# Modified Teukolsky equation for spectral shifts

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Nonlinear Aspects of General Relativity
Princeton
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(Based on arXiv:2206.10653 with Asad Hussain)

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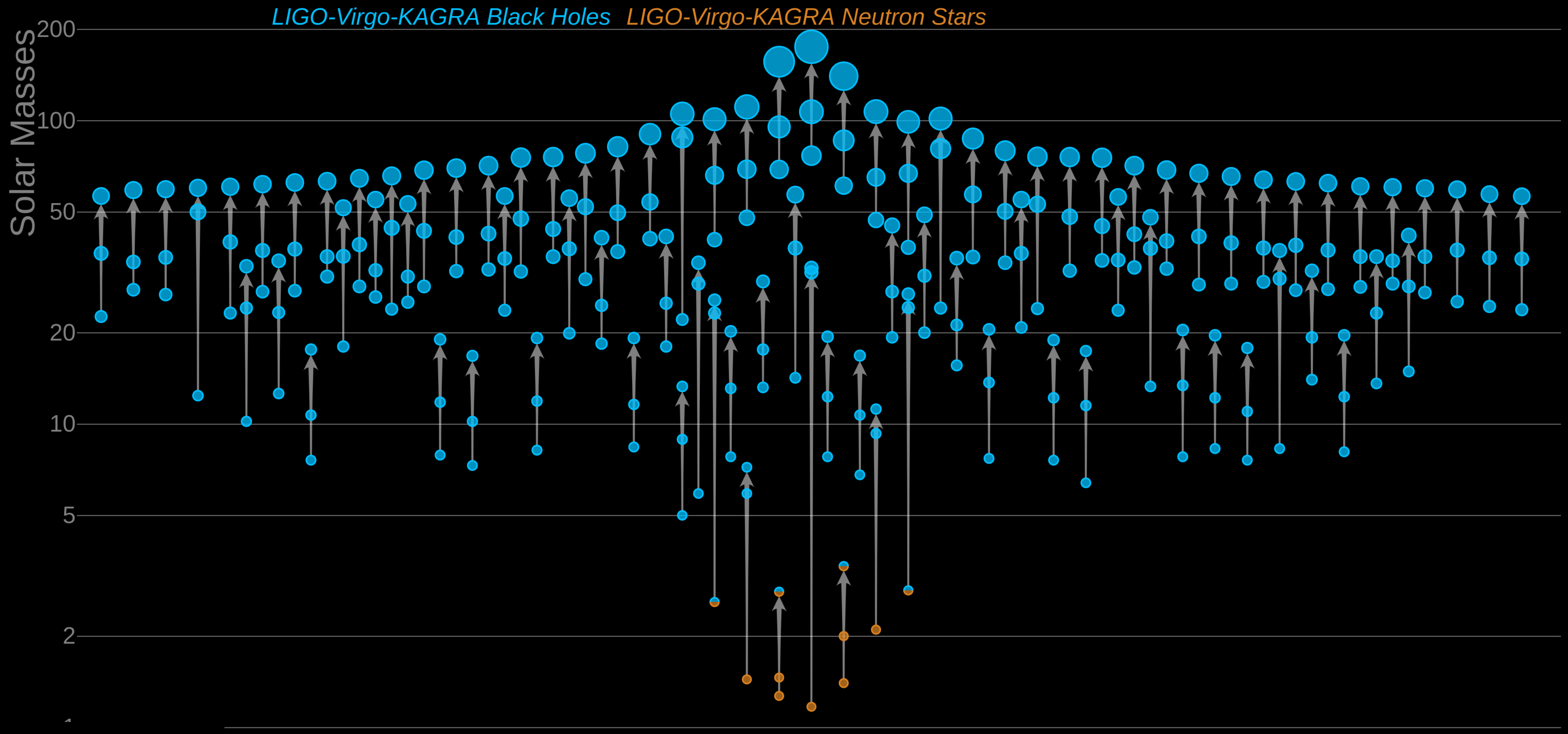
# Motivation

# Masses in the Stellar Graveyard

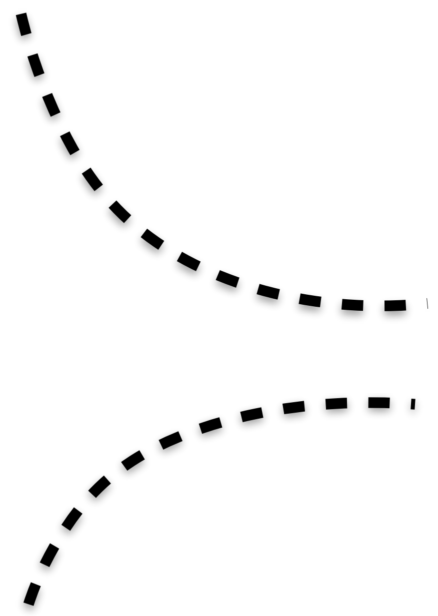



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

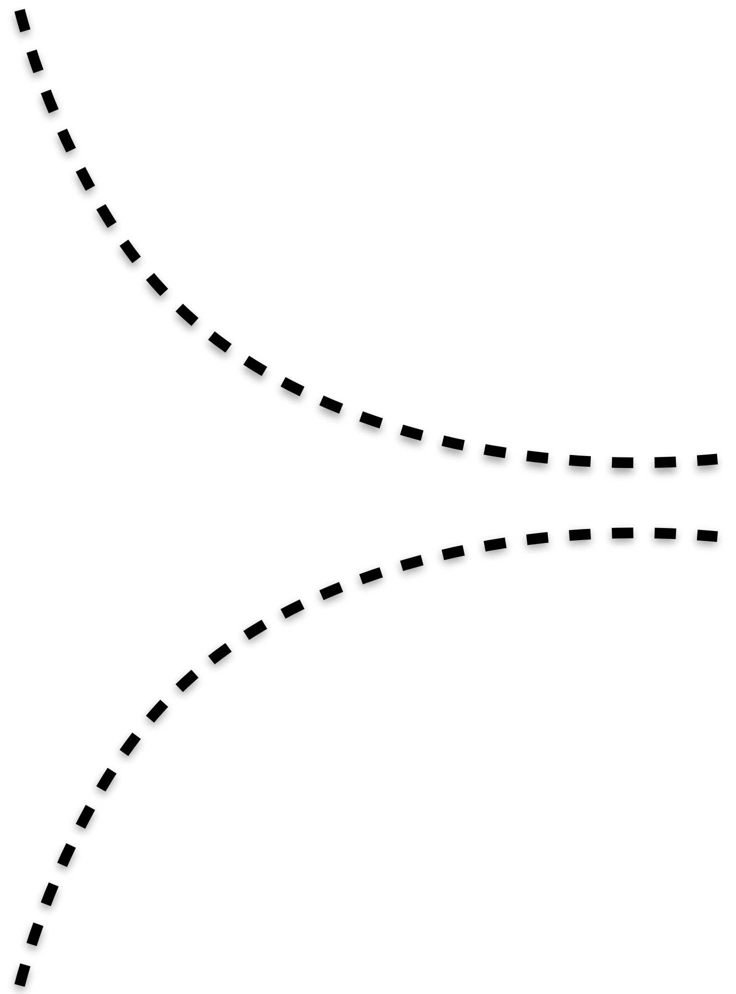
LVK arXiv:2111.03606

# GWs from binary black holes

**GW150914**



**GW190521**



LVC PRL 116, 061102 (2016)

LVC PRL 125, 101102 (2020)

# Ringdown in binary black holes

LCV PRL 116, 221101 (2016)

LVC ApJL 900, L13 (2020)

# Black hole spectroscopy

Berti,Cardoso, Will (2006)

- Spectra determined by mass and spin
- Mass sets overall frequency scale

$$Q = \omega\tau/2$$

- Low quality oscillator: hard to measure ringdown
- One mode: mass and spin
- Two modes: clean test of Kerr spacetime

$\chi$

# Multiple modes in ringdown

**GW150914**

**GW190521**

Isi, Gielser+ PRL 123, 111102 (2019)
c.f. Cotesta+ PRL 129, 111102 (2022)

Capano, Cabero+ arXiv:2105.05238
c.f. LVK PRD 103, 122022 (2021)

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# Constraining deviations

- Primarily null tests

- Each event weakly constraining
- Combine multiple constraints (AZ, Haster, Chatziioannou 2019)
  - Population model
  - Specific theory



Isi, Chatziioannou, Farr, arXiv:1904.08011

# Ringdown tests from O3a

Full waveform, no overtones

Ringdown only

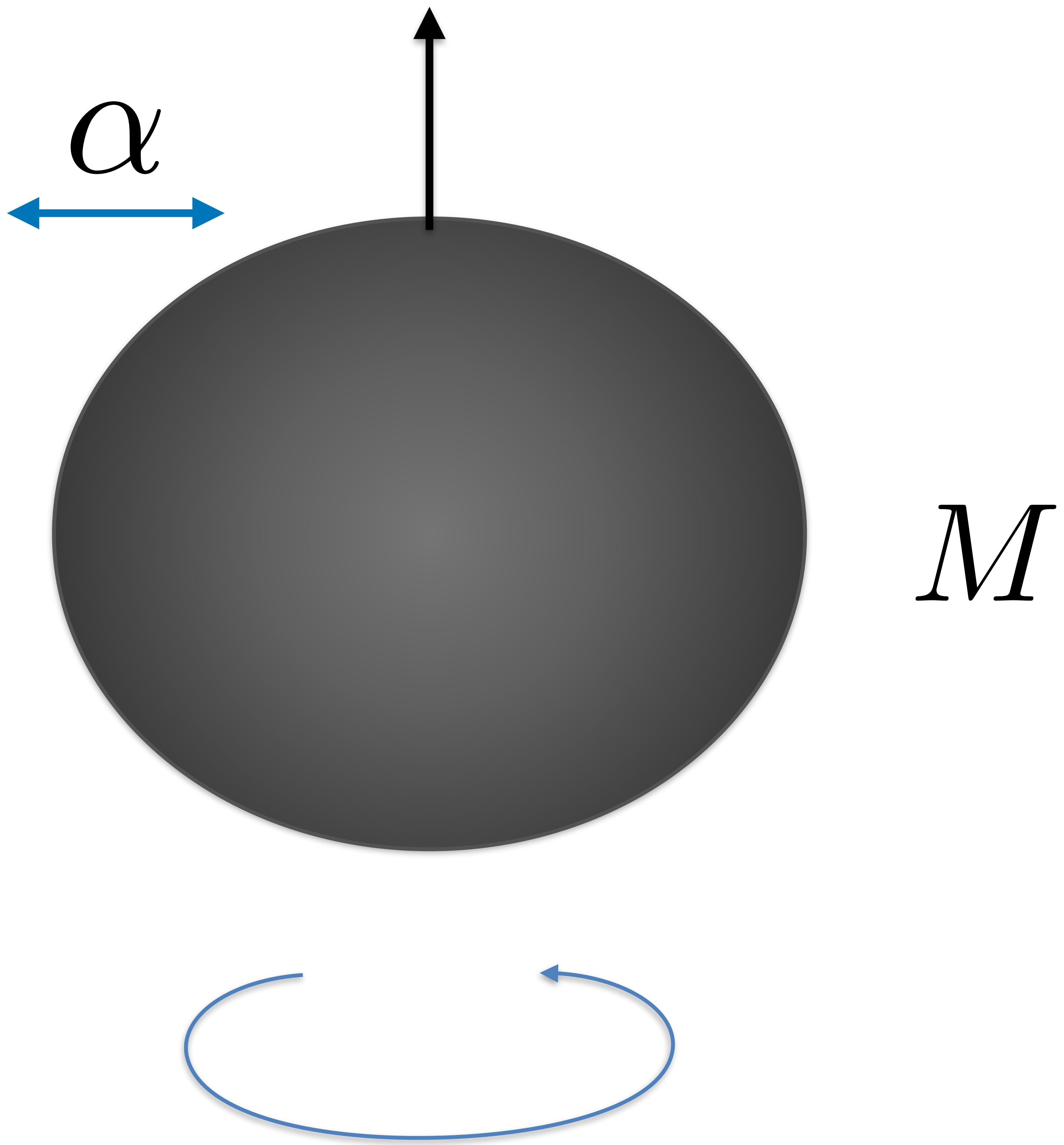
**PyRING**

**pSEOBNRv4HM**

LVK arXiv:2112.06861

# Towards precision tests

- Test specific theories
  - Constraints mapped to theory params
  - Incorporate higher harmonics and overtones
- Why ringdown? Seems tractable, needed anyway
- Much work on QNMs beyond-GR, expansions in small spin, e.g.
  - McManus+ arXiv:1906.05155
  - Pierini & Gualtieri arXiv:2207.11267
  - Cano, Fransen, Hertog arXiv:2005.03671
- But merged black holes have

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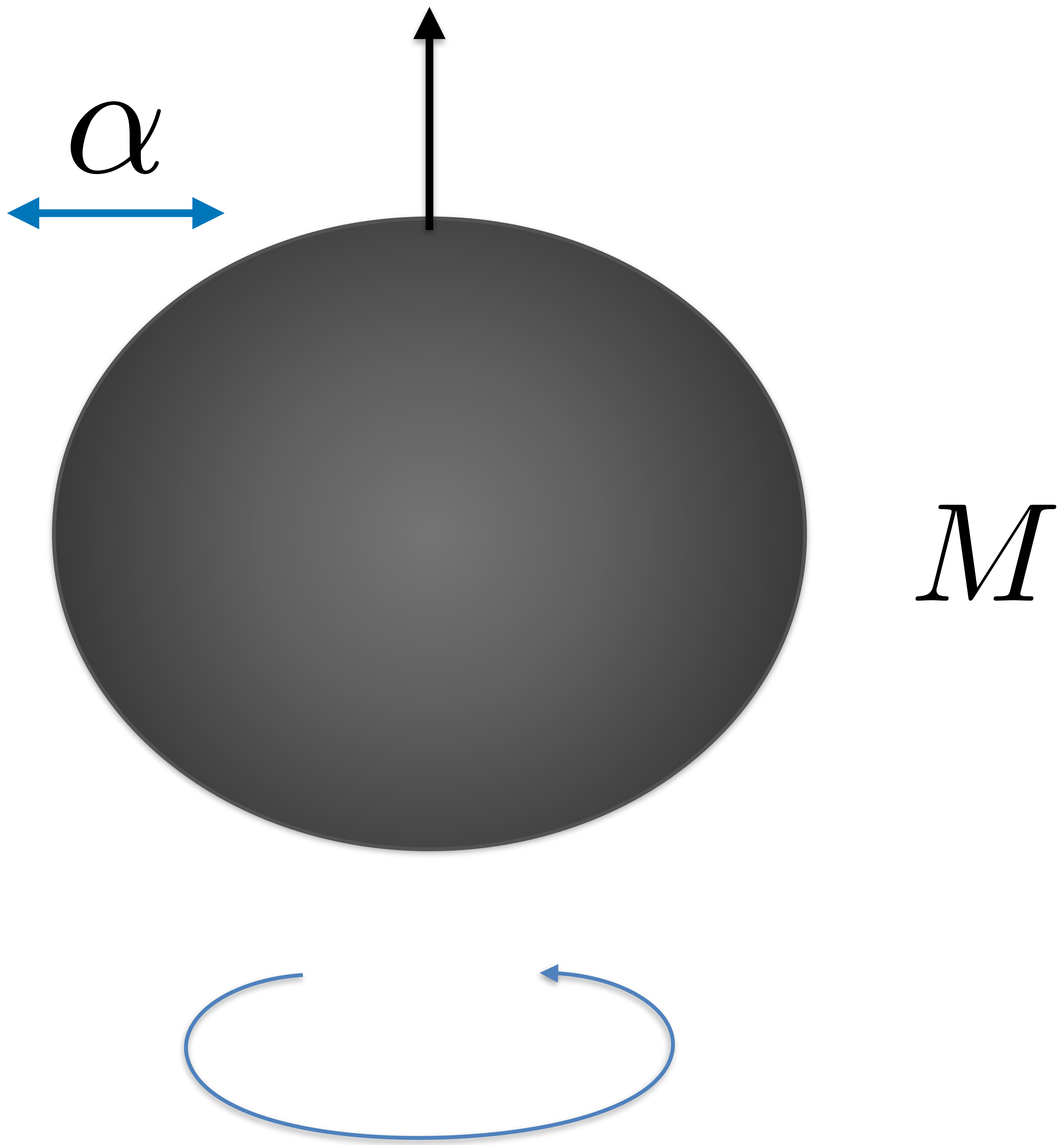
# Towards precision tests

- Need a method that handles high spins
- Strategy: derive master wave equation in theories beyond-GR
- Use eigenvalue perturbation theory to compute ringdown spectrum
- Much recent work, e.g.
  - Cano + (2023a,b)
  - Li+ arXiv:2310.YYYYY



# Ringdown in Kerr

# Gravitational perts for Kerr

- Scalar wave equation separates, metric perts don't separate or decouple

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- Teukolsky (1973): Use Newman-Penrose eqns to decouple scalar quantites

$\longrightarrow$ $\longrightarrow$

# Gravitational perts for Kerr

- Master eqn separates (Teukolsky 1973):

# Gravitational perts for Kerr

- Master eqn separates (Teukolsky 1973):

- Operator picture (Wald 1978):

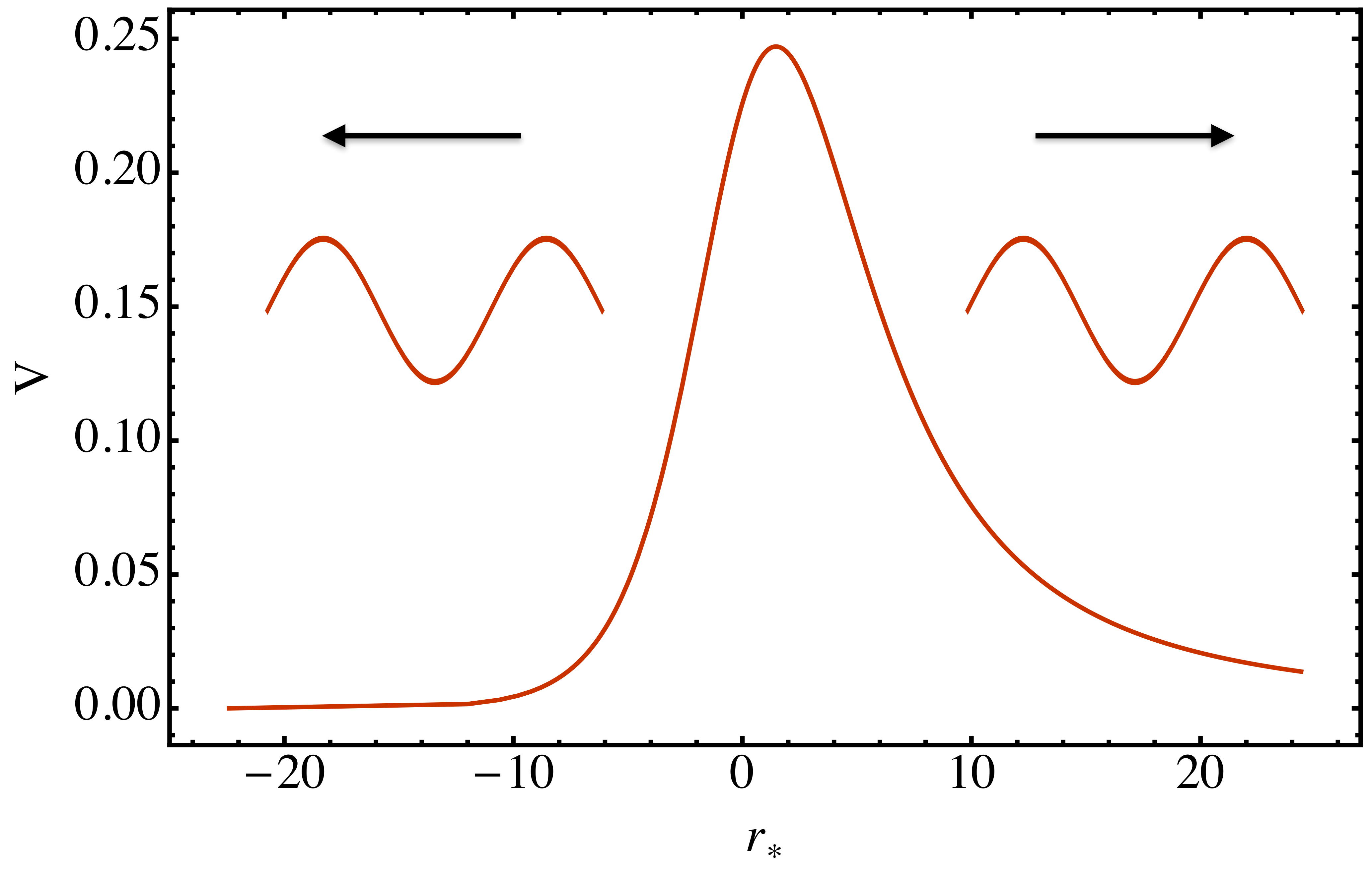
# Gravitational perts for Kerr

- Master eqn separates (Teukolsky 1973):

- Operator picture (Wald 1978):

- Metric can be reconstructed (in special gauges)

# Quasinormal modes



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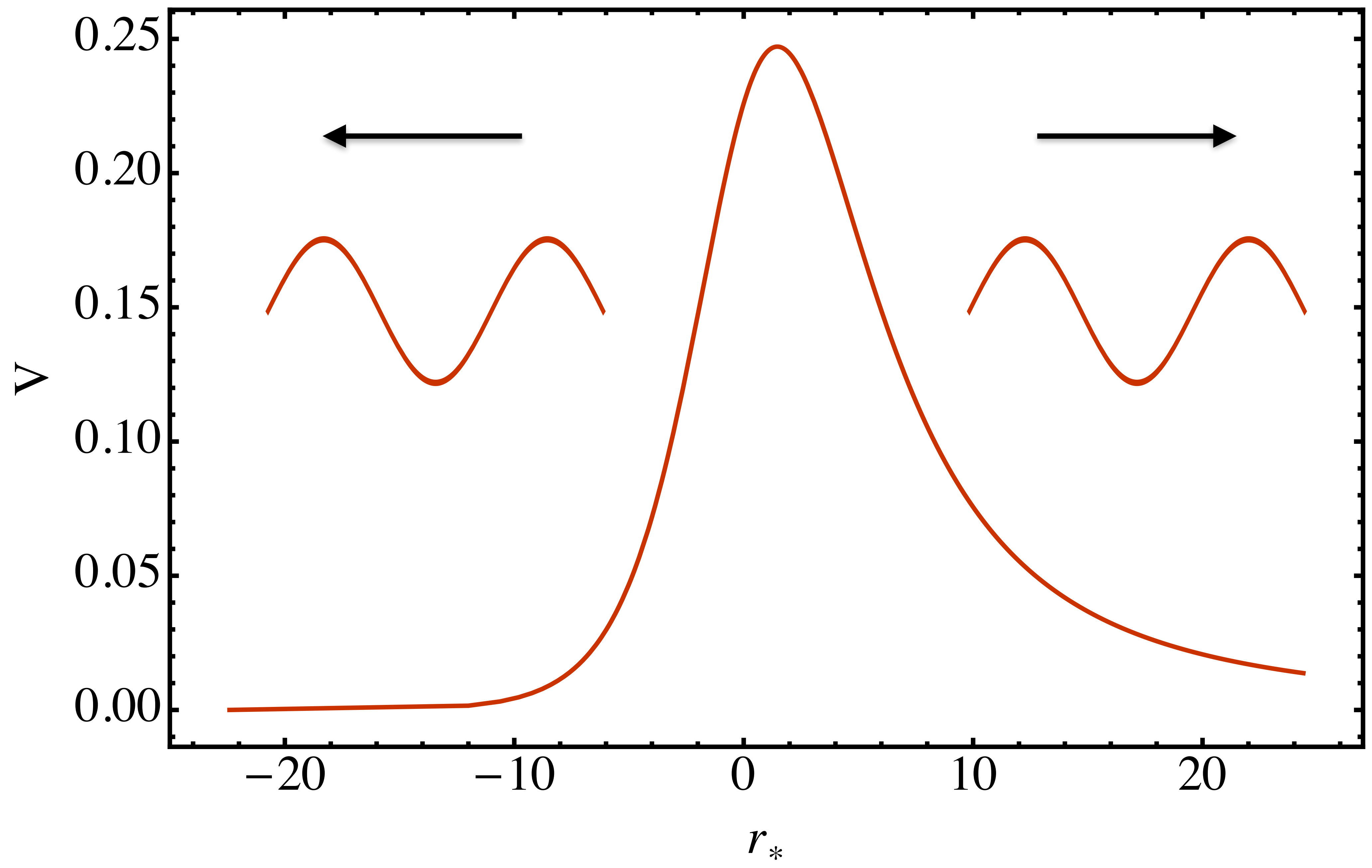


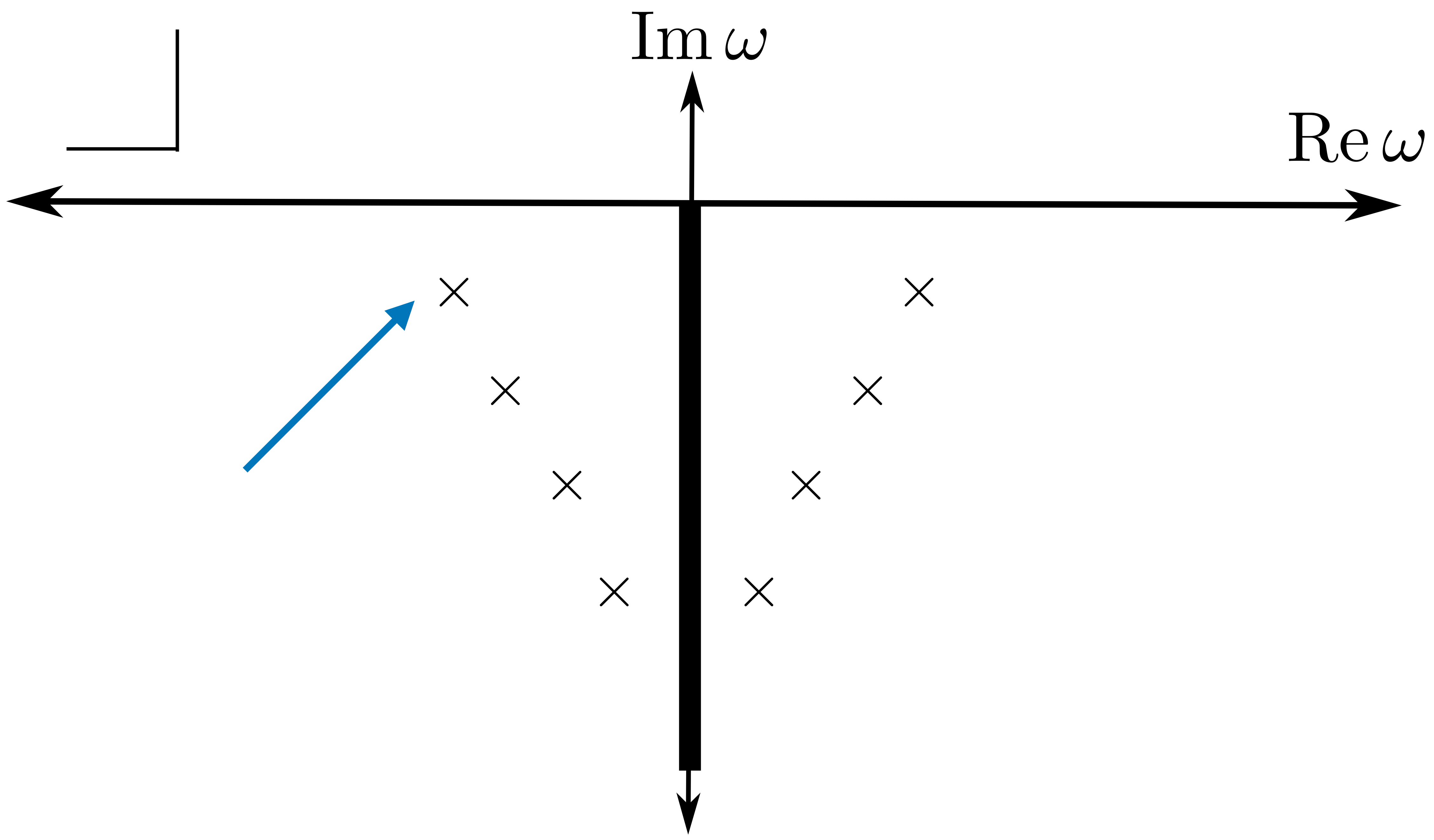

# Ringdown beyond Kerr

# Black holes beyond GR

- Focus on theories which perturb off GR in decoupling limit



Hussain & AZ arXiv: 2206.10653

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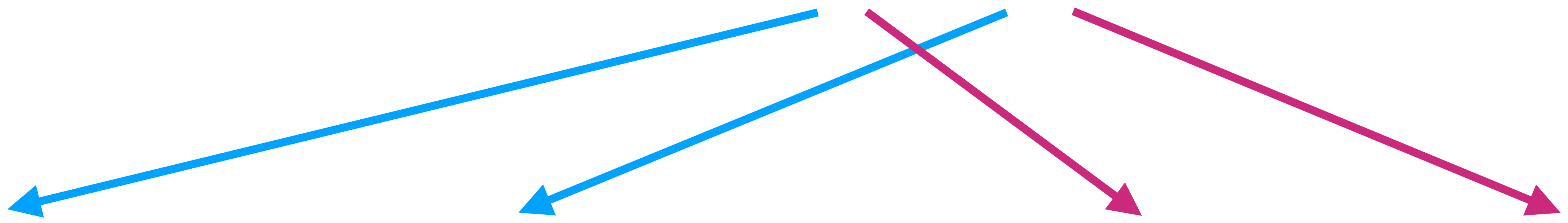

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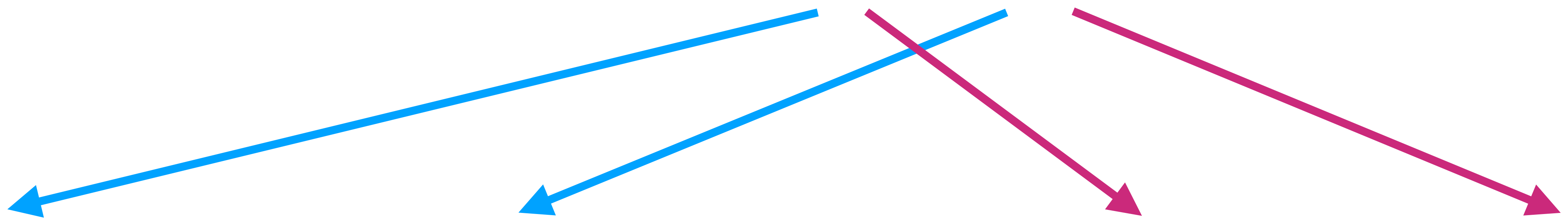
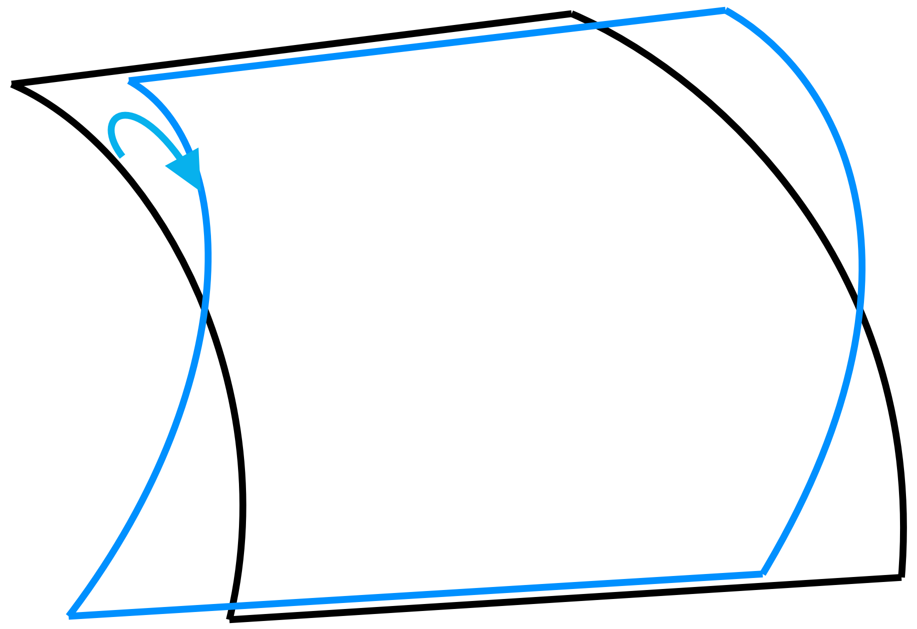


- Solve order by order for equilibrium solution





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# Quadratic gravity example: dCS

- Dynamical Chern-Simons: couple total derivative to scalar field

- New length scale



Stein arXiv:1407.2350

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- Dynamical Chern-Simons: couple total derivative to scalar field

- New length scale

- Stationary BH solutions

- known in slow spin expansion (Cano et al. 2019),

- Numerical solution tractable (Stein 2014)



Stein arXiv:1407.2350

# Quadratic gravity example: dCS

- Stationary BH solutions
- Post-Newtonian predictions (Yagi+ 2012)
- Binary black hole simulations (Okounkova+ 2019)
- Strong constraints from NICER (Silva+ 2021)

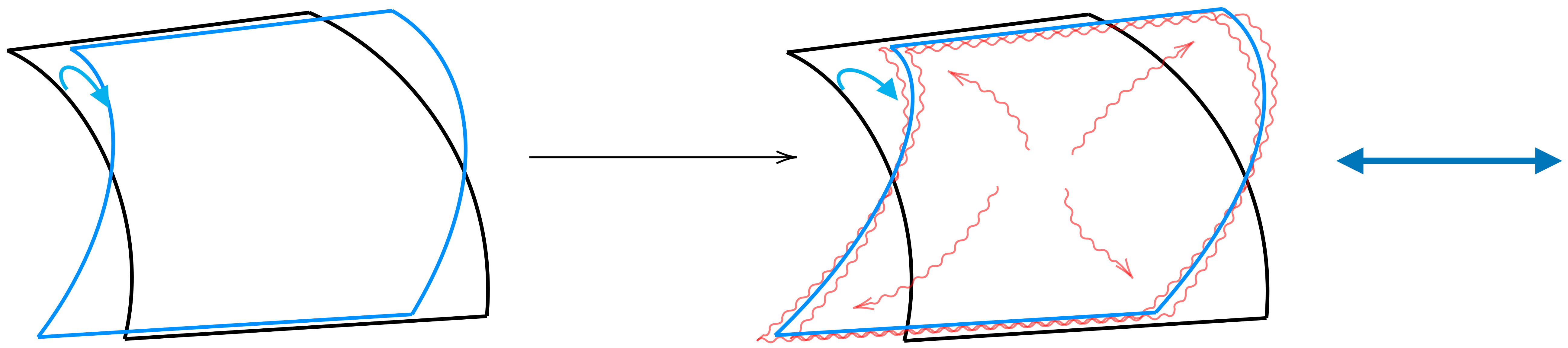
- Slow-spin expansion for ringdown (Cano+ 2020; Wagle+ 2021; Srivastava+ 2021)
- But parameter inference requires results at high spins



Wagle, Yunes & Silva
arXiv:2103.09913

# Perturbed black holes beyond Kerr

- Now add dynamical perturbations to all fields





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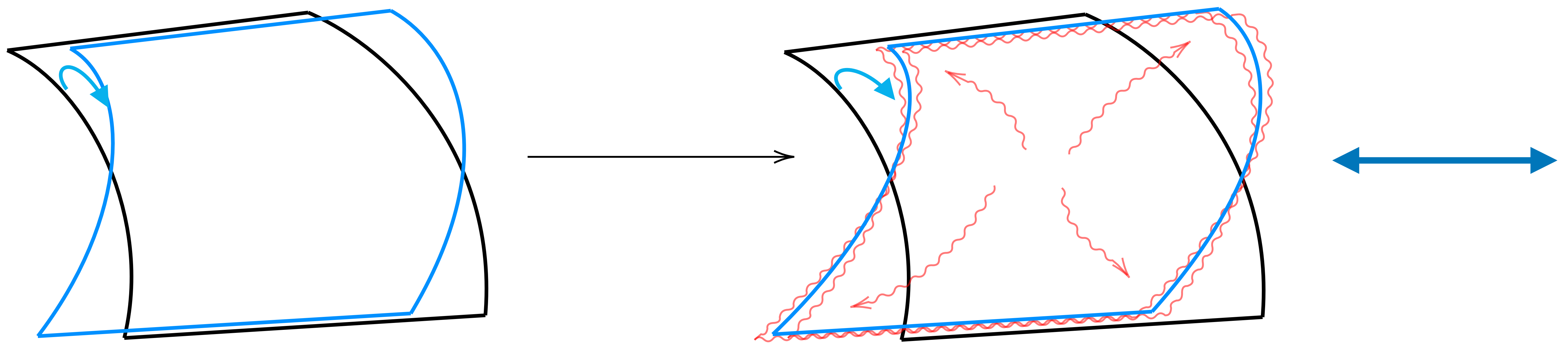

- Resulting equations are coupled and not separable



Hussain & AZ arXiv:2206.10653

# Partial decoupling



Hussain & AZ arXiv:2206.10653

# Partial decoupling

- Two bases to perturb around


Hussain & AZ arXiv:2206.10653

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- Two bases to perturb around

- First gives full decoupling, second gives partial decoupling

# Modified Teukolsky equation

- Gravitational case: derive perturbations to Teukolsky equation

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- Direct derivation from NP approach involved (Li, Wagle + 2022, Hussain & AZ 2022)
  - Track modifications to null tetrad, spin coefficients, curvature quantities

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Leading order in metric
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# Perturbations of quasinormal modes

# Eigenvalue perturbations

- For a spacetime deformed from Kerr, can apply perturbative approach



Mark, Yang, AZ, Chen, arXiv:1409.5800
AZ+ arXiv:1406.4206

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$\longrightarrow$



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# Eigenvalue perturbations

- For a spacetime deformed from Kerr, can apply perturbative approach

$$\longrightarrow$$

- Conceptually extends to QNMs



Mark, Yang, AZ, Chen, arXiv:1409.5800
AZ+ arXiv:1406.4206

# Eigenvalue perturbations

- Need finite product where wave operator is self-adjoint





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# Scalar example: parametric resonance

- Nearly extremal BHs: QNMs nearly evenly spaced

- “Background” grav QNM drives scalar QNMs

Yang, AZ, Lehner, arXiv:1402.4859

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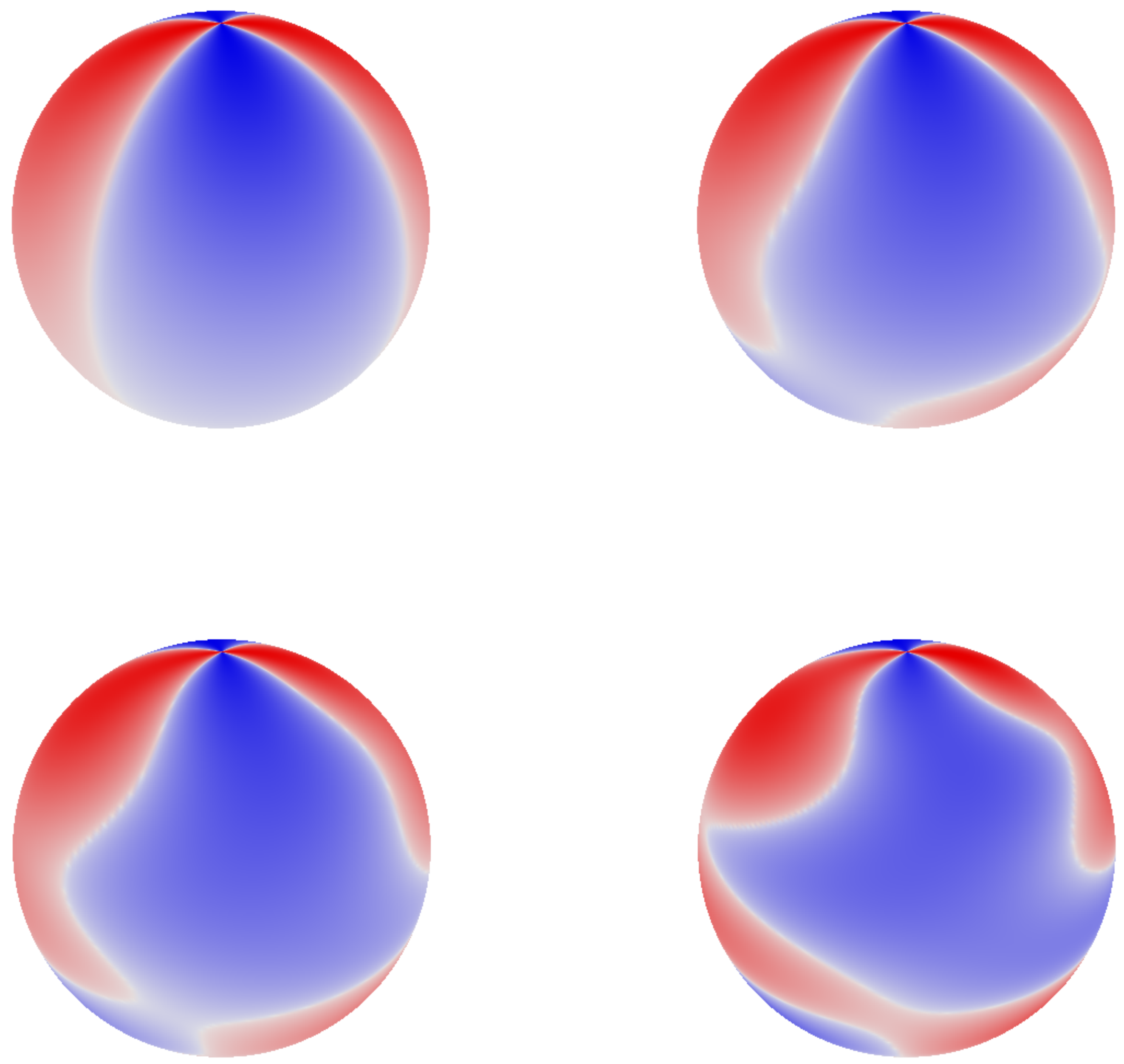


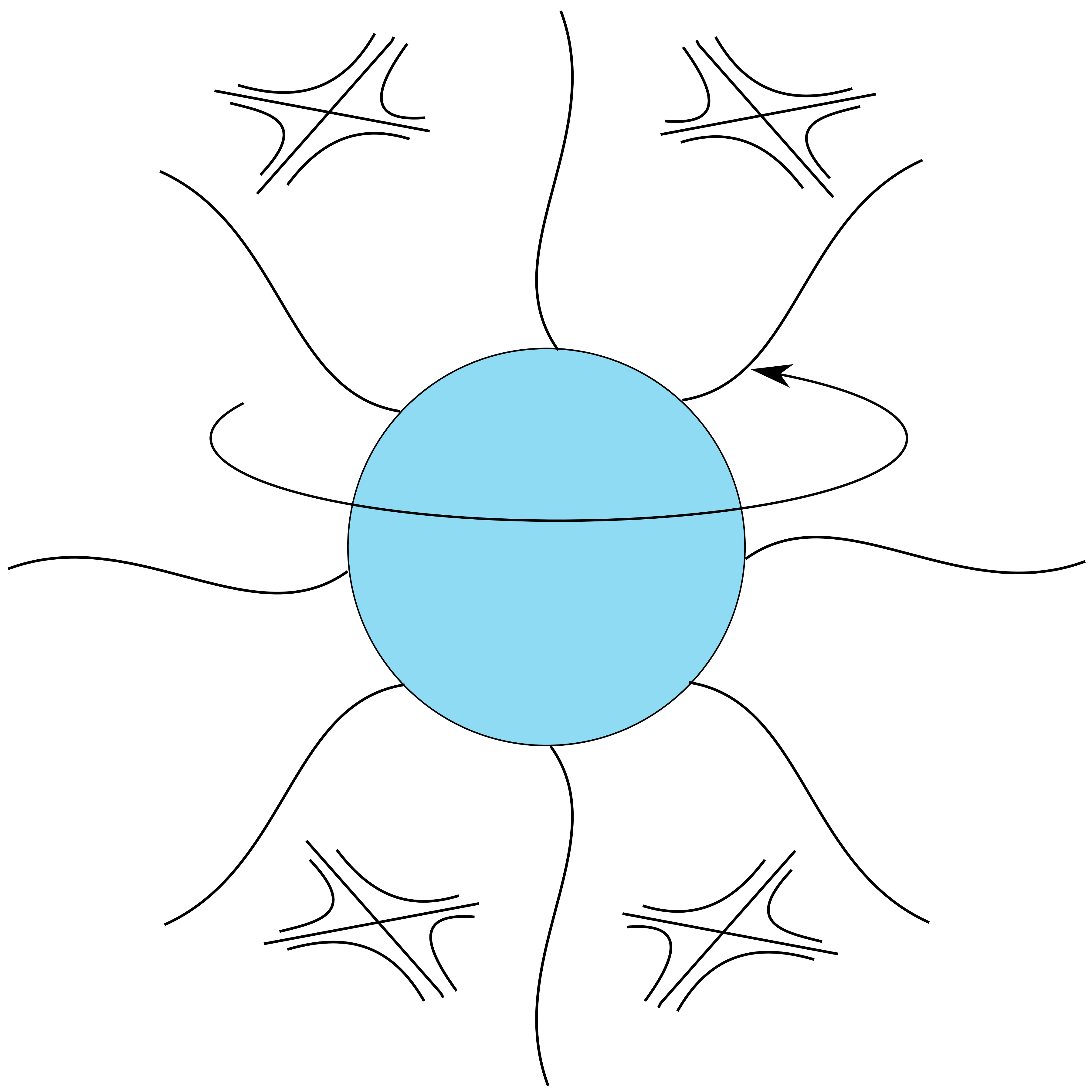
Transient “turbulence” of scalar perts

Yang, AZ, Lehner, arXiv:1402.4859

# Gravitational example: charged black holes

- Coupled equations

- Cannot decouple and separate: gravito-electromag perturbations

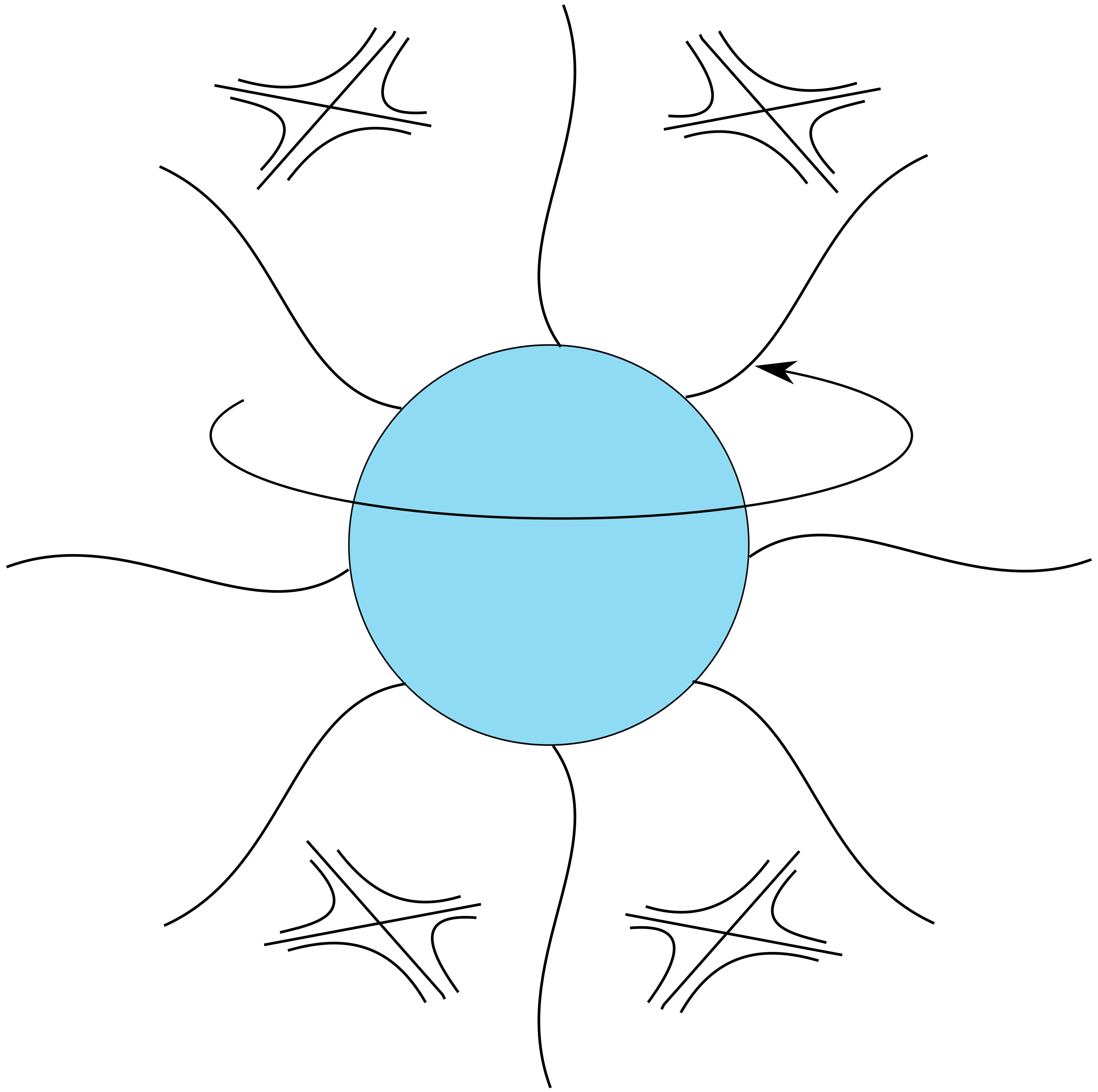


# Gravitational example: charged black holes

- Coupled equations

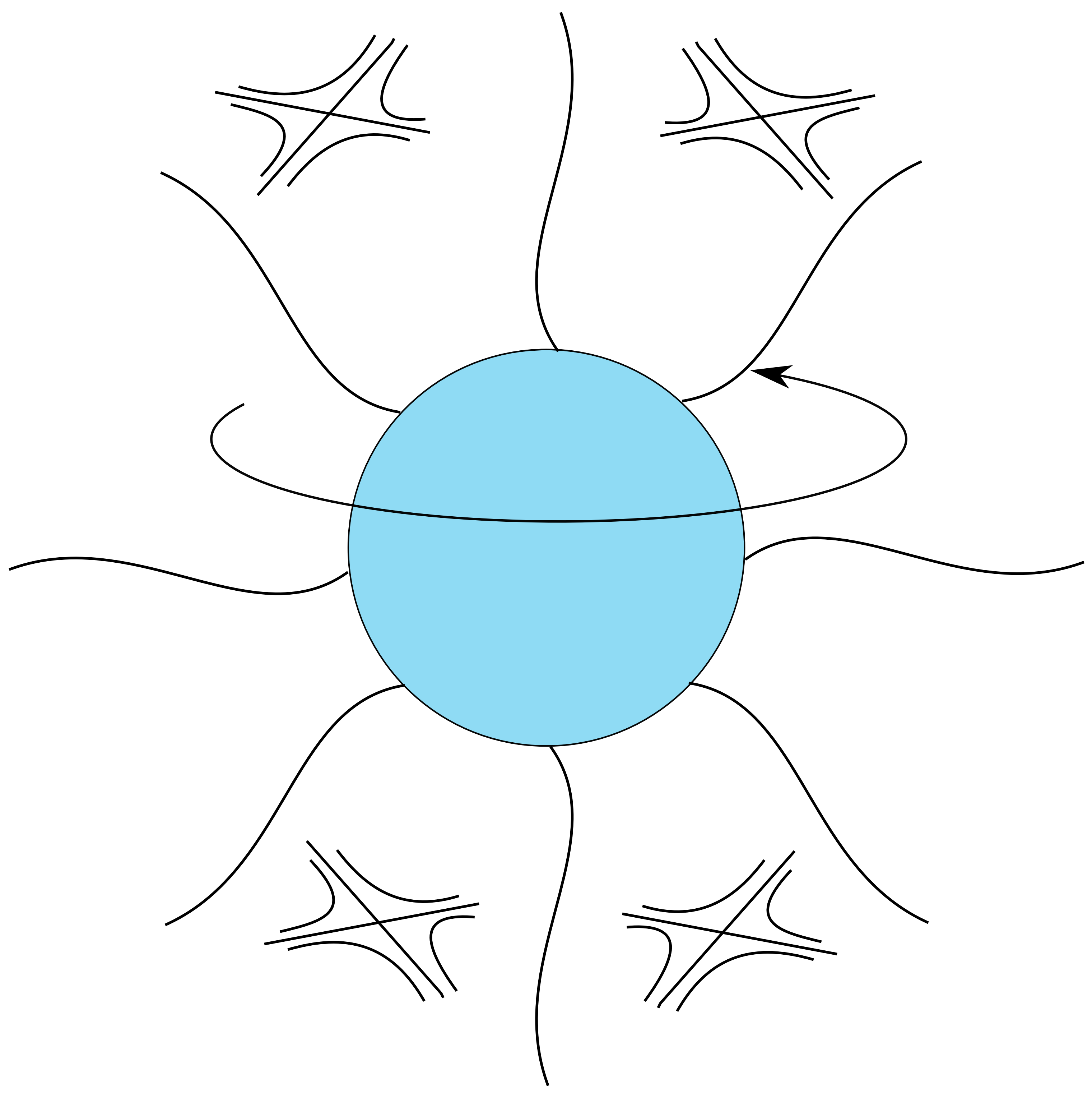
- Cannot decouple and separate: gravito-electromag perturbations

- Small charge: can decouple and apply EVP



# Gravitational example: charged black holes

- Chandrasekhar: NP derivation





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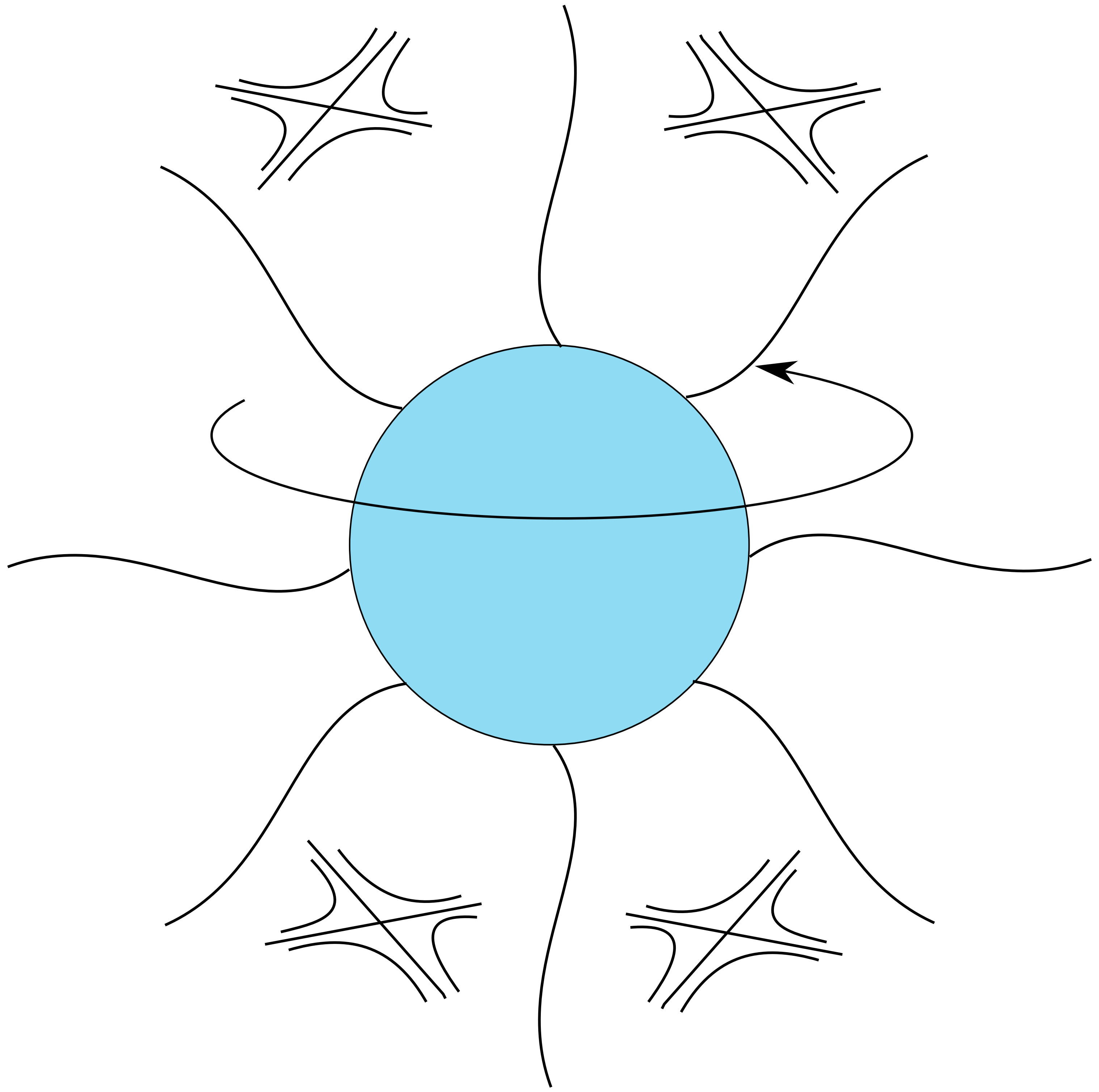
# Gravitational example: charged black holes

- Chandrasekhar: NP derivation

- We know the eigenmodes for Q = 0





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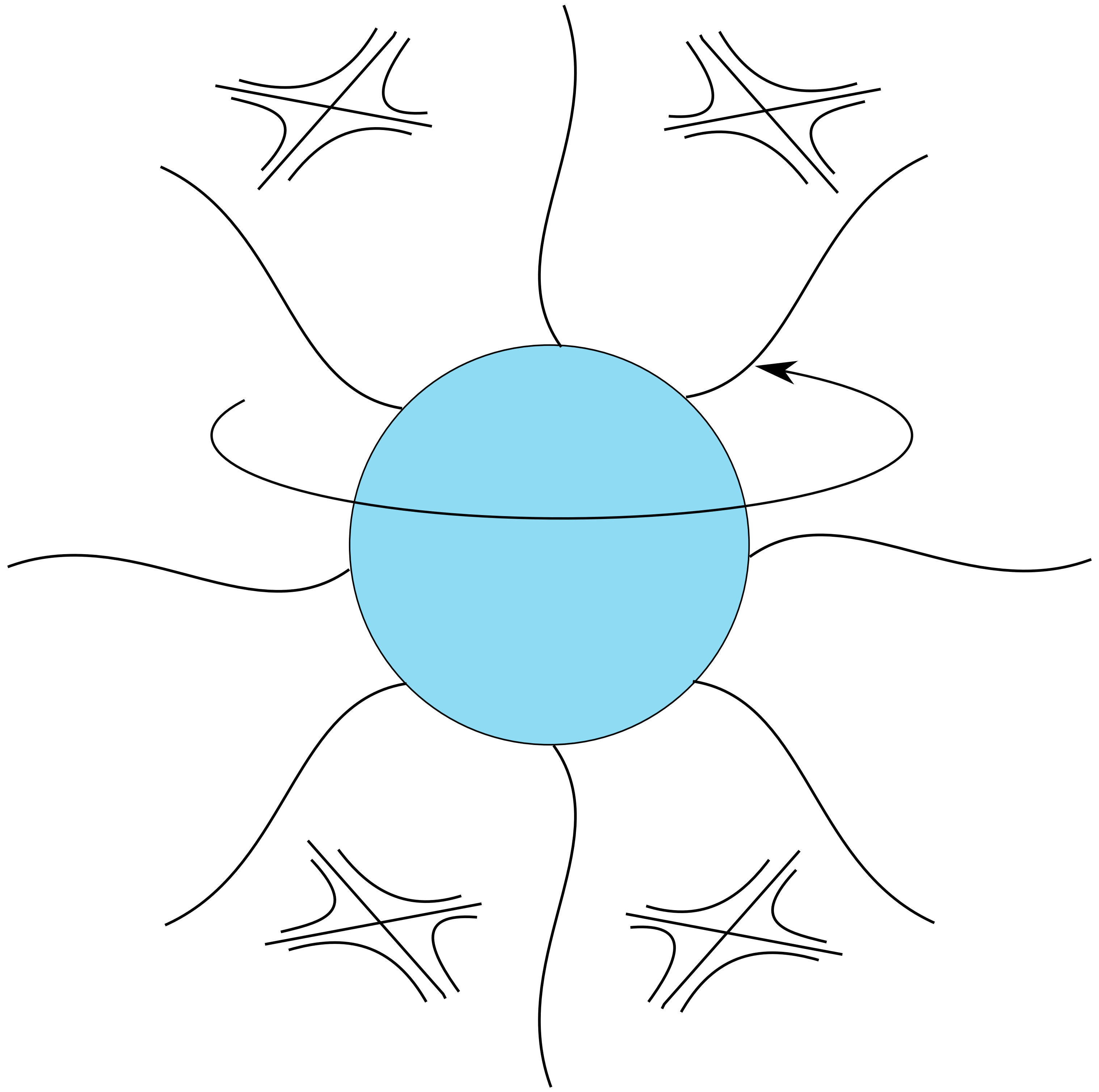
# Gravitational example: charged black holes

- Chandrasekhar: NP derivation
- We know the eigenmodes for Q = 0
- This decouples everything







Mark, Yang, AZ, Chen, arXiv:1409.5800

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- We know the eigenmodes for Q = 0

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Dias, Godazgar, Santos, arXiv:1501.04625
Carullo et al. arXiv:2109.13961

# Frequency shifts beyond Kerr

- Application to ringdown modes is direct:

Mark, Yang, AZ, Chen, arXiv:1409.5800
AZ +, arXiv:1406.4206
Hussain & AZ arXiv: 2206.10653

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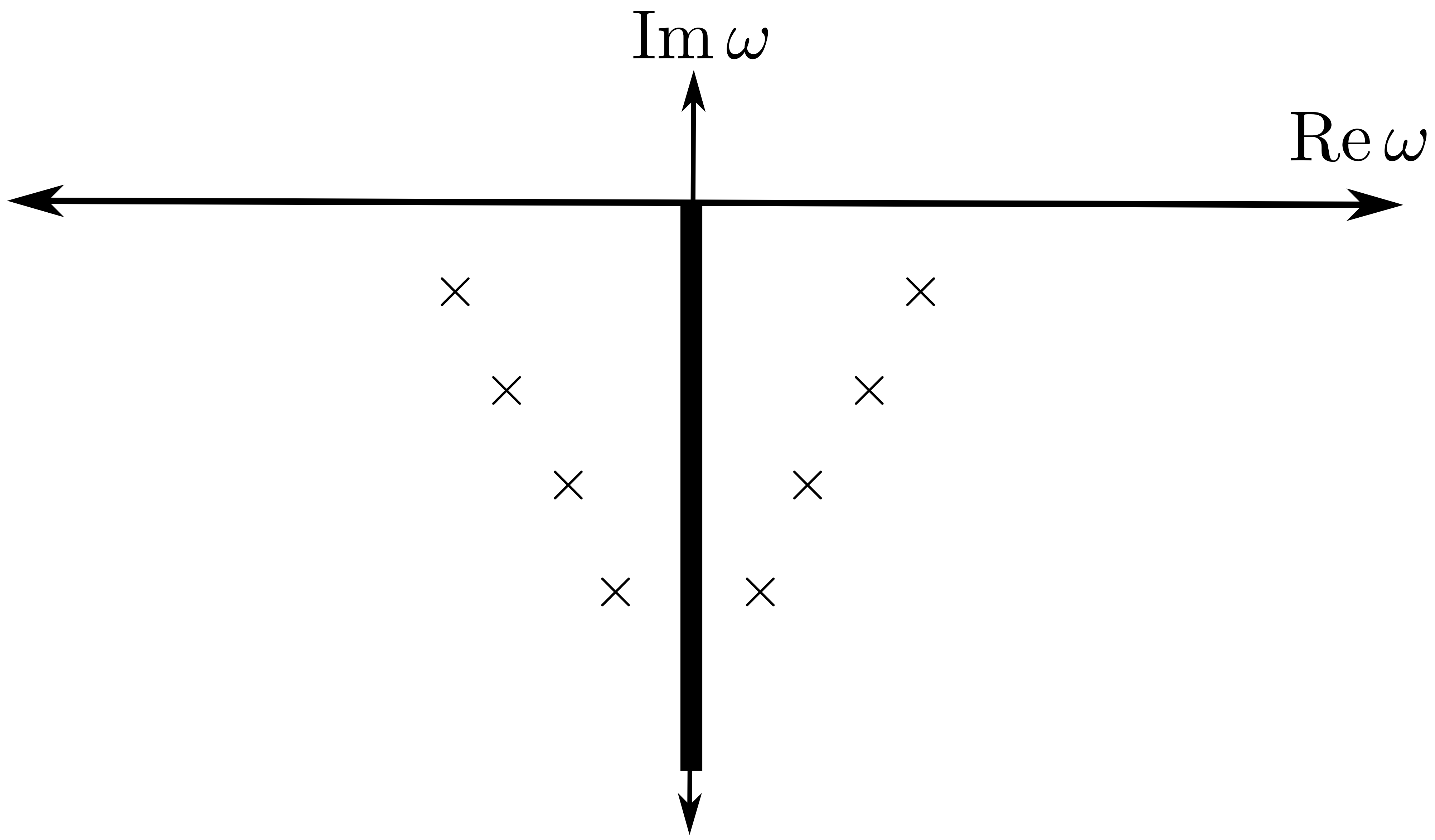
# Frequency shifts beyond Kerr

- Application to ringdown modes is direct:

- … except it is not
- Conceptual issue: metric reconstruction couples $\psi$ and
- Couples two families of modes: and

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AZ +, arXiv:1406.4206
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# Degenerate EVP

- Formally write metric reconstruction as

- Consider superposition of states that don’t mix

- Apply EVP approach

Hussain & AZ arXiv:2206.10653

# Isospectrality

- Connected to Kerr isospectrality (Chrzanowski 1976, Nichols + 2012)
- Definite-parity perturbations constructed from Hertz potential

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↑

- Equality of modes means Kerr is isospectral
- Perts to Kerr break isospectrality generically (Li + arXiv:2310.YYYYY)

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# Roadmap

Choose theory

Stationary deformation

Choose mode

Reconstruct metric

Coupled dynamical fields

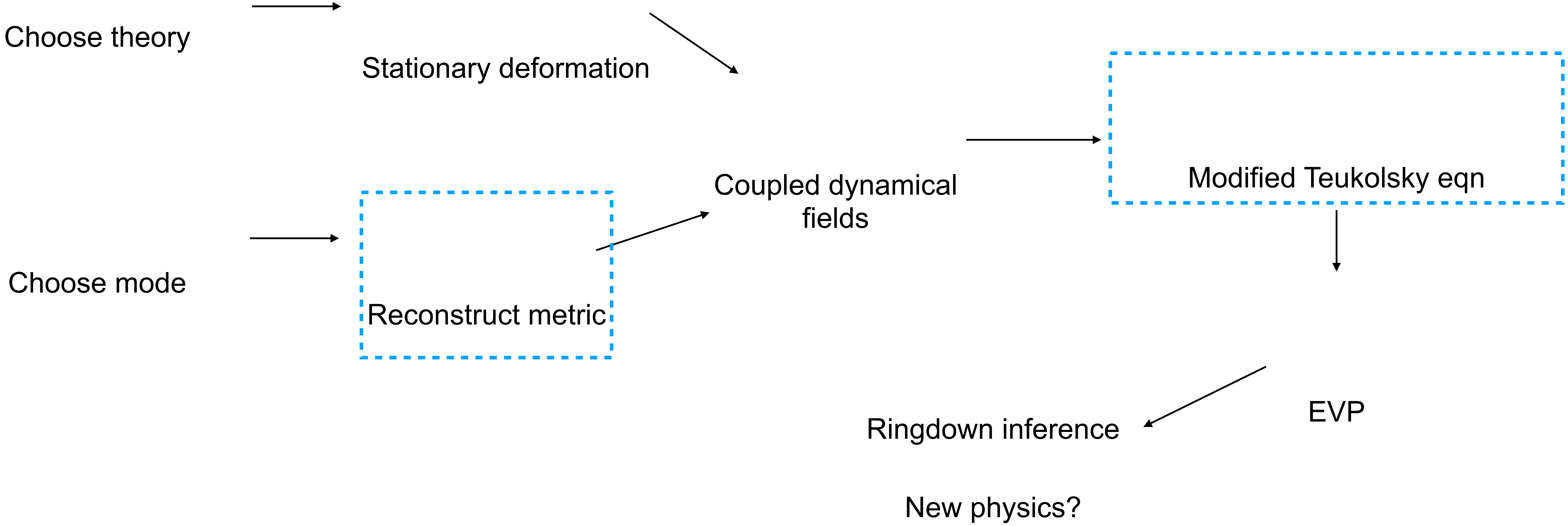
Modified Teukolsky eqn

EVP

Ringdown inference

New physics?

# Roadmap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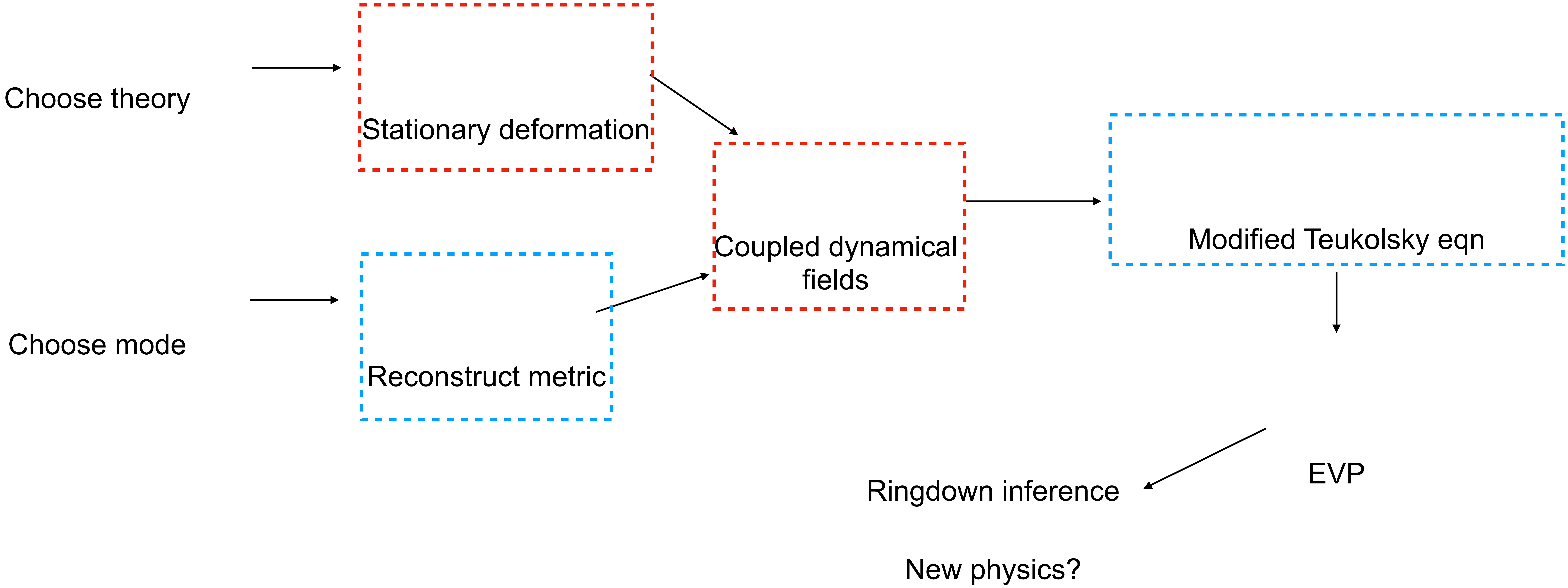
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# Looking ahead

- Predicting QNMs may allow for multi-mode ringdown tests of Kerr
  - Modified Teukolsky eqn
  - EVP method: allows for high spins
  - Several challenges ahead in implementation
- Many detections in the coming years
  - Combine constraints
- 3rd gen and LISA: precision predictions needed



LVK arXiv:2111.03606

# Extras

# Combining events

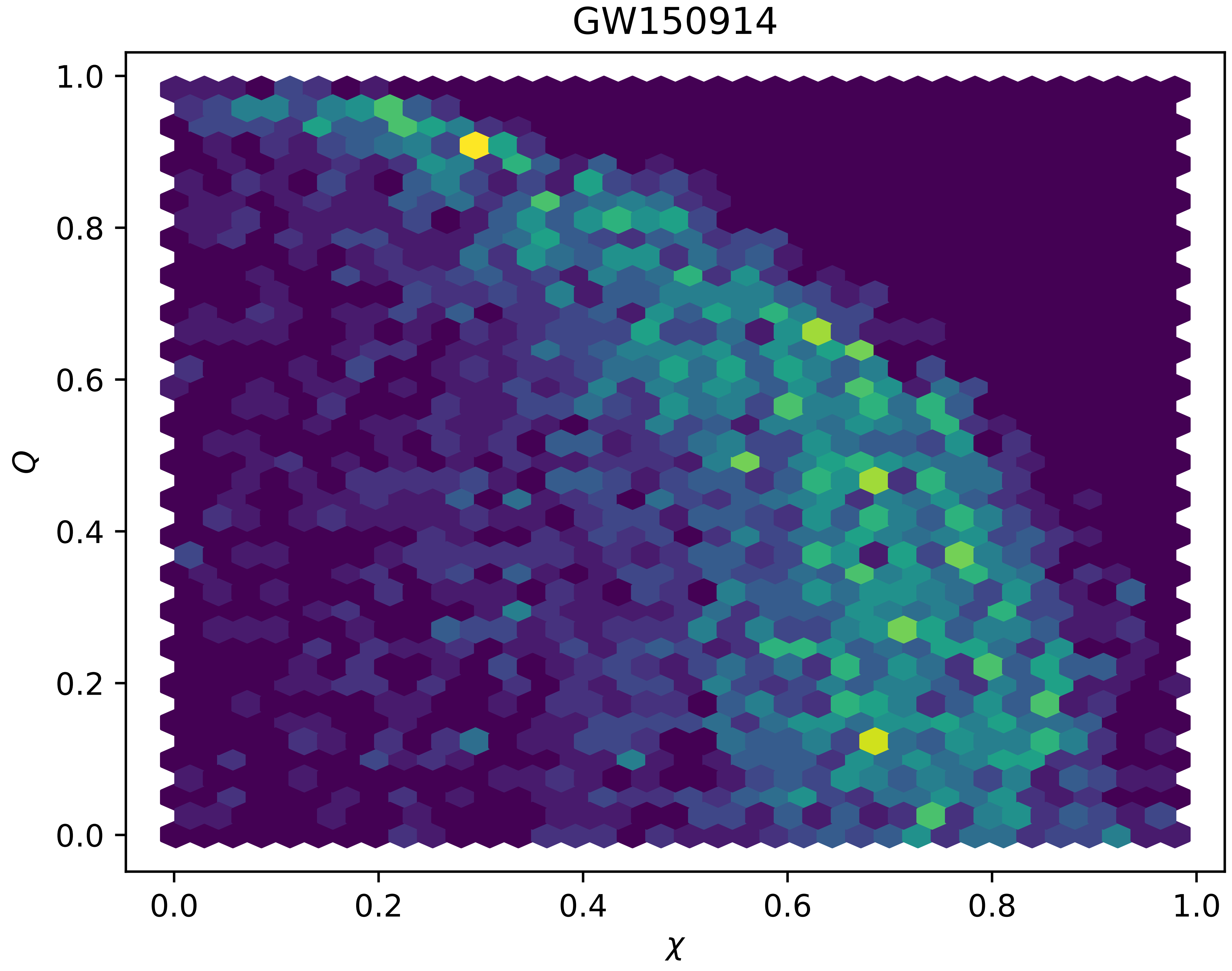
- Beyond-GR parameter common to all events: combine constraints directly
- Beyond-GR parameter varies per event
  - Need population modeling (hierarchical modeling) to combine events
  - Modeling needs to account for degeneracies

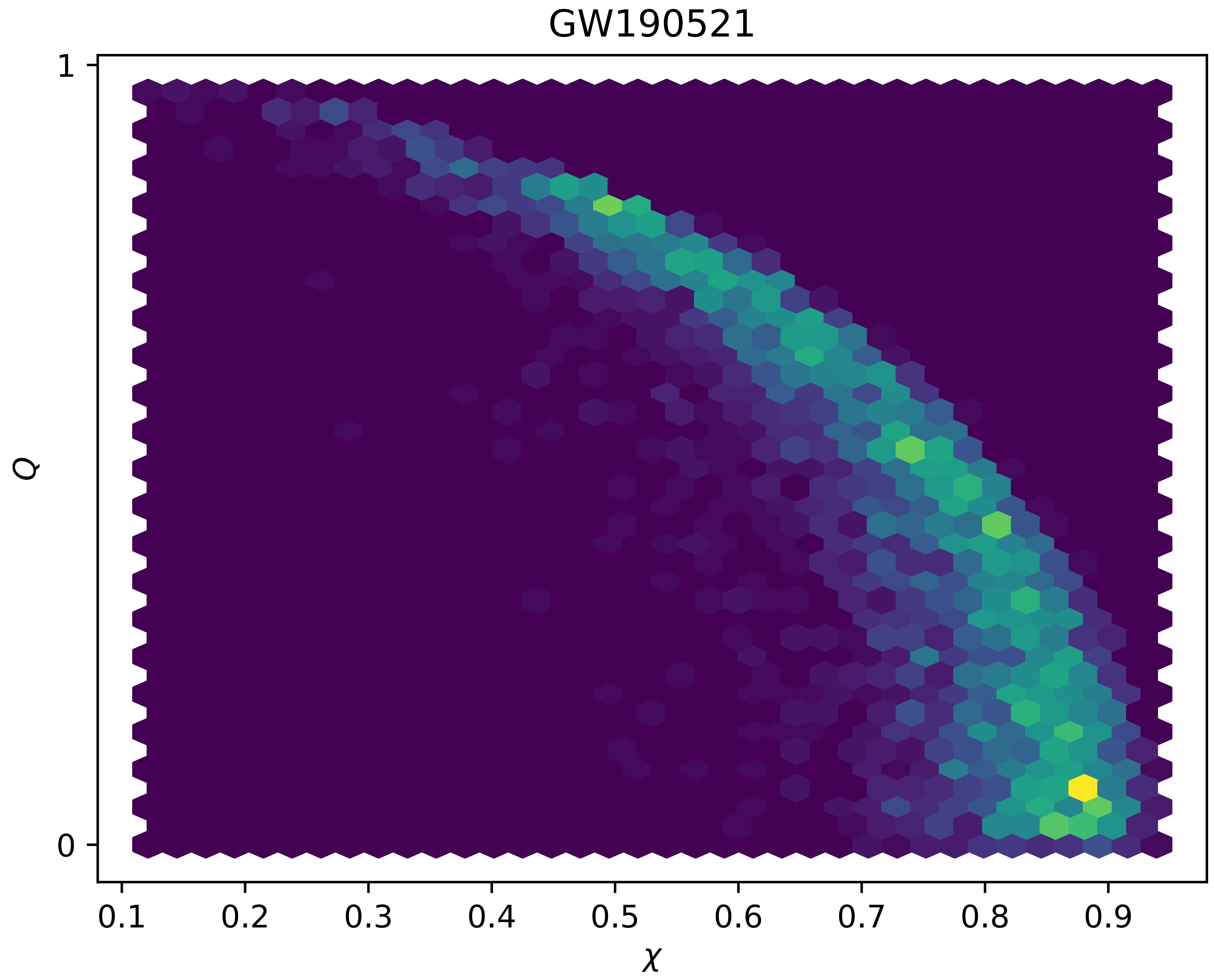
- Example: charged black holes
  - Use `ringdown` package (Isi, Farr)
  - Use multiple tones, infer
  - Start from peak of full IMR waveform



Hussain, Isi, AZ in prep

# Example: Charged BHs

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**See also Carullo + arXiv: 2109.13961**

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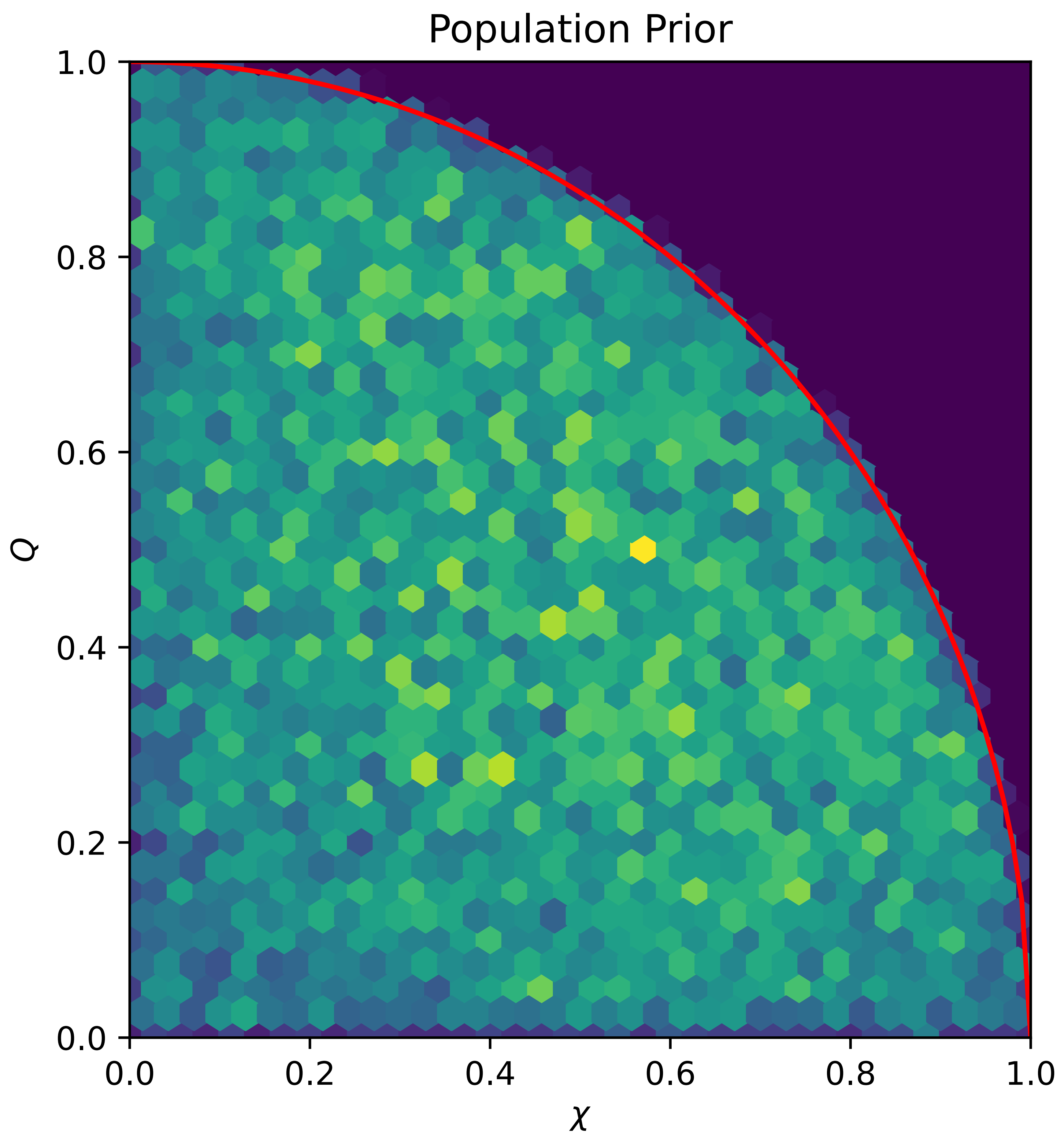


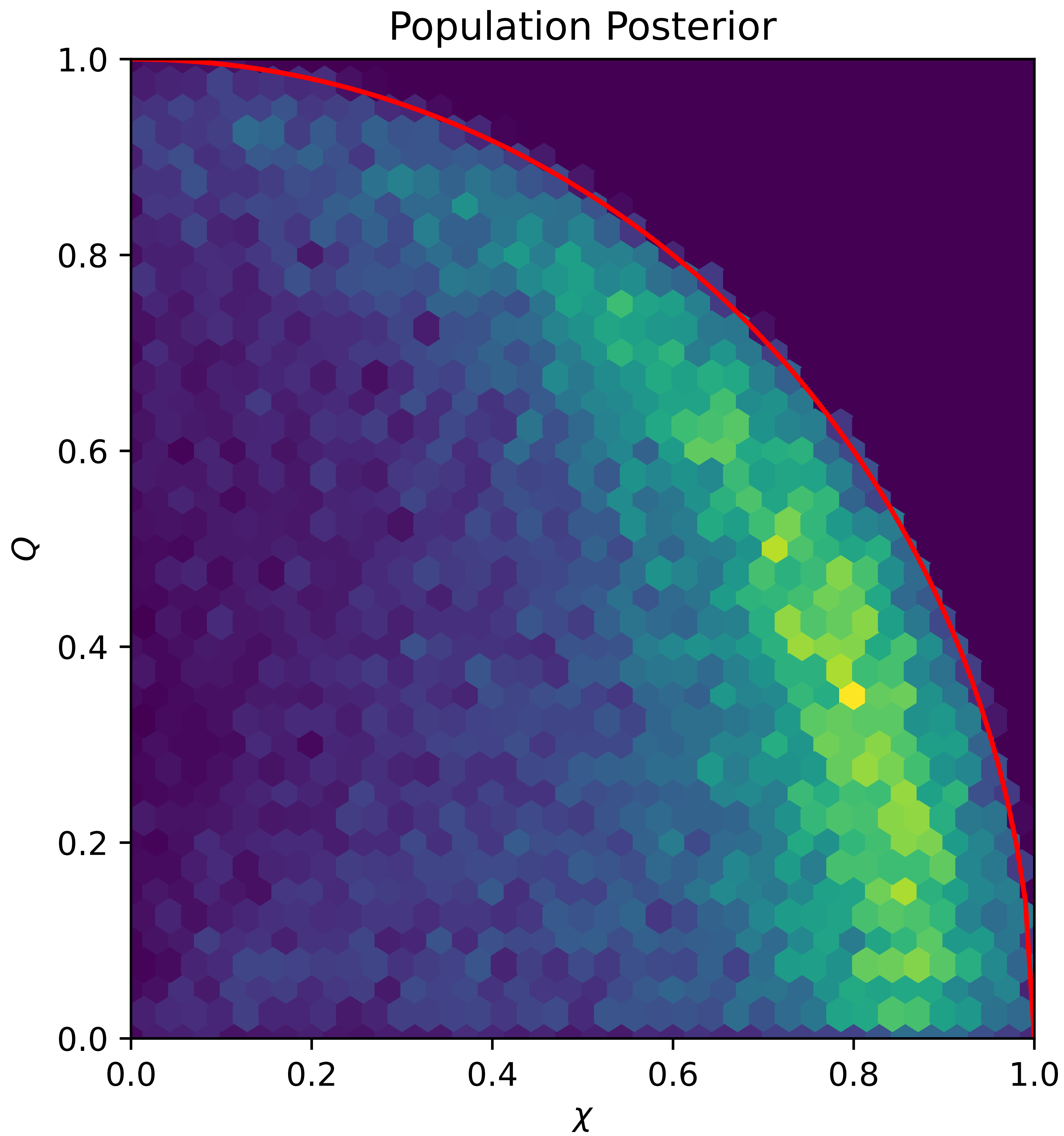


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Hussain, Isi, AZ in prep

# Example: Charged BHs

# Gravitational perts for Kerr

- Angular equation: (spin-weighted) spheroidal harmonics

- Standard Sturm-Liouville eigenvalue problem

# Gravitational perts for Kerr

- Radial equation: Schrodinger-like with complex potential

# Gravitational perts for Kerr

- Radial equation: Schrodinger-like with complex potential