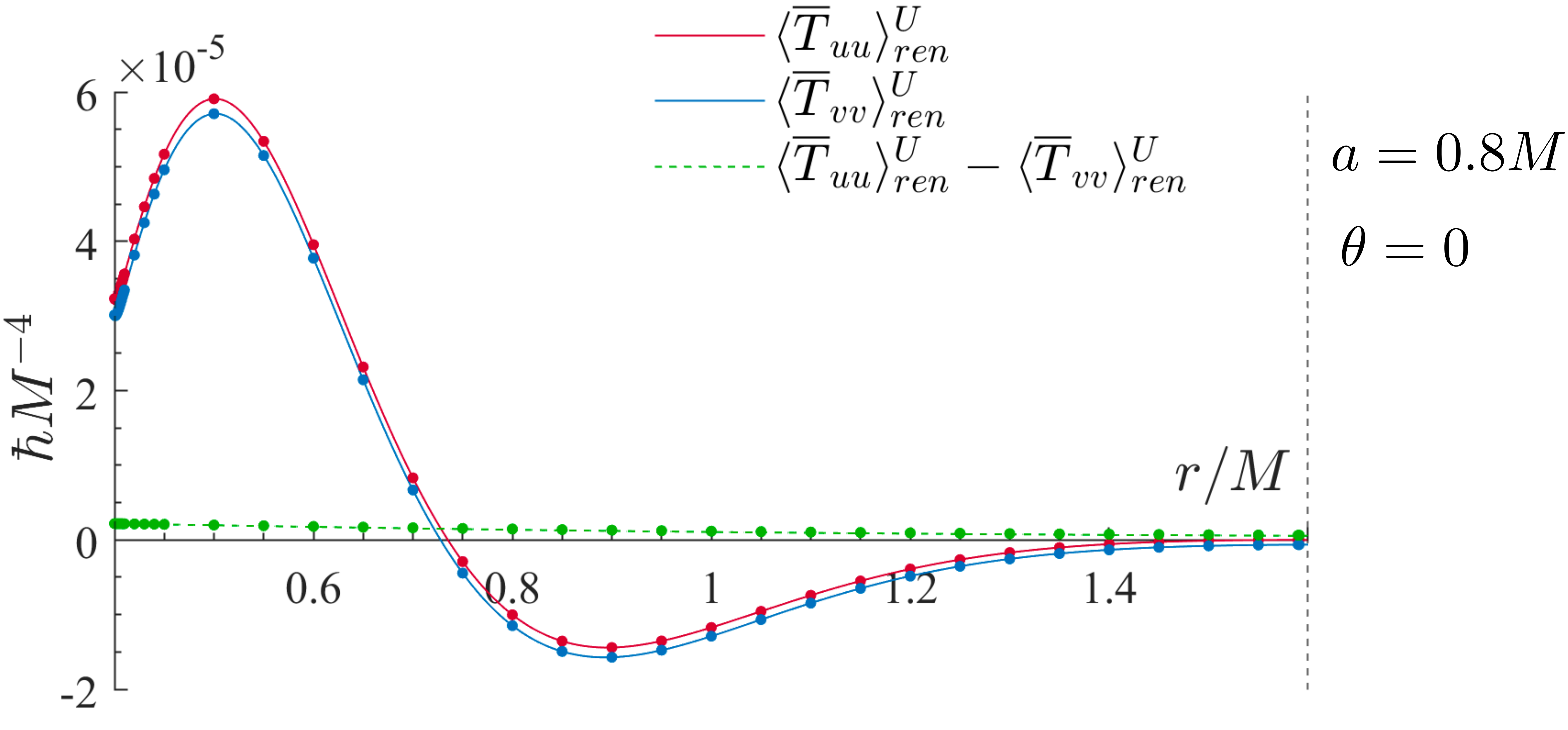


$a = 0.8M$

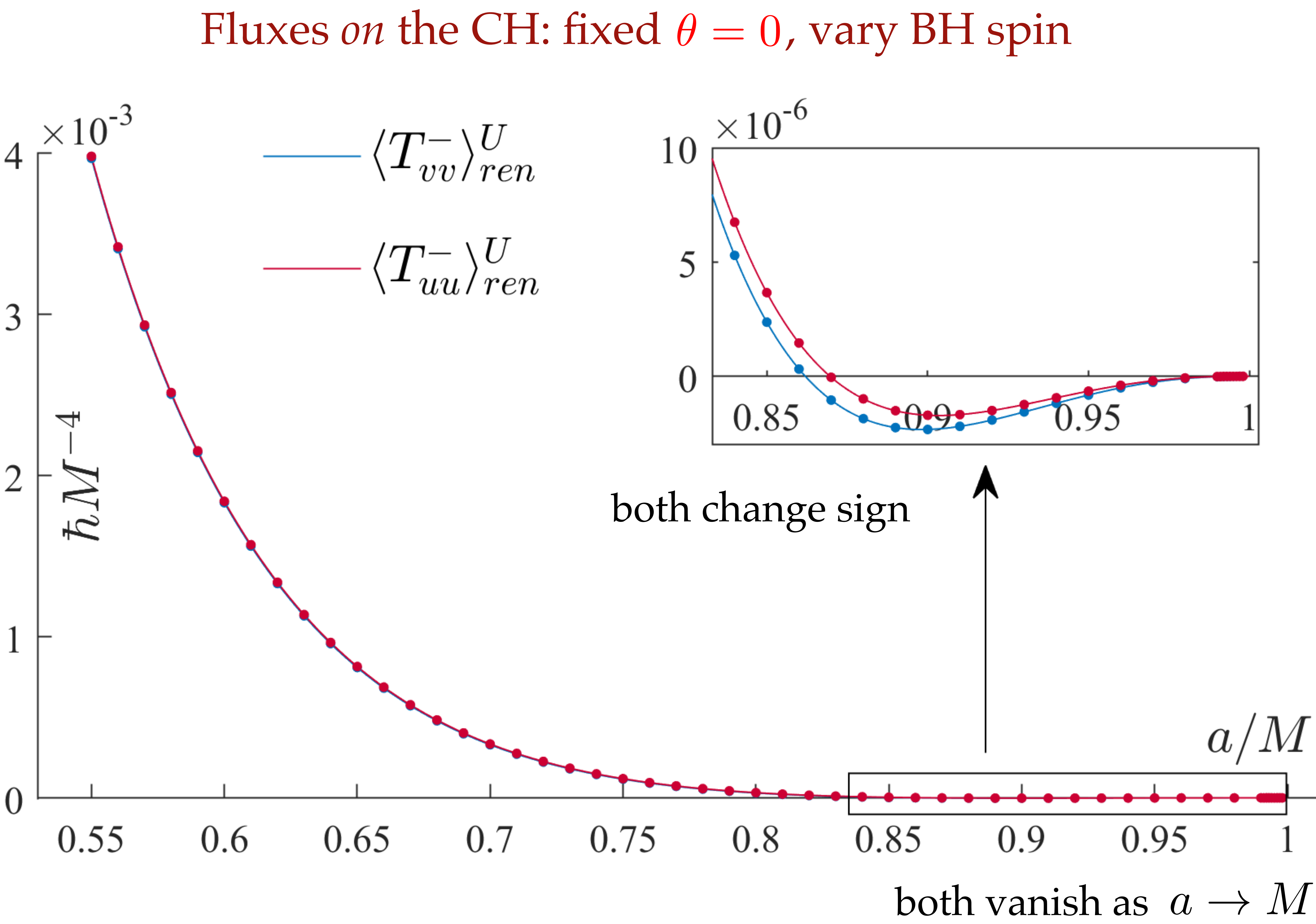
$\theta = 0$

Finite value at CH:





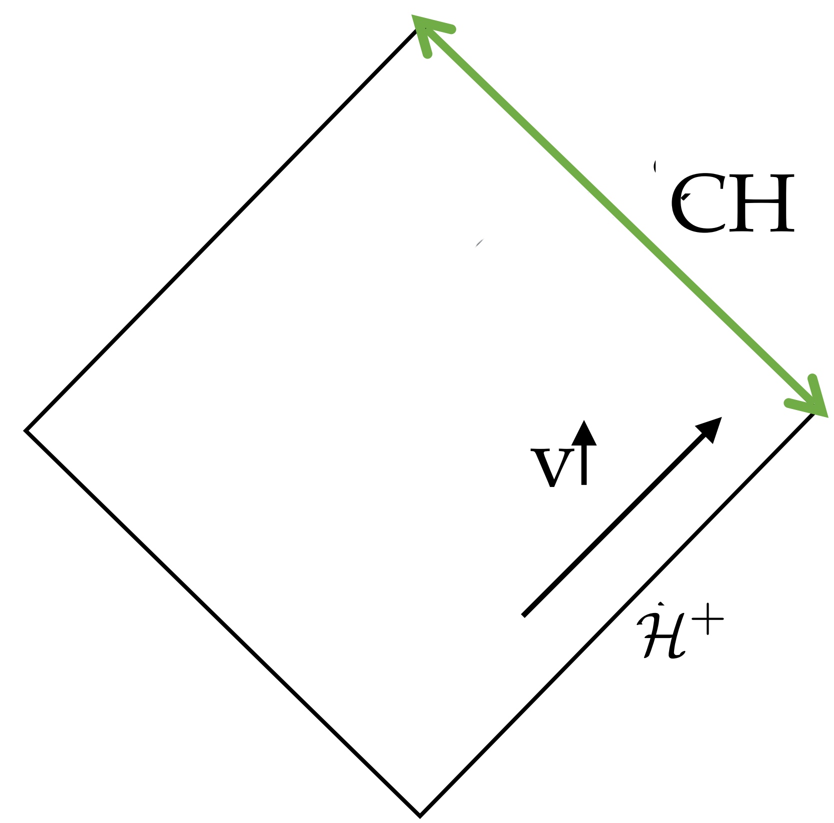
$\langle \overline{T}_{uu} \rangle^U_{ren}$
$\langle \overline{T}_{vv} \rangle^U_{ren}$
$\langle \overline{T}_{uu} \rangle^U_{ren} - \langle \overline{T}_{vv} \rangle^U_{ren}$
$\times 10^{-5}$
6
4
2
0
-2
$\hbar M^{-4}$
0.6
0.8
1
1.2
1.4
r/M
$a = 0.8M$
$\theta = 0$


Fluxes *on* the CH: fixed $\theta = 0$, vary BH spin
$\times 10^{-3}$
$\langle T^{-}_{vv} \rangle^{U}_{ren}$
$\langle T^{-}_{uu} \rangle^{U}_{ren}$
$\hbar M^{-4}$
$\times 10^{-6}$
both change sign
a/M
both vanish as $a \to M$
0
1
2
3
4
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
10
5
0
0.85
0.9
0.95
1

$$\left\langle \hat{T}^{-}_{vv} \right\rangle^{U}_{\mathrm{ren}} \neq 0 \longrightarrow \left\langle \hat{T}^{-}_{VV} \right\rangle^{U}_{\mathrm{ren}} \propto e^{2\kappa_- v} \left\langle \hat{T}^{-}_{vv} \right\rangle^{U}_{\mathrm{ren}} \sim e^{2\kappa_- v}$$

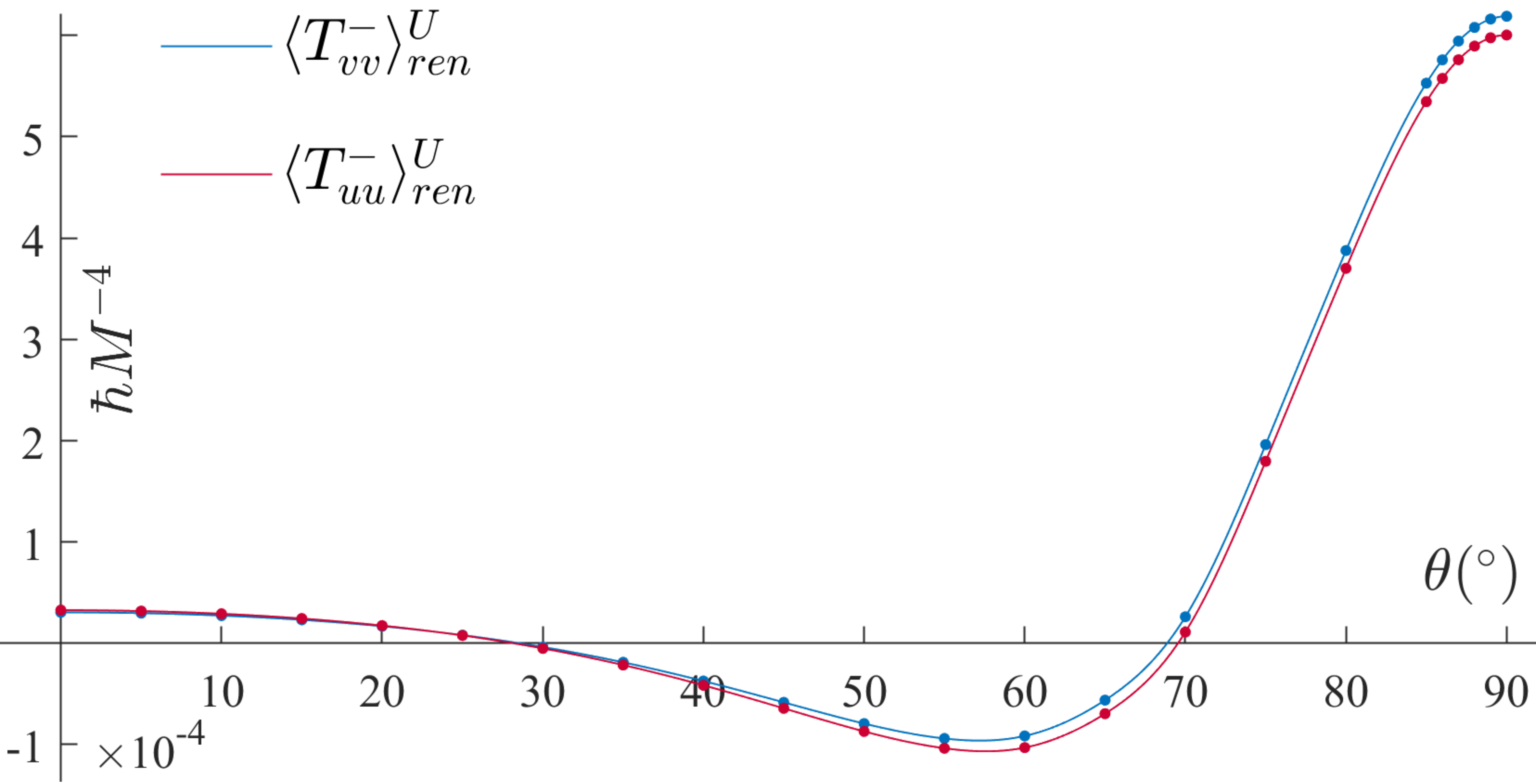
$$v \to \infty$$

(*stronger* than the classical $v^{-n} e^{2\kappa_- v}$)



We found the *dominant* divergence on CH of astrophysical BHs (which is due to quantum backreaction)

Fluxes *on* the CH: fixed $a = 0.8M$, vary θ



NB: Integral of $\left\langle \hat{T}^{-}_{uu} \right\rangle^{U}_{\text{ren}}$ over angle is *positive*

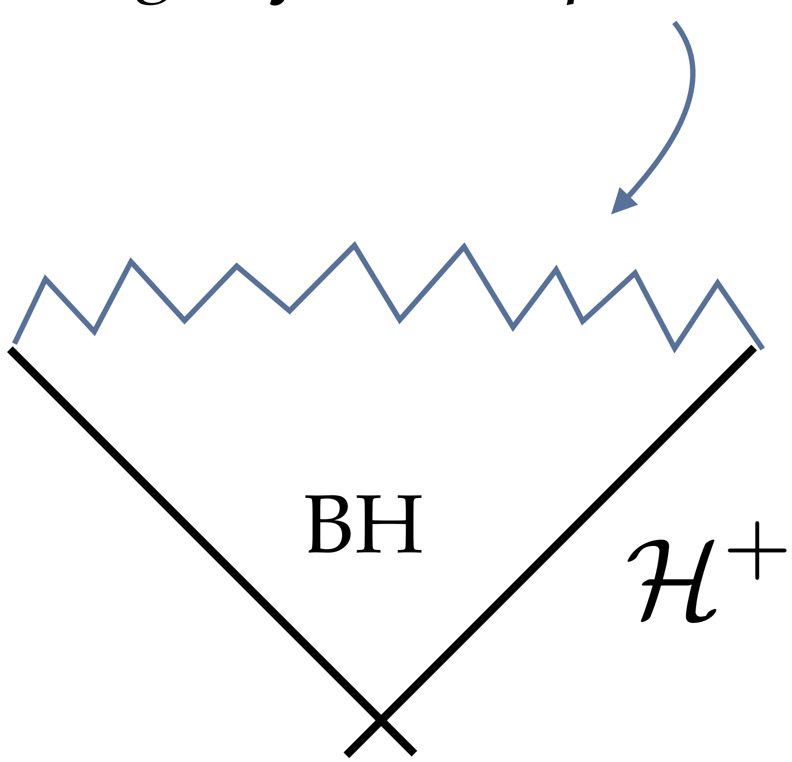
Tentative backreaction

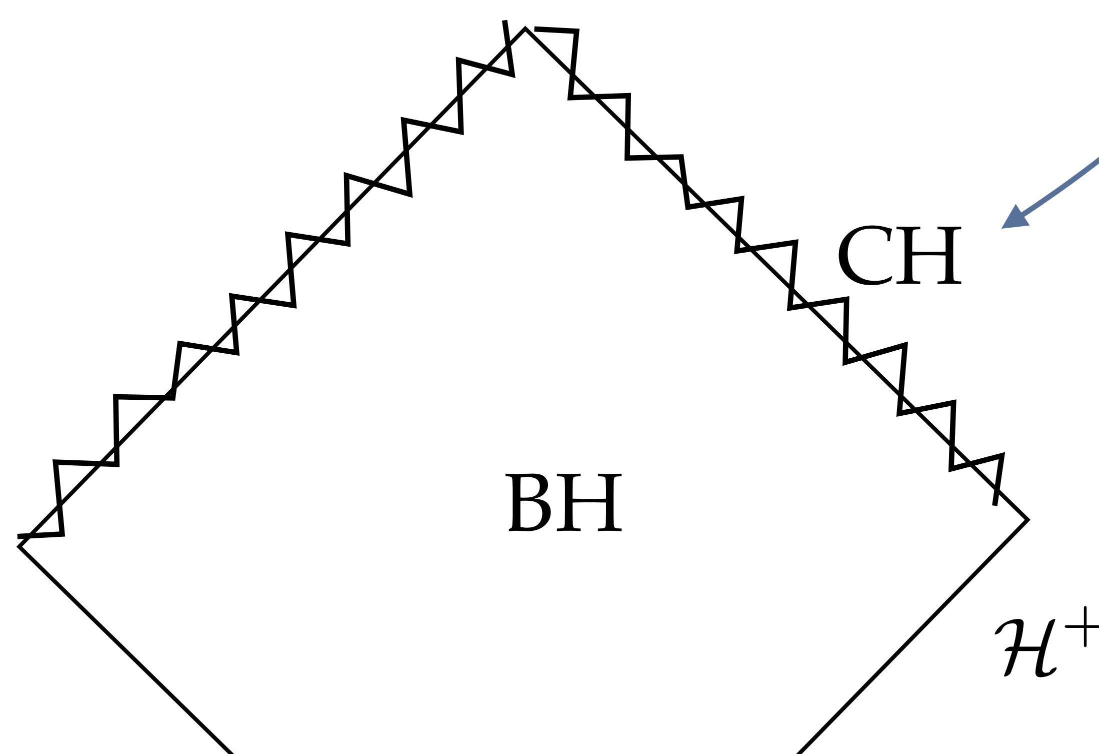
For future investigation, so *tentatively*:

Sign of $\left\langle \hat{T}^{-}_{uu} \right\rangle^{U}_{\mathrm{ren}}$ averaged over angle is expected to determine whether the sing. on the CH is spacelike (if +ive) or null (if -ive)

Our results show

that *might* yield a *spacelike* sing. , as opposed to the classical case (null)





Results for quantum effects inside a Kerr-*dS* BH

Together with C. *Klein, M. Soltani & S. Hollands* [In preparation]

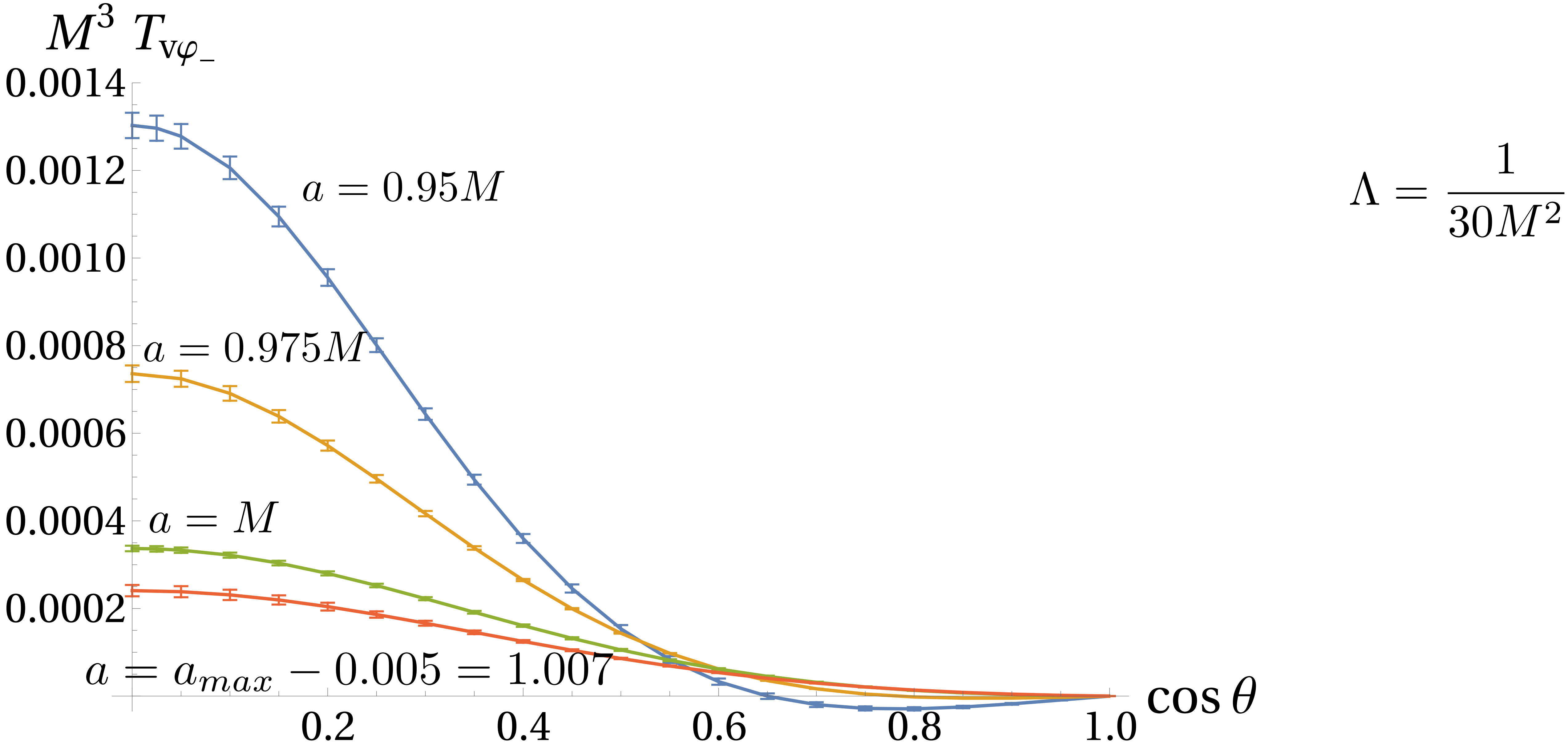
Angular momentum (Komar) of a sphere as it approaches the CH ($v \to \infty$) of Kerr-dS diverges as

$$v \cdot \left(\text{angle average of} \left\langle \hat{T}^{-}_{v\varphi_-} \right\rangle^{U}_{\text{ren}} \right)$$

ang. mom. current at CH

Its sign might determine if the ``ang. mom. of CH" grows or diminishes

Quantum angular momentum current of CH



The angle-average for all curves is *positive* (subject to double-checking sign!) -> *increase* in “ang. mom. of CH” -> what does it involve for backreaction?

1. Cauchy horizon

2. Classical perturbations of CH

3. Semiclassical gravity

4. Semiclassical effects on CH of Kerr

5. Conclusion

Conclusions

Irregularity of CH / predictability of GR/ SCC is hypothesis to be verified

Quantum effects on the CH are typically *stronger* than classical effects:

In de Sitter BHs, CH seems to remain regular enough under classical perturbations but quantum effects act as a strong censor for Reissner-Nordstrom-dS

In Kerr(-dS): RSET diverges (dominant over classical); angle-average of fluxes are +ve -> slike sing. (as opposed to null under classical perts.); angle-average of ang. mom. current *increases* (?) ang. mom. of CH…

To do: extend analysis beyond the weak backreaction domain

Thank you!