Nonlinear black hole spectroscopy

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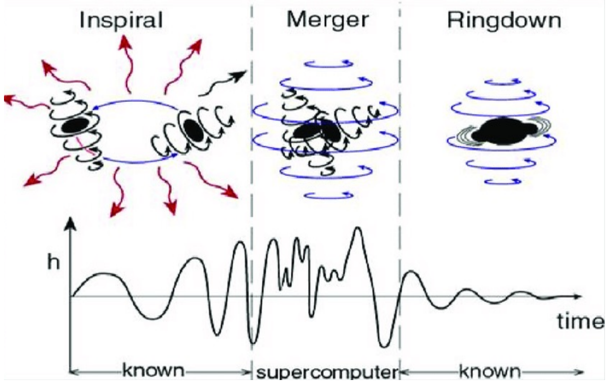
Nonlinear Aspects of General Relativity
Princeton Center for Theoretical Science
October 10 2023



Talk based largely on:

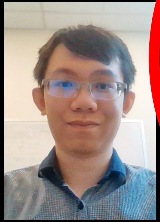
Cotesta+ 2201.00822, Cheung+ 2208.07374,
Baibhav+ 2302.03050, Redondo-Yuste+ 2308.14796, Cheung+ 2310.04489

JOHNS HOPKINS UNIVERSITY

Group effort! In particular M.Cheung, V.Baibhav, R.Cotesta (+S.Yi, in prep.)

JHU postdocs

K.Ng



T.Helfer

R.Cotesta

T.Edwards

A.Antonelli

+

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JHU students

M.Caliskan

M.Cheung

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Z.Wang

S.Yi

Black hole spectroscopy: from theory to experiment
Overtones, nonlinearities, and elephants
Agnostic black hole spectroscopy (now automated!)
Have we observed overtones?*
Pseudospectra and quasinormal mode instabilities*

Black hole spectroscopy: from theory to experiment

The Schwarzschild metric

November 18, 1915:
Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$r = 0$: physical curvature singularity

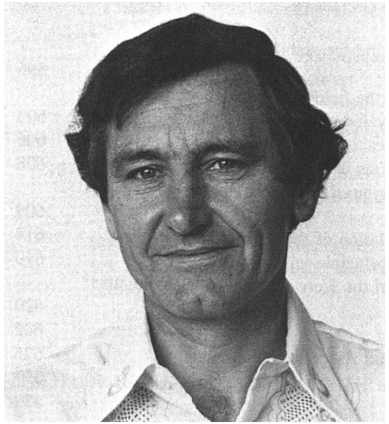
$r = \dfrac{2GM}{c^2}$: "Schwarzschild radius"

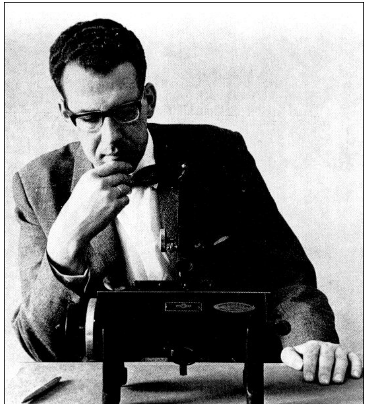
Key questions:

1) **Is the Schwarzschild "singularity" the end point of gravitational collapse?**
   1939: Oppenheimer-Snyder, yes (for dust, in spherical symmetry)
   1963: Lifshitz-Khalatnikov, not generically
   Wheeler (following Schmidt's discovery of a quasar): does the nuclear equation of state halt collapse?
   Answer: Penrose-Hawking singularity theorems

2) **If so, is the Schwarzschild solution stable?**
   Answer: black hole perturbation theory, quasinormal modes (QNMs) and black hole spectroscopy

Are black holes stable? The Golden Age (1963-1970s)





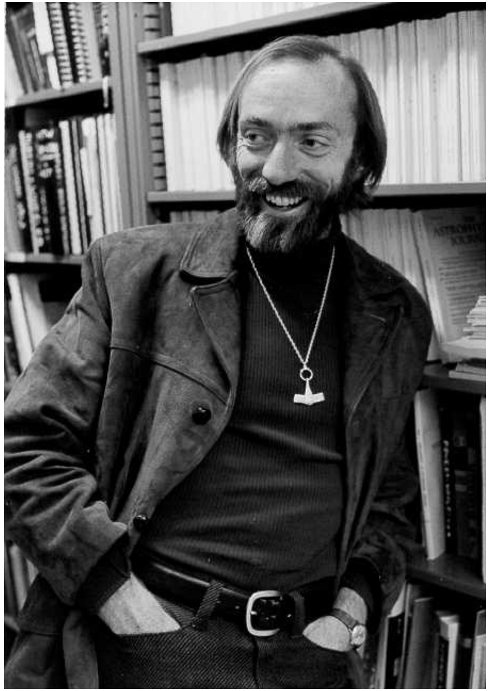
1963:

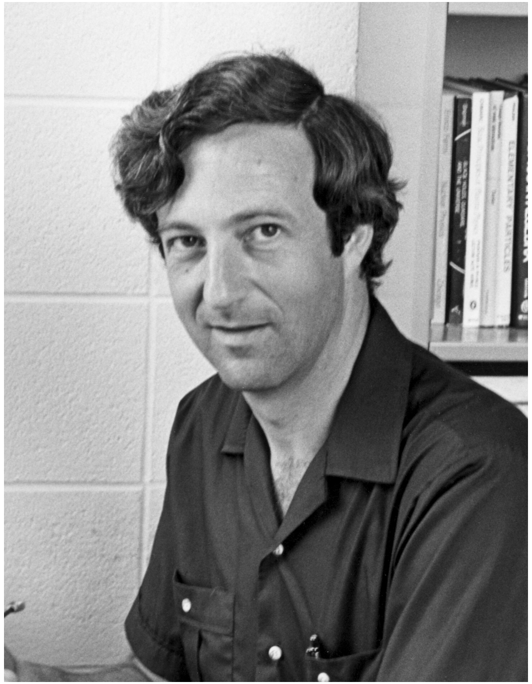
- ✓ Roy Kerr: **rotating black holes**

- ✓ Maarten Schmidt at Caltech discovers the first quasar, 3C273 at z=0.15 – extragalactic!
- ✓ Must be compact and outshines the brightest galaxies: first supermassive black hole
- ✓ Giacconi-Gursky propose orbital satellite to study X-ray sources
  1964: Cygnus X-1, first stellar-mass black hole

Late 1960s and 1970s:

- ✓ “Golden Age” of black hole physics
- ✓ Misner-Thorne-Wheeler, “Gravitation”
- ✓ Kip Thorne and students (including Saul Teukolsky) lay the foundations to understand black hole stability and dynamics





QNMs and overtones: some milestones. Phase 1 – theory development

1957 – Regge-Wheeler axial (odd-parity) perturbations as a scattering problem, boundary conditions not understood
1970 – Zerilli polar (even-parity) perturbations, much harder!

Scalar, electromagnetic and gravitational perturbations of a Schwarzschild BH: Regge-Wheeler/Zerilli equations

$$f\frac{d}{dr}\left(f\frac{d\Phi}{dr}\right) + \left[\omega^2 - fV_\pm\right]\Phi = 0$$

$$V_s = \frac{\ell(\ell+1)}{r^2} + \left(1 - s^2\right)\frac{r_H}{r^3}$$

$$V_- = \frac{\ell(\ell+1)}{r^2} - \frac{3r_H}{r^3}$$

$$V_+ = \frac{9\lambda r_H^2 r + 3\lambda^2 r_H r^2 + \lambda^2(\lambda+2)r^3 + 9r_H^3}{r^3(\lambda r + 3r_H)^2}$$

QNMs and overtones: some milestones. Phase 1 – theory development

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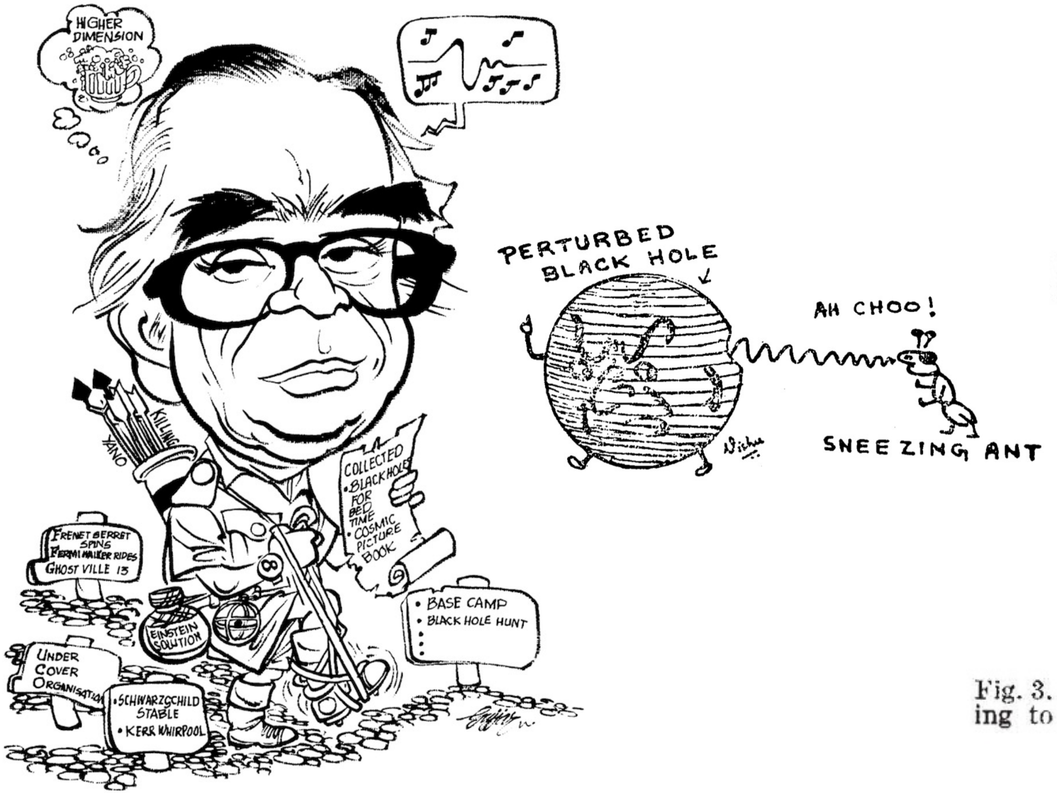
1970 – Vishveshwara now boundary conditions are clear: scattering experiment, "ringdown waves"



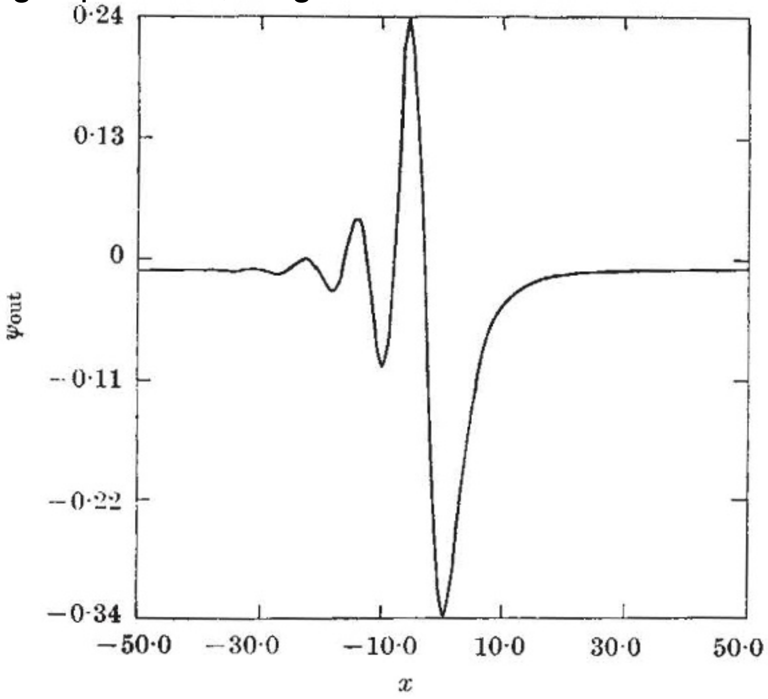


Fig. 3. The outgoing wave packet $\psi_{out}(x)$ at spatial infinity corresponding to the incident Gaussian wave packet $\psi_{in}(x) = e^{-ax^2}$ with $a = 1$.

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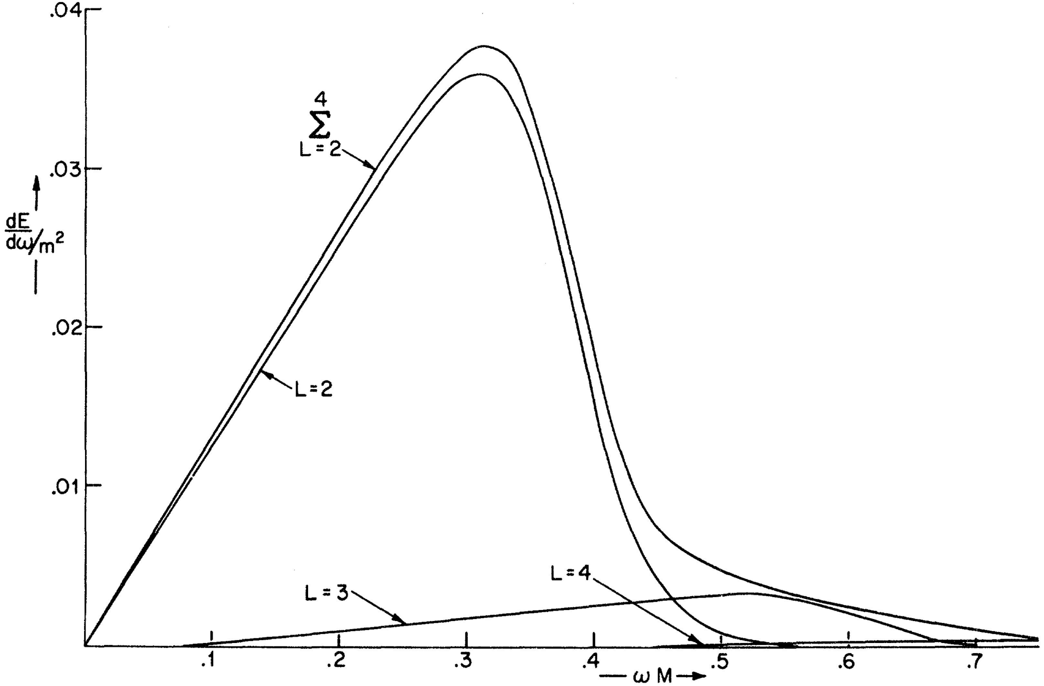
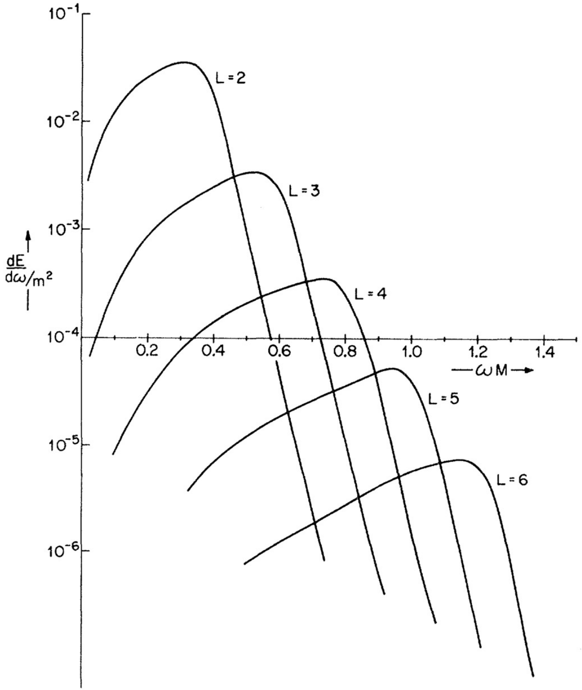
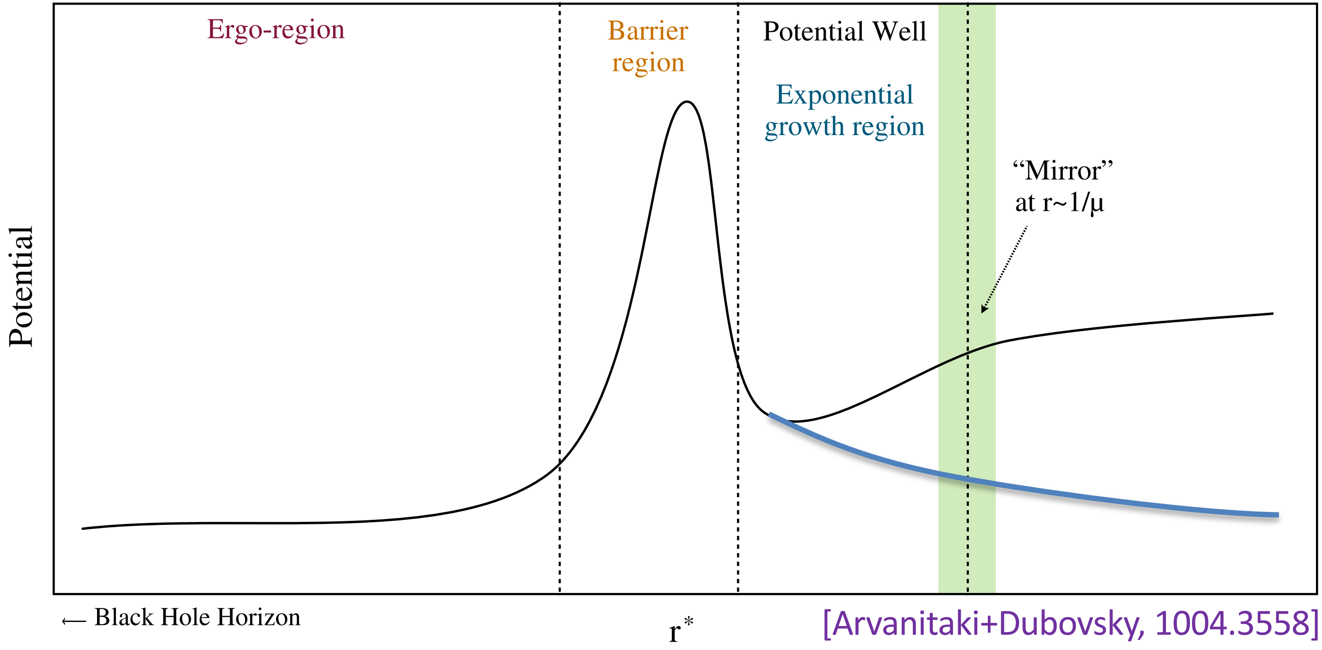
1971 – Press ringdown waves are free oscillation modes of the black hole
1971 – Davis-Ruffini-Press-Price these modes are excited when radially falling particles cross the light ring
1973 – Teukolsky formalism for Kerr perturbations





QNMs and overtones: some milestones. Phase 2 – overtones and spectroscopy

$$\left(\frac{d^2}{dr_*^2}+\omega^2\right)\Phi = V\Phi$$



[Arvanitaki+Dubovsky, 1004.3558]

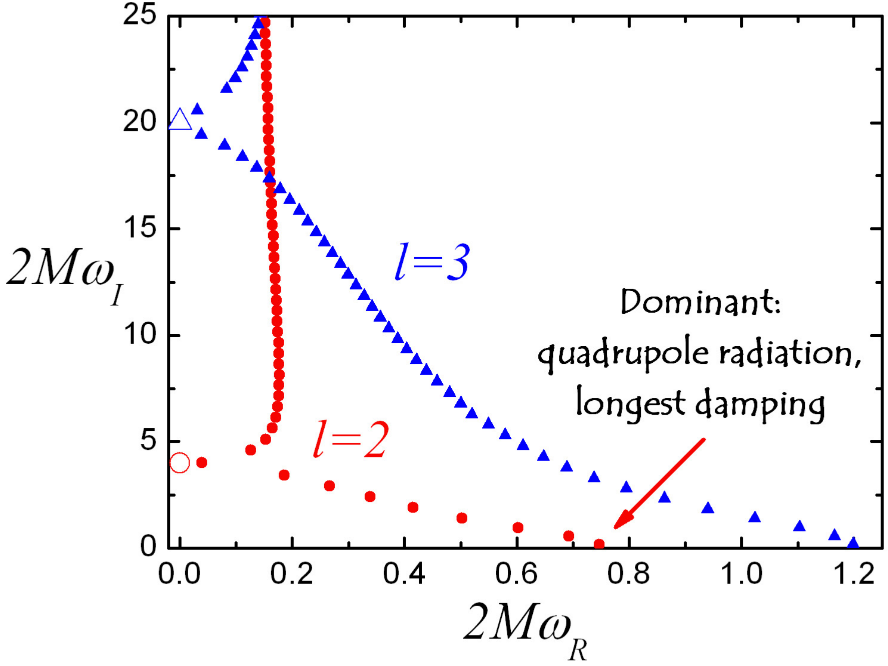
Quasinormal modes:

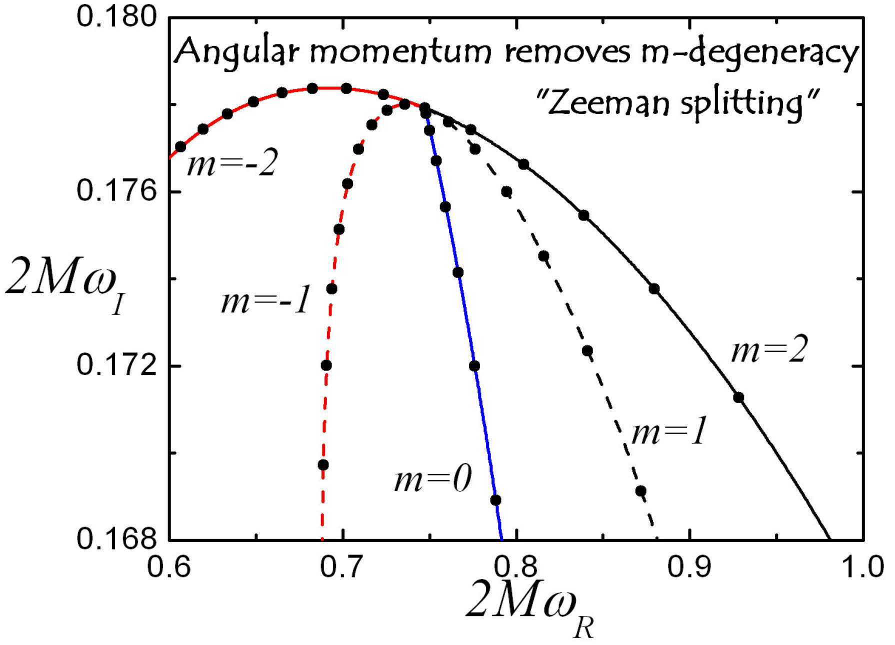
- Ingoing waves at the horizon, outgoing waves at infinity
- Spectrum of damped modes ("ringdown")

Massive scalar field:

- Superradiance: black hole bomb when $0 < \omega < m\Omega_H$ [Press-Teukolsky 1972]
- Hydrogen-like, unstable bound states [Detweiler 1980, Zouros+Eardley, Dolan…]

QNMs and overtones: some milestones. Phase 2 – overtones and spectroscopy



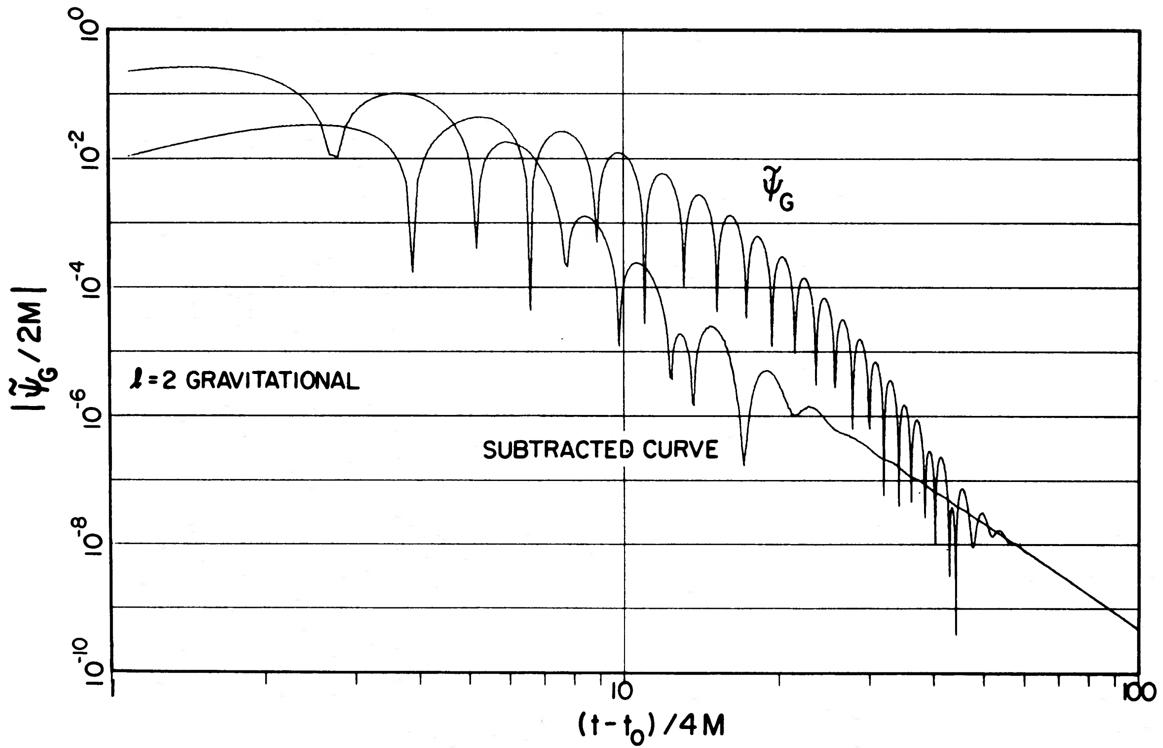


- **One mode** fixes mass and spin – and the whole spectrum!
- **N modes:** N tests of GR dynamics…**if** they can be measured
- **Measurement requires understanding of QNM excitation** (as in atomic physics!)
- **Retrograde modes** (too often ignored)

[Berti-Cardoso-Will, gr-qc/0512160; EB+, gr-qc/0707.1202]

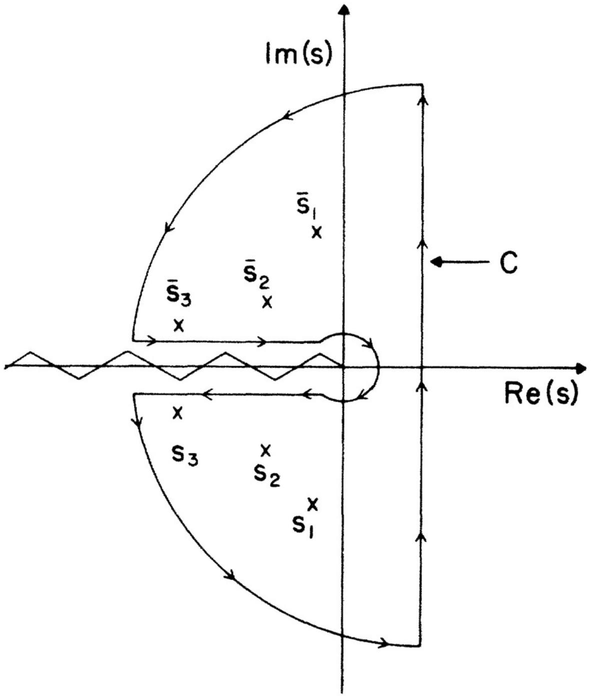
QNMs and overtones: some milestones. Phase 2 – overtones and spectroscopy

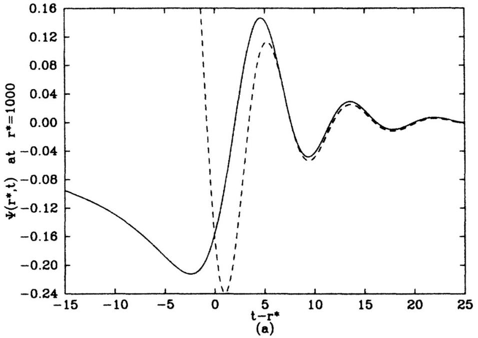
1975 – Chandrasekhar-Detweiler first numerical calculation of overtones in Schwarzschild, with limited accuracy
1978 – Cunningham-Price-Moncrief observe overtones in perturbative calculation of collapse to Schwarzschild
1979 – Detweiler first complete calculation of the Kerr spectrum, **"black hole spectroscopy"**
"After the advent of gravitational wave astronomy, the observation of [the black hole's] resonant frequencies might finally provide direct evidence of black holes with the same certainty as, say, the 21 cm line identifies interstellar hydrogen."



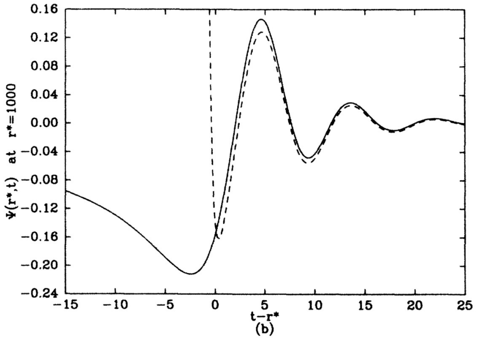
QNMs and overtones: some milestones. Phase 3 – excitation, pre-NR

1986 – Leaver Green's function, continued fractions, excitation factors (also **Andersson**) – **by analogy with H_2^+ ion!**
1989 – Echeverria quantifies how well you can measure mass and spin from a single mode
1998 – Flanagan-Hughes ringdown may have as much SNR as inspiral
2002 – Hod-Dreyer are QNMs related with Bekenstein's ideas on area quantization and LQG?
2003 – Dreyer+ revive/rebrand Detweiler's idea of "black hole spectroscopy"
2005 – Berti-Cardoso-Will SNRs, measurability, QNM frequencies+fits, **overtones vs. higher multipoles**





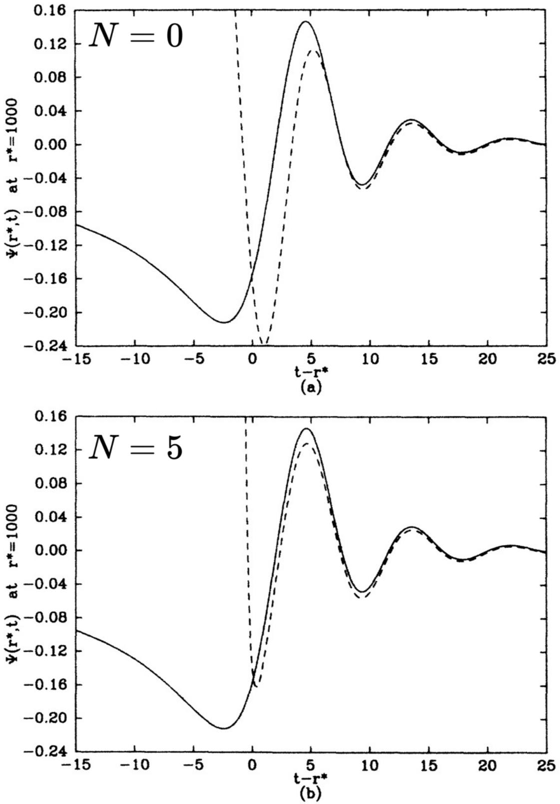
Radial infall vs. one mode

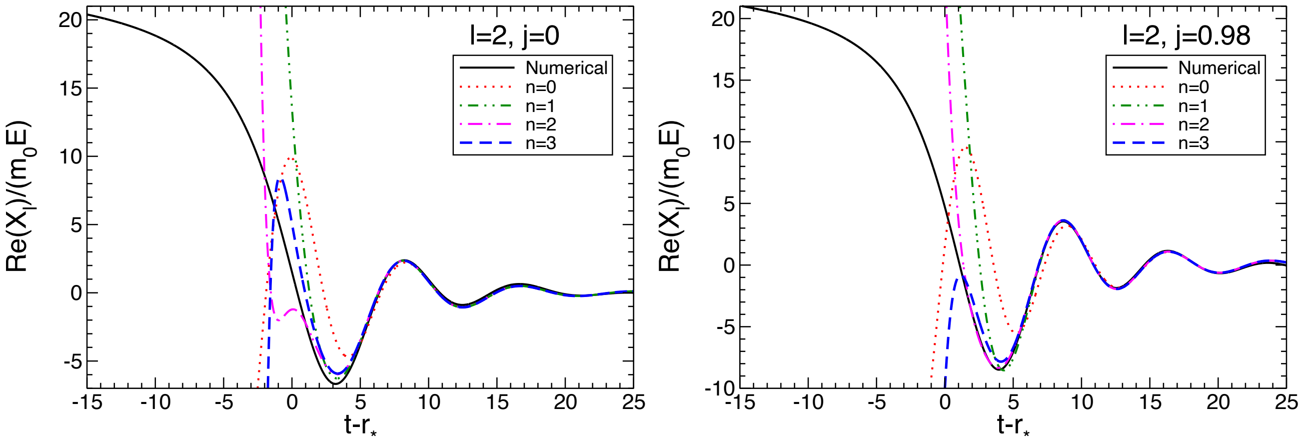


Radial infall vs. six modes

Overtone excitation can be computed (no fits!) in linear perturbation theory

Leaver (1986): Green's function in Schwarzschild. Overtones: agreement well before peak
Zhang+ (2013): extension to Kerr (here for an ultrarelativistic infall along the z axis)





[Zhang+, 1305.4306]

"Excitation factors" in Kerr known
"Excitation coefficients" depend on initial data
Difficult, **unsolved** problem for comparable-mass mergers
See recent work by Oshita

[Leaver, PRD, 1986]

[EB+Cardoso, gr-qc/0605118; Oshita, 2109.09757]

QNMs and overtones: some milestones. Phase 4 – excitation, post-NR

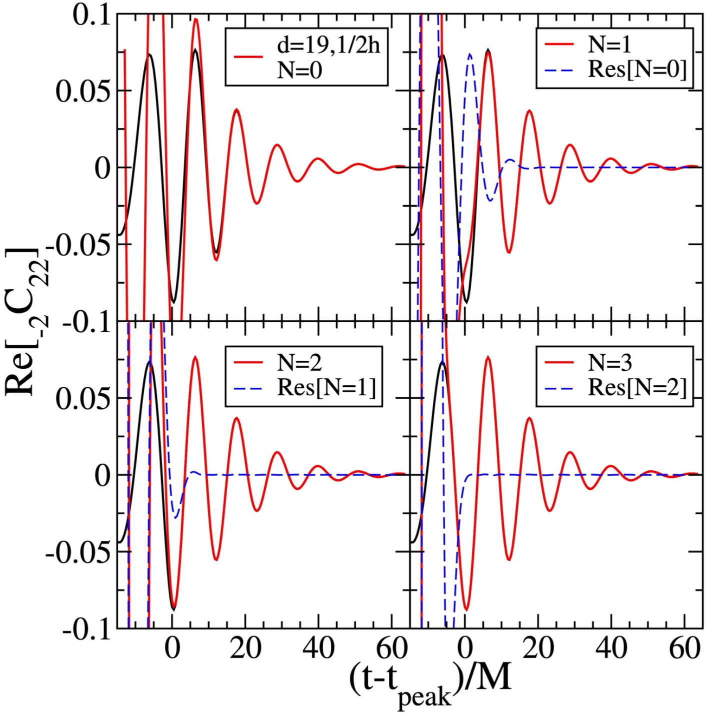
2005 – Pretorius numerical relativity breakthrough: merger simulations. Soon after Brownsville/RIT, Goddard…

2006 – Berti-Cardoso systematic calculation of Kerr excitation factors

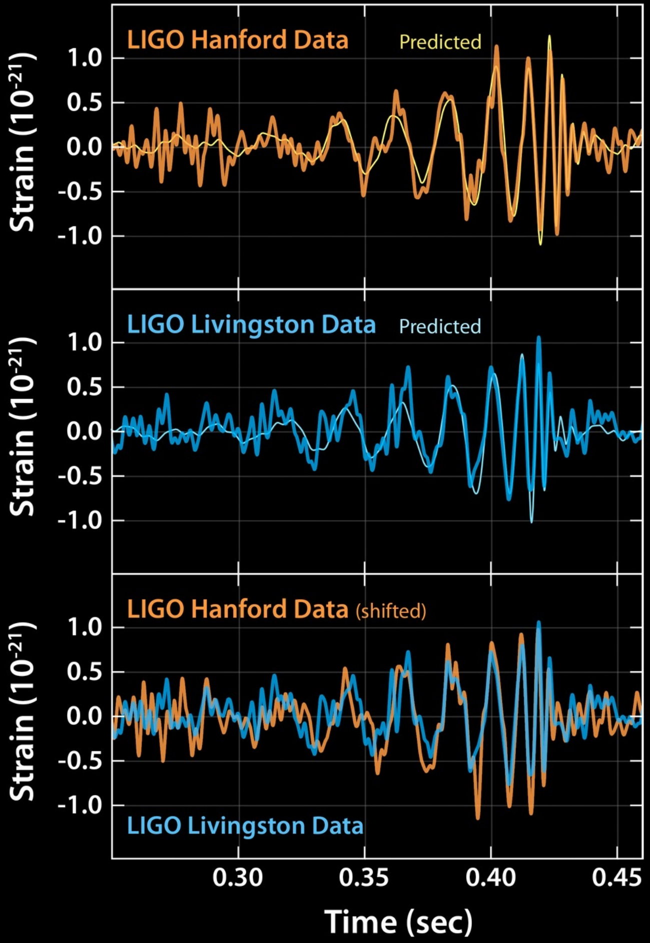
2006 – Buonanno-Cook-Pretorius fit overtones to Pretorius' equal-mass simulations – but are they physical?
Spherical-spheroidal mixing: numerical simulations use the "wrong" basis (Berti-Cardoso-Casals 2005)

2007 – Berti+ quantify excitation of higher multipole QNMs in unequal-mass, nonspinning mergers

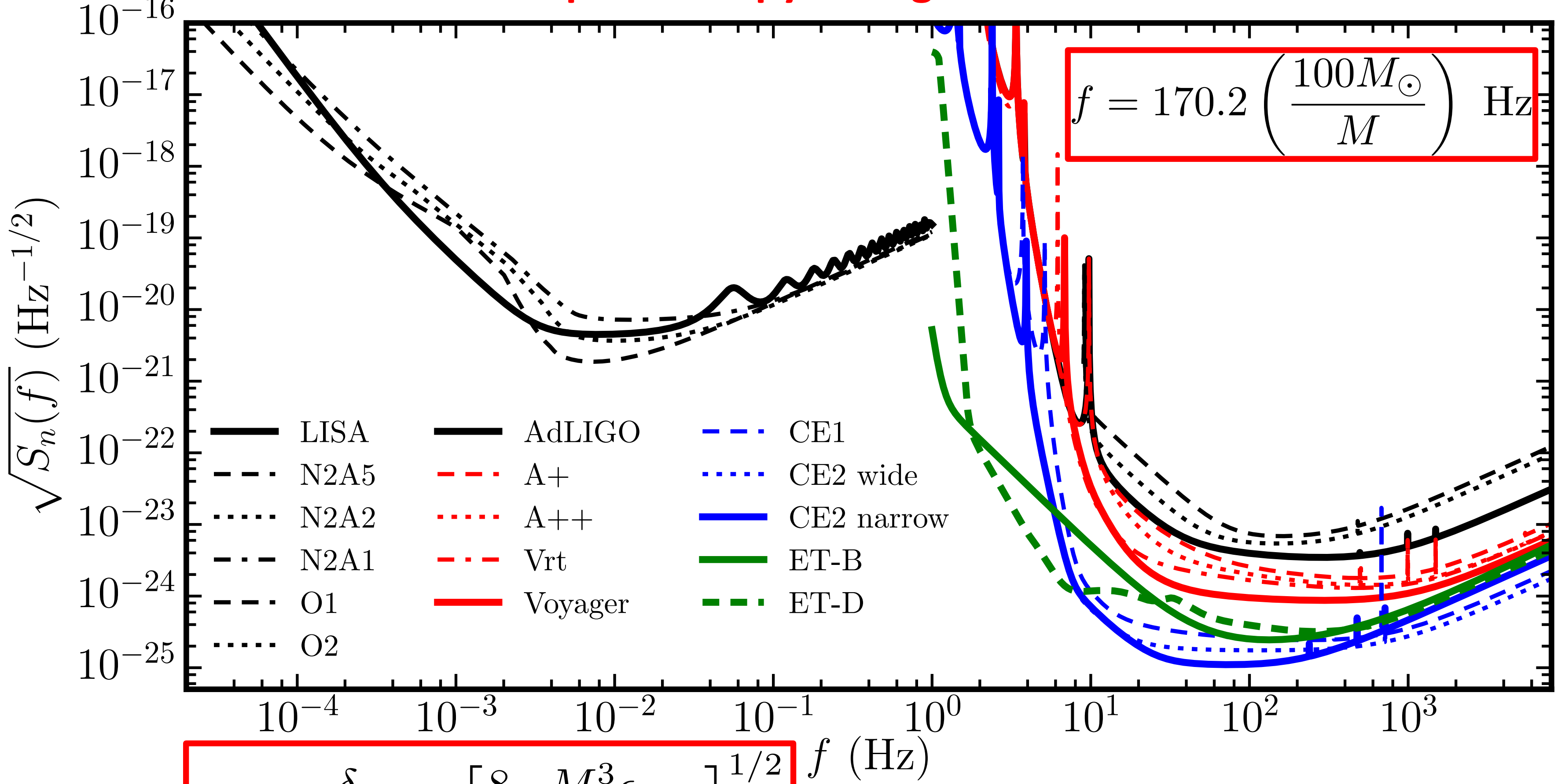
2012, 2014 – Gossan+, Meidam+ first Bayesian study of ringdown



A new Golden age: GW150914 – SNR~7 in ringdown



Black hole spectroscopy with gravitational waves

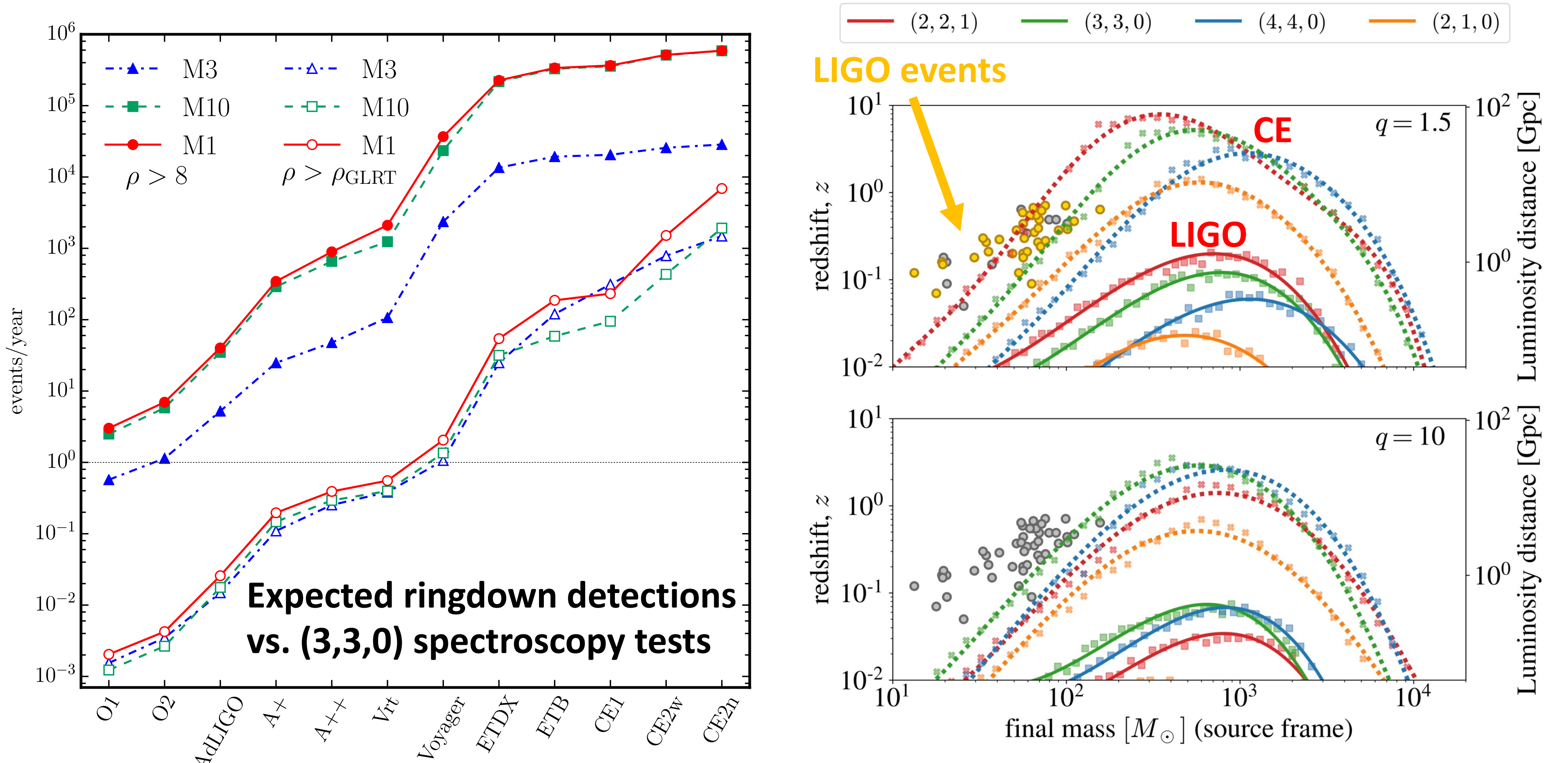


$$f = 170.2 \left(\frac{100 M_\odot}{M}\right) \text{ Hz}$$

$$\rho = \frac{\delta_{\rm eq}}{D_{\rm L} \mathcal{F}_{lmn}} \left[\frac{8}{5} \frac{M_z^3 \epsilon_{\rm rd}}{S_n(f_{lmn})}\right]^{1/2}$$

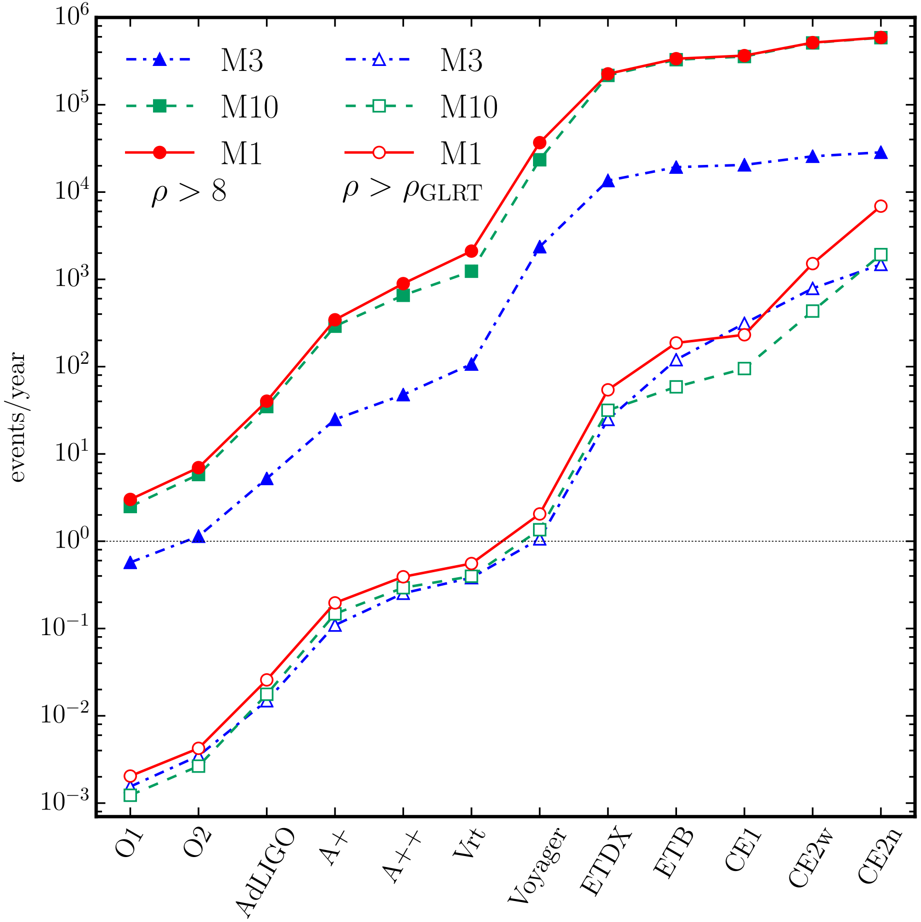
[EB+, 1605.09286]

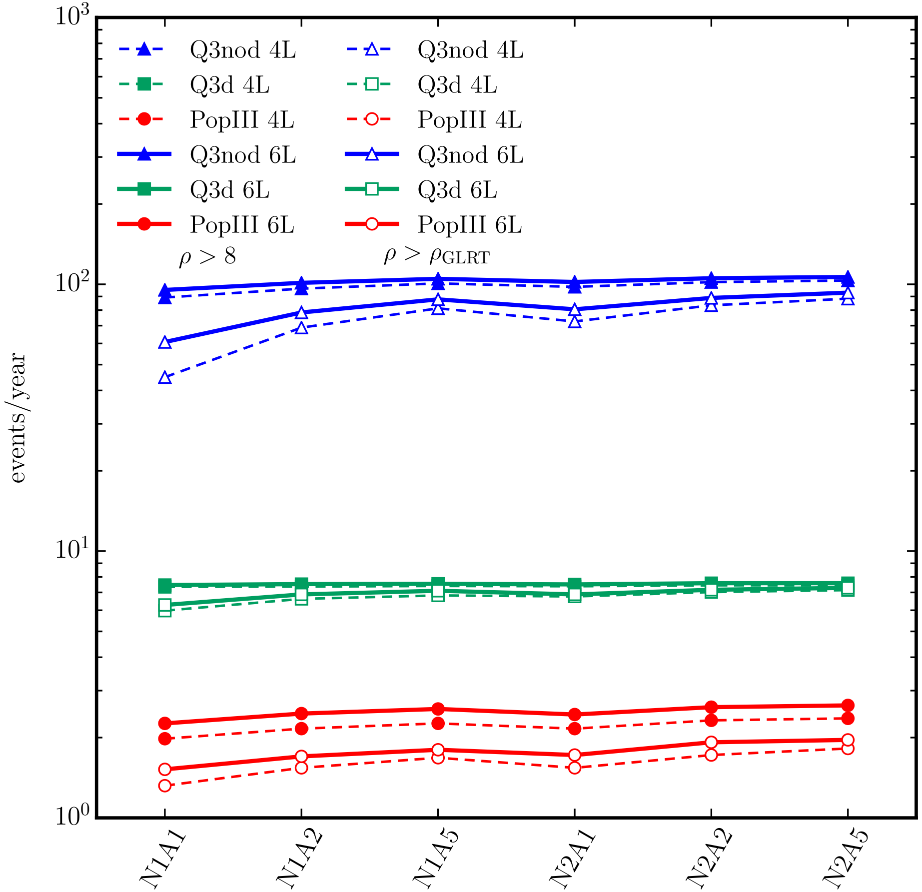
Black hole spectroscopy horizons



[EB+, 1605.09286; Jiménez Forteza+, 2005.03260; Ota-Chirenti, 2108.01774…]

Earth vs. space-based: ringdown detections and black hole spectroscopy

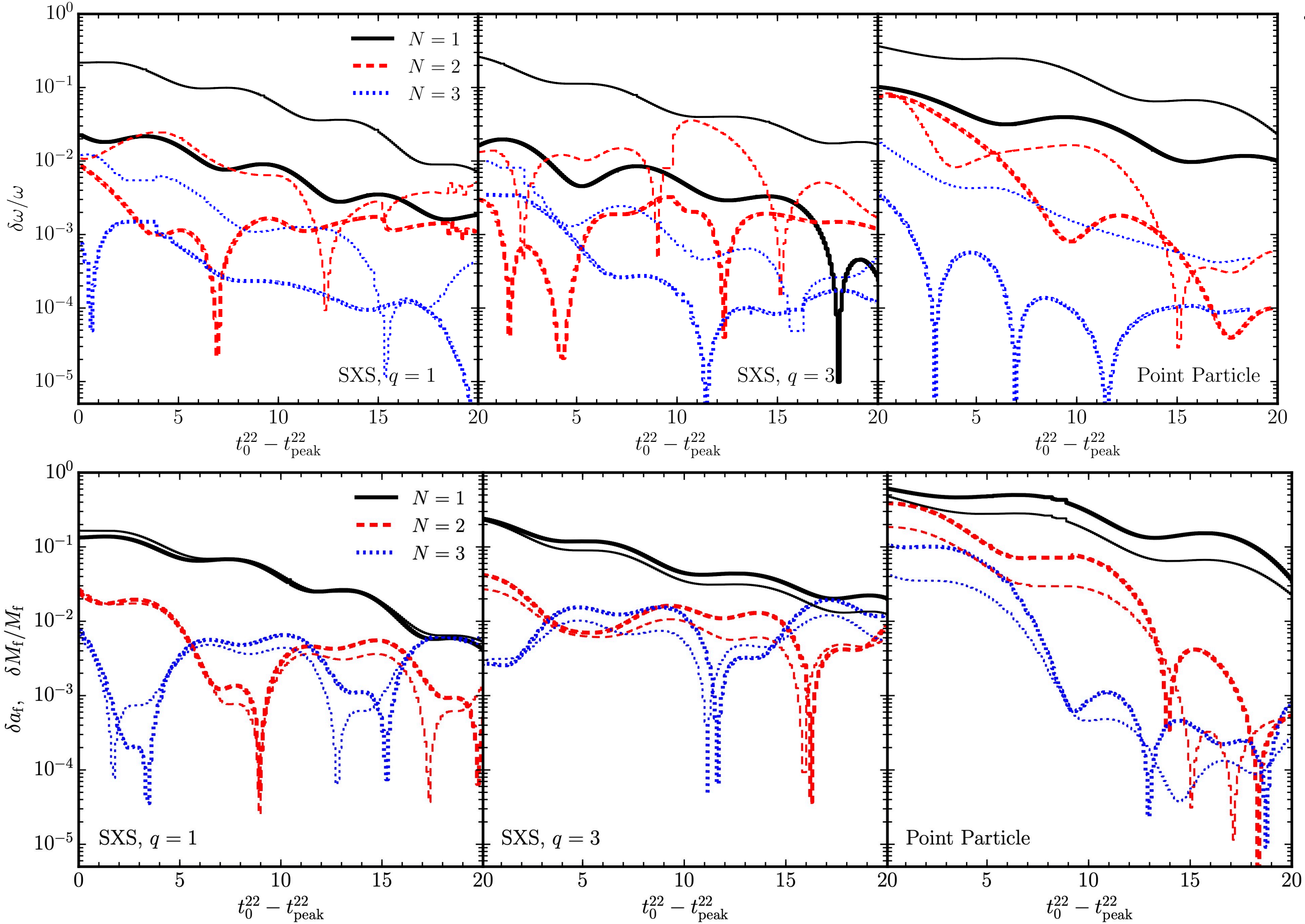




[EB+, 1605.09286]

Overtones, nonlinearities, and elephants:
Is the merger linear?
Is it all much easier than this?

From point particles to NR: overtones needed to reduce mass/spin errors



Top:
real part (thick)
imaginary part (thin)

1% determination of ω_{220} needs one overtone (better if two or three)

Bottom:
spin (thick)
mass (thin)

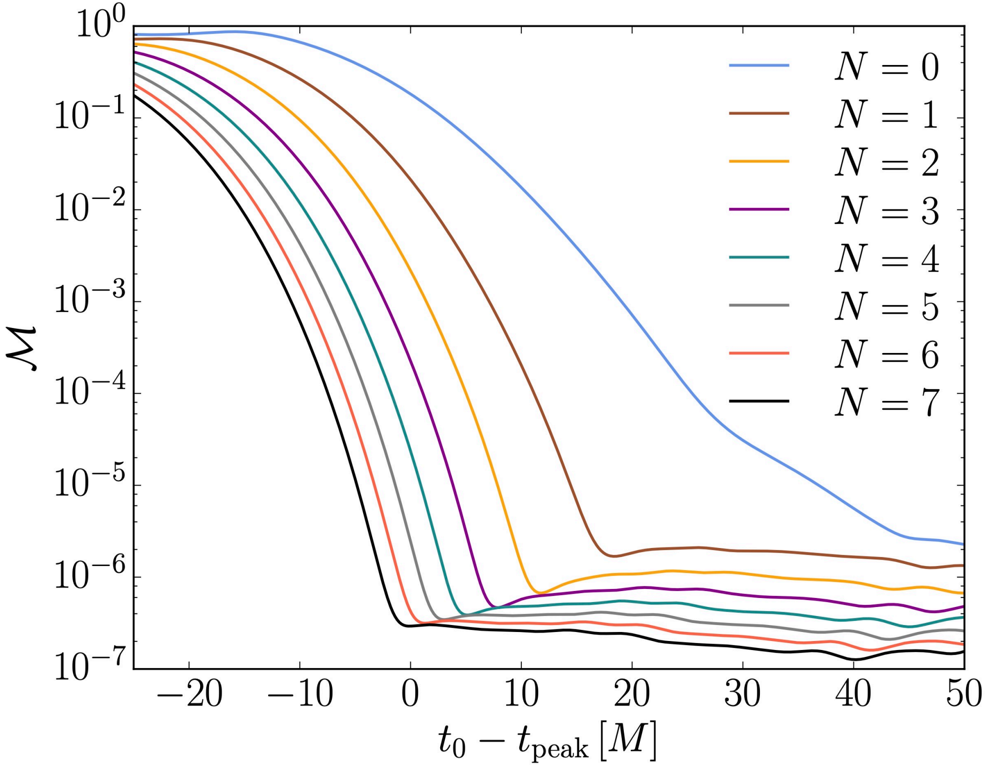
1% determination of mass and spin needs at least two modes

[Baibhav+, 1710.02156]

Nonlinear merger: is it just a superposition of linear QNMs?

"Including overtones allows for the modeling of the ringdown signal for all times beyond the peak strain amplitude, indicating that **the linear quasinormal regime starts much sooner than previously expected**. This implies that the spacetime is **well described as a linearly perturbed black hole with a fixed mass and spin as early as the peak**"

$$\mathcal{M} = 1 - \frac{\langle h_{22}^{\mathrm{NR}}, h_{22}^{N} \rangle}{\sqrt{\langle h_{22}^{\mathrm{NR}}, h_{22}^{\mathrm{NR}} \rangle \langle h_{22}^{N}, h_{22}^{N} \rangle}}$$



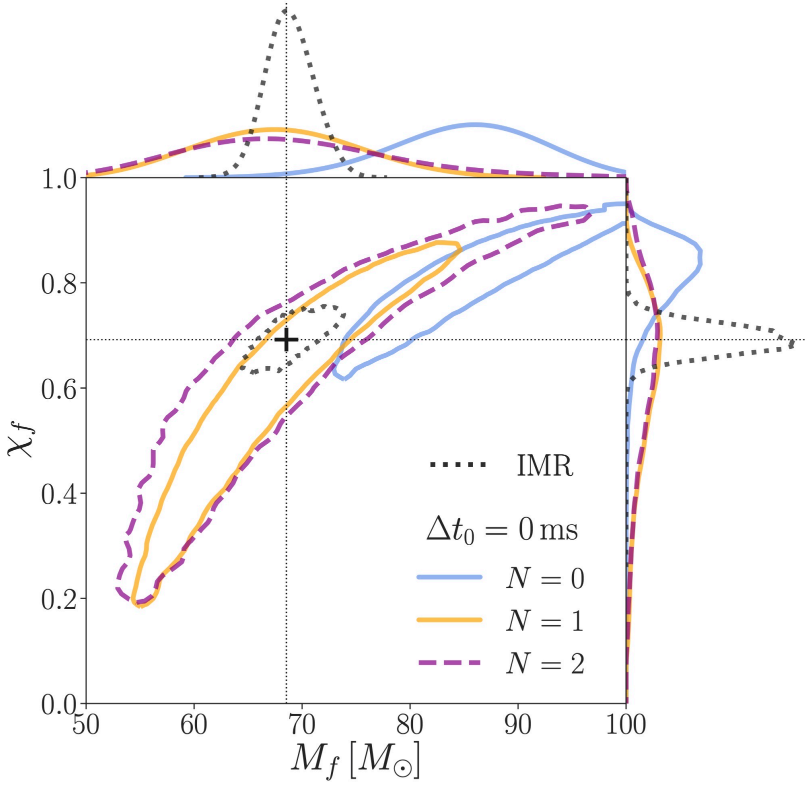
Does it?

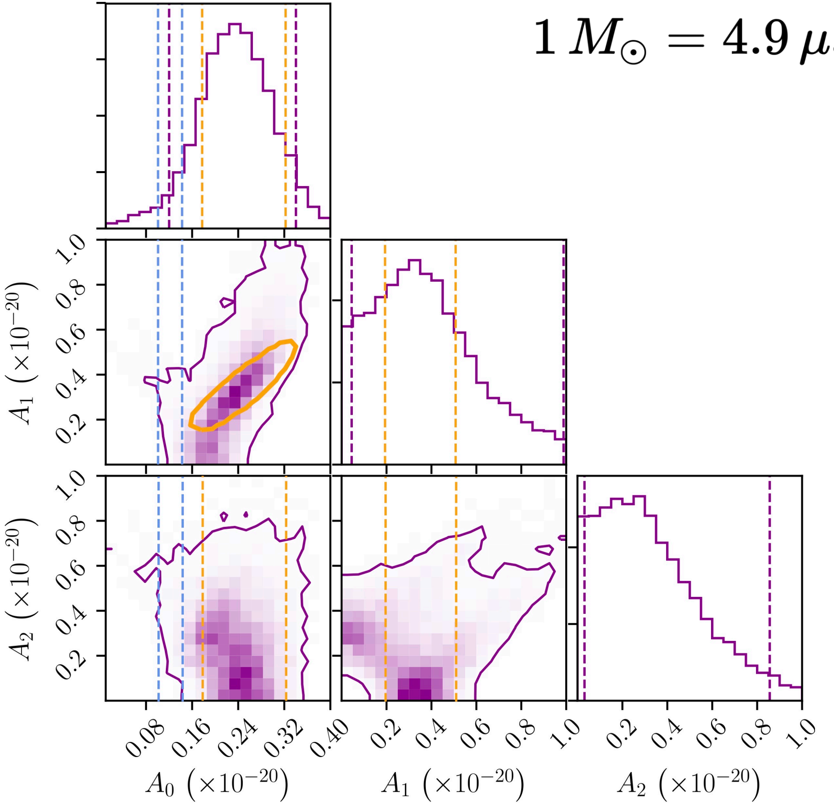
[Giesler+, 1903.08284]

GW150914 tests of the no-hair theorem with the first overtone?

Overtones improve quality of consistency tests for GW150914

Is the overtone detection robust? Assumes t_{start}=1126259462.423 s



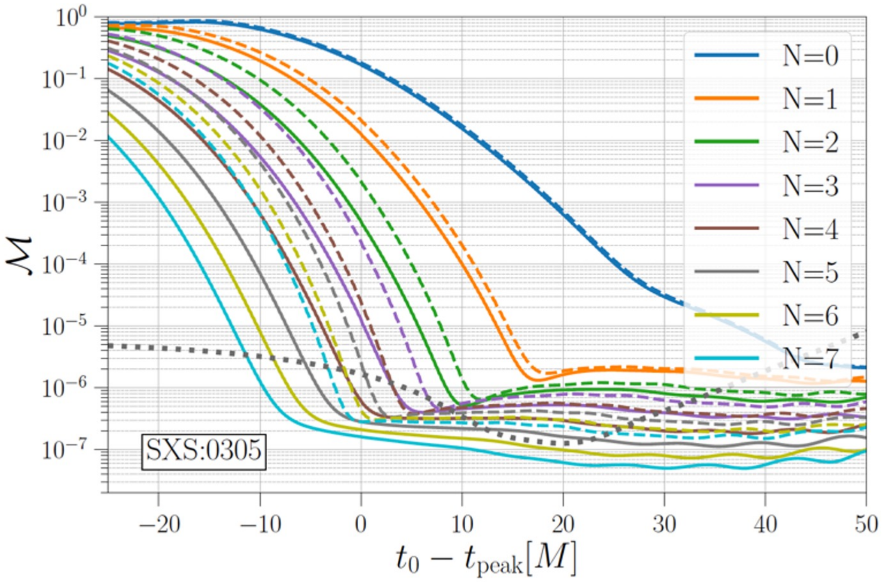


$$1\,M_\odot = 4.9\,\mu s$$

[Isi+, 1905.00869]

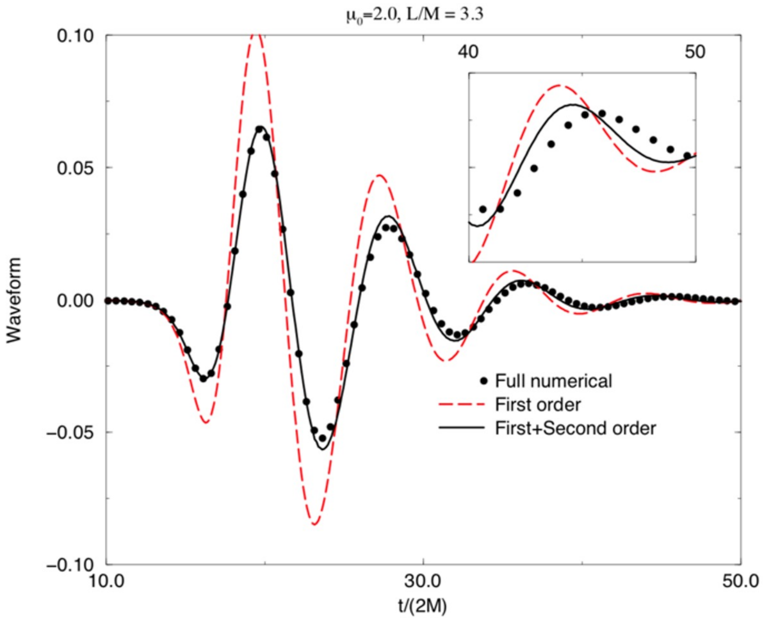
Why would the full merger be linear? Two counterarguments (credits: Carullo)

Counterrotating modes fit well before the peak even for systems with positive aligned spins.
This clashes with point-particle understanding [Bernuzzi+, 1003.0597; Barausse+, 1110.3081]
Why? Because lower frequencies match inspiral: see EOB "pseudo-QNMs"



[Dhani, 2010.08602]

Post-merger head-on collisions are nonlinear
Need second-order perturbation theory
First order has larger peak amplitude in both cases
Second-order corrections decrease it



[Gleiser+, gr-qc/9609022]

Search for nonlinearities and nonlinear modes

Two stages

Before the 2005 NR breakthrough: perturbation theory to the rescue

Close limit approximation [e.g. Gleiser+ gr-qc/9609022…]

“Lazarus project”, second-order Kerr [e.g. Campanelli-Lousto gr-qc/9811019]

After the 2005 NR breakthrough:

Where are all the nonlinearities?

[Zlochower+, gr-qc/0306098; Ioka-Nakano, 0704.3467 + 0708.0450; Brizuela+, 0903.1134; Pazos+, 1009.4665]

$$\psi = \epsilon\psi_{(1)} + \epsilon^2\psi_{(2)}$$

$$\mathcal{L}\psi_{(2)} \propto \psi_{(1)}^2 \sim A_1 A_2 e^{i(\phi_1+\phi_2)}$$

Pioneering search for nonlinearities in the Georgia Tech NR catalog

[London+, 1404.3197]

Recent explosion of activity – analytical and numerical

[Loutrel+, 2008.11770; Ripley+, 2010.00162; Magana-Zertuche+, 2110.15922; Sberna+, 2112.11168; Ma+, 2207.10870; Lagos-Hui, 2208.07379; Cheung+, 2208.07374; Mitman+, 2208.07380; Zhu+, 2309.13204; Kehagias+, 2301.09345 + 2302.01240; Nee+, 2302.06634; Perrone+, 2308.15886; Bucciotti+, 2309.08501…]

Extrapolating linear theory to the nonlinear merger: are we fitting elephants?

"Including overtones allows for the modeling of the ringdown signal for all times beyond the peak strain amplitude, indicating that **the linear quasinormal regime starts much sooner than previously expected**. This implies that the spacetime is **well described as a linearly perturbed black hole with a fixed mass and spin as early as the peak**"

N	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	$t_{\rm fit} - t_{\rm peak}$
0	0.971	...	...	...	...	...	...	...	47.00
1	0.974	3.89	...	...	...	...	...	...	18.48
2	0.973	4.14	8.1	...	...	...	...	...	11.85
3	0.972	4.19	9.9	11.4	...	...	...	...	8.05
4	0.972	4.20	10.6	16.6	11.6	...	...	...	5.04
5	0.972	4.21	11.0	19.8	21.4	10.1	...	...	3.01
6	0.971	4.22	11.2	21.8	28	21	6.6	...	1.50
7	0.971	4.22	11.3	23.0	33	29	14	2.9	0.00

[Giesler+, 1903.08284]

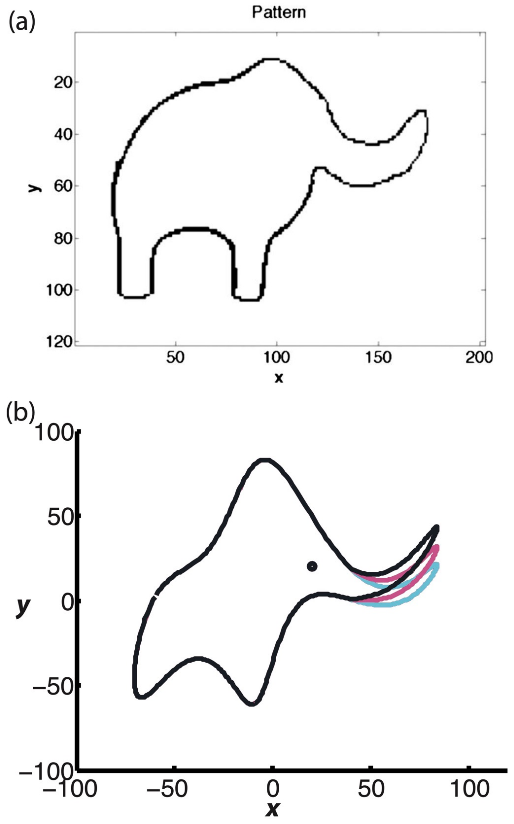
Crucial: (complex) frequencies are fixed! Fitted mode amplitudes change a lot with the number of overtones

[Bhagwat+, Mourier+, Jimenez-Forteza+, Sberna+…]

Extrapolating linear theory to the nonlinear merger: are we fitting elephants?

Freeman Dyson: https://www.youtube.com/watch?v=hV41QEKiMlM

One of the big turning points in my life was a meeting with Enrico Fermi in the spring of 1953. In a few minutes, Fermi politely but ruthlessly demolished a programme of research that my students and I had been pursuing for several years. He probably saved us from several more years of fruitless wandering along a road that was leading nowhere. I am eternally grateful to him for destroying our illusions and telling us the bitter truth. […] He delivered his verdict in a quiet, even voice. "There are two ways of doing calculations in theoretical physics", he said. "One way, and this is the way I prefer, is to have a clear physical picture of the process that you are calculating. The other way is to have a precise and self-consistent mathematical formalism. You have neither."

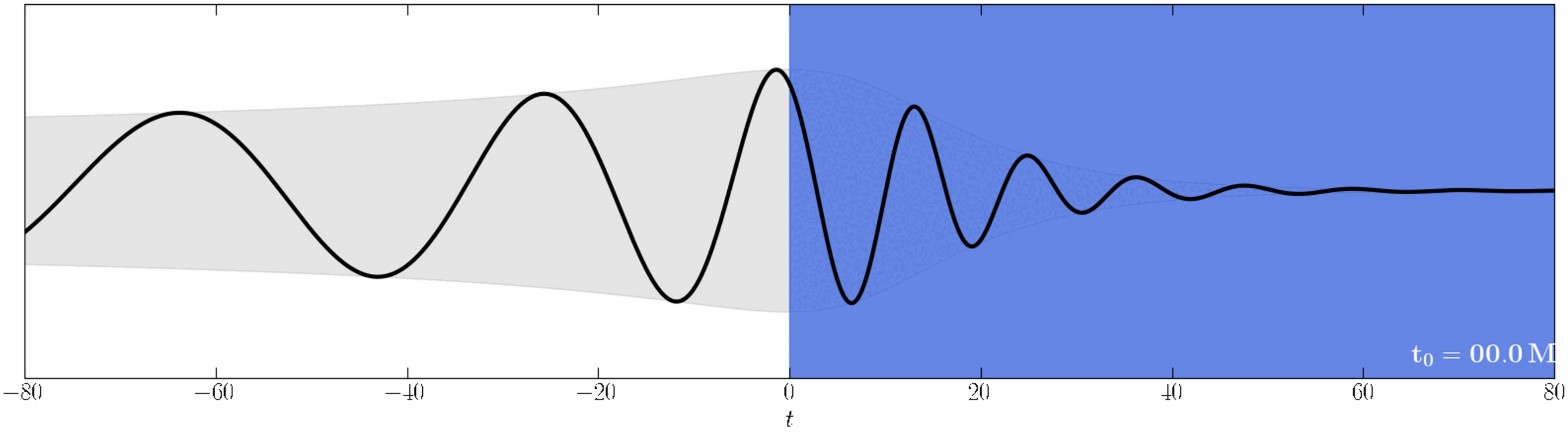


In desperation I asked Fermi whether he was not impressed by the agreement between our calculated numbers and his measured numbers.
He replied, "How many arbitrary parameters did you use for your calculations?"
I thought for a moment about our cut-off procedures and said, "Four."
He said, "I remember my friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiggle his trunk."
With that, the conversation was over.

[Freeman Dyson, Nature 427, 297 (2004); Mayer+, Am. J. Phys. 78 (6), 2010]

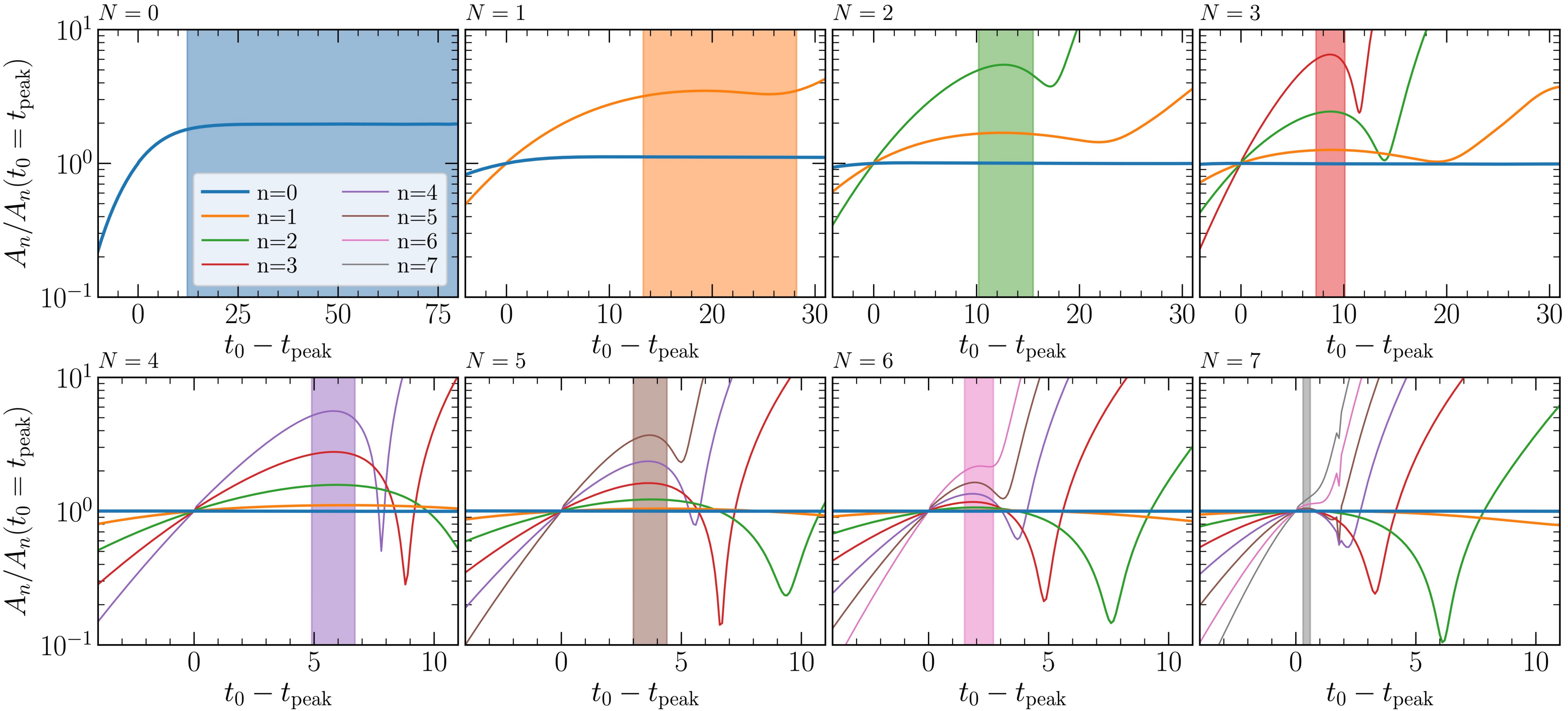
Is the linear model consistent when we change the fitting window?

$$h = \sum \boxed{A} e^{-\omega_i(\chi, M)t} \cos(\omega_r(\chi, M)t + \boxed{\phi})$$



Nonlinear merger QNM amplitudes are not constant near the peak

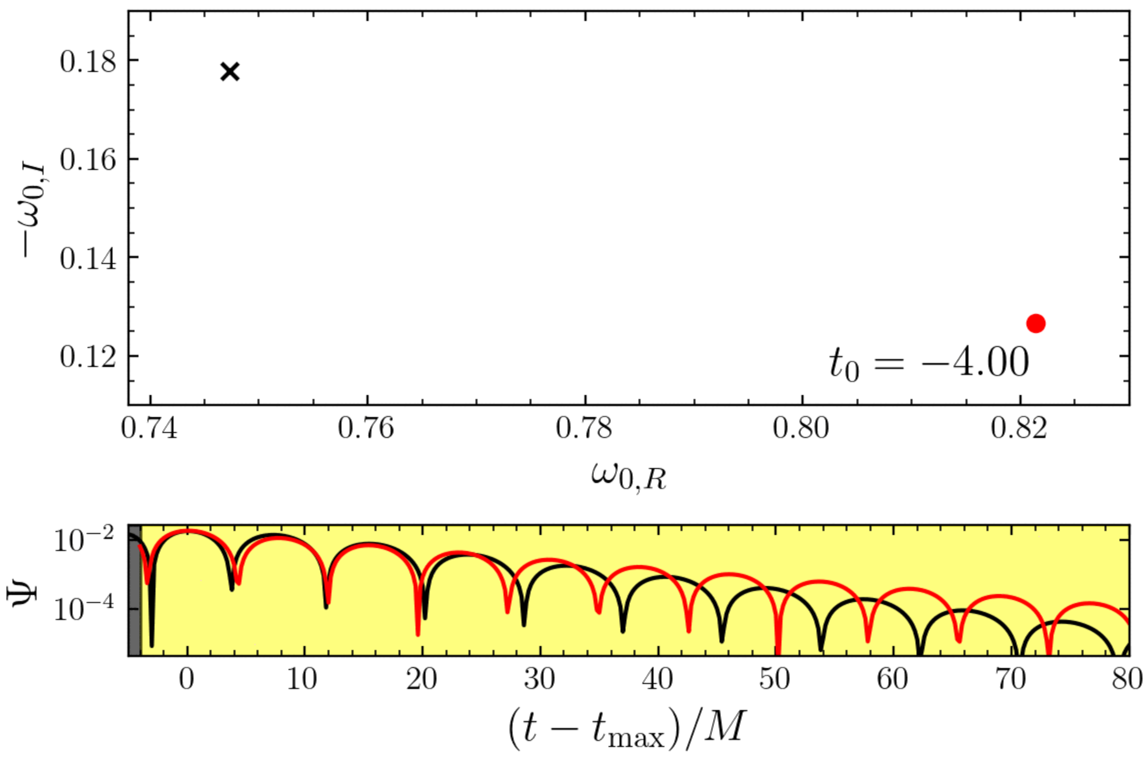
"Fixed-frequency" fits as in Giesler (weaker test). Bands show regions where amplitudes are constant within 10%

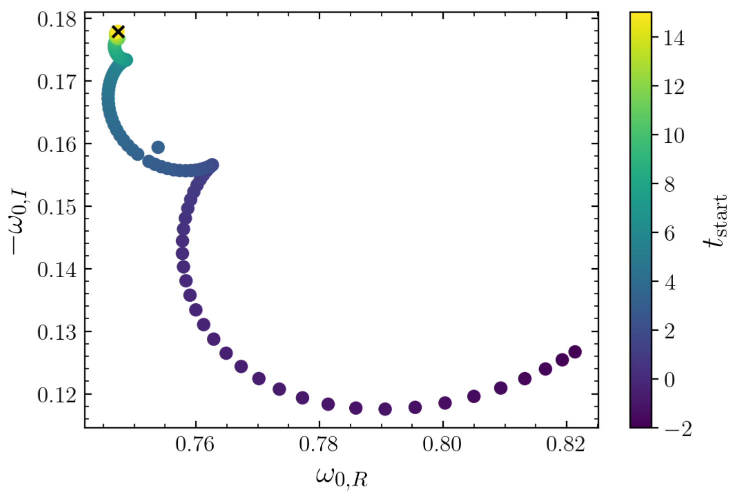


[Baibhav+, 2302.03050; see also Bhagwat+, 1910.08708]

Back to basics: Vishveshwara scattering experiment, fundamental mode

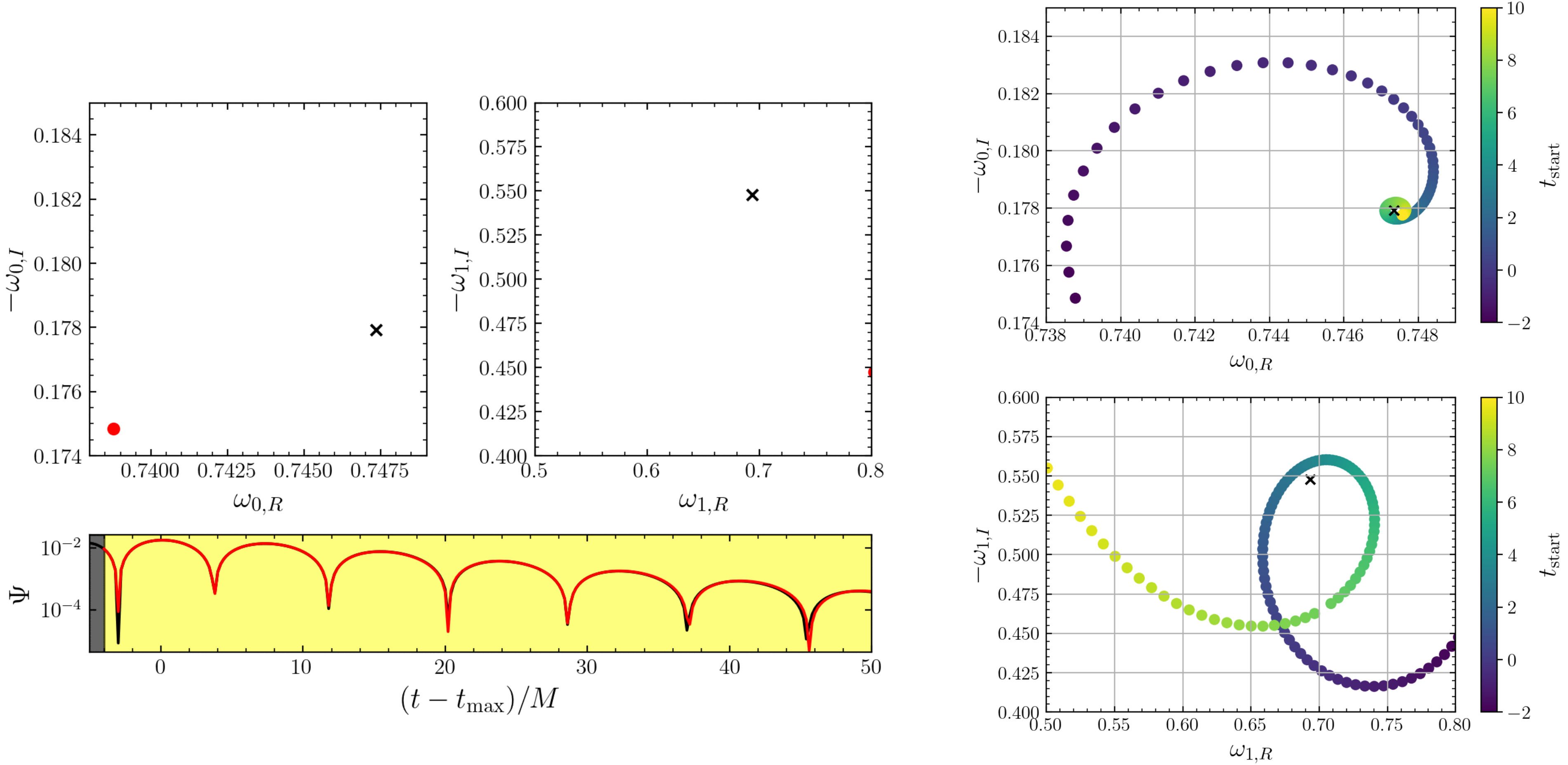
Preliminary question: can we recover overtone frequencies agnostically? **No**





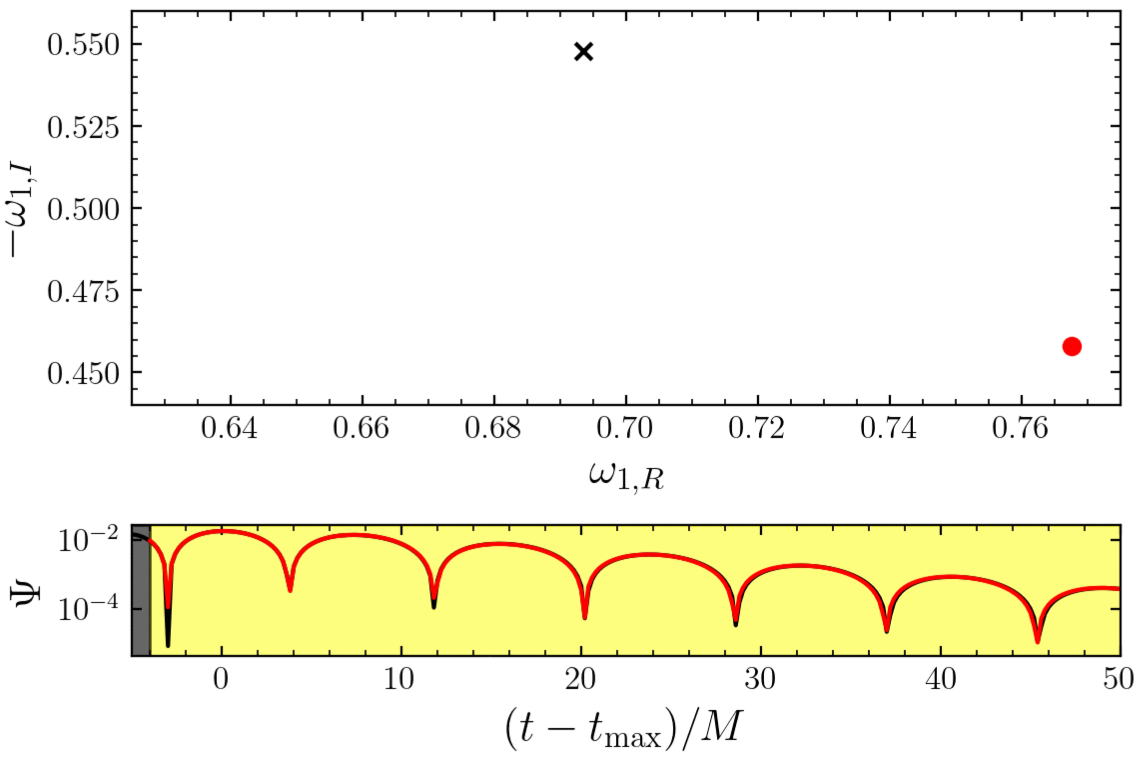
[Baibhav+, 2302.03050]

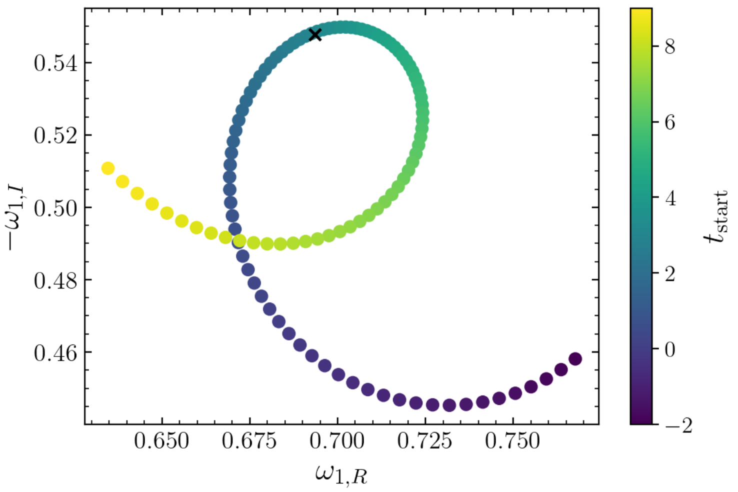
Vishveshwara scattering experiment, first overtone: N=1; n=0 and n=1 free



[Baibhav+, 2302.03050]

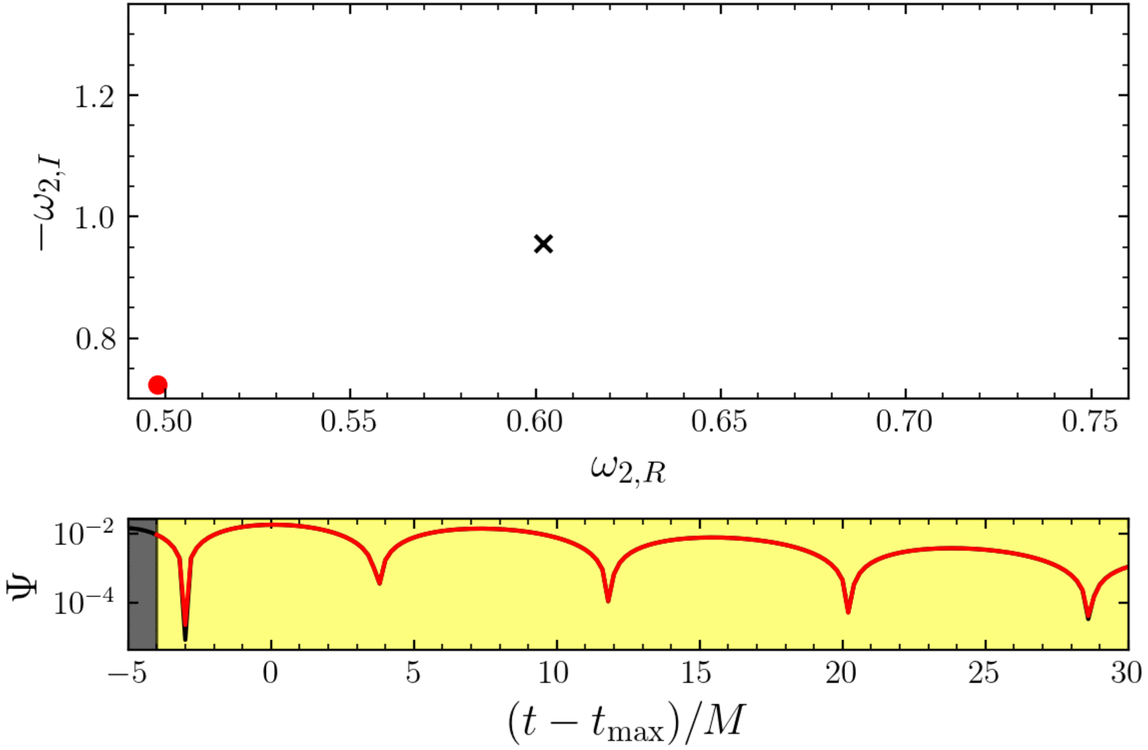
Vishveshwara scattering experiment, first overtone: N=1; n=0 fixed; n=1 free

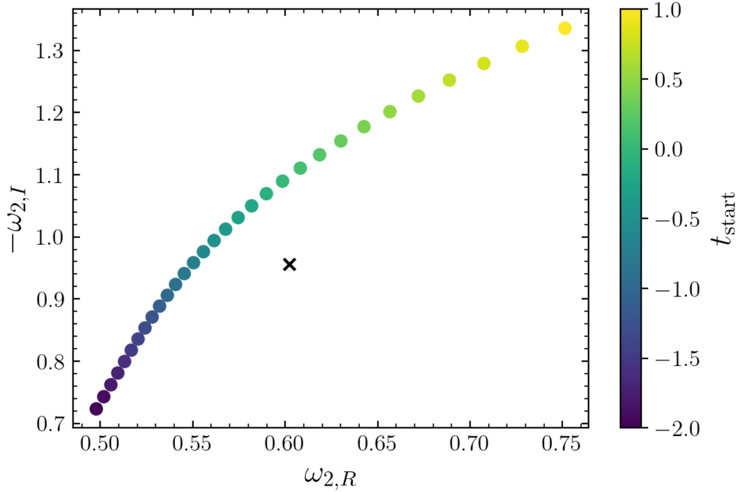




[Baibhav+, 2302.03050]

Vishveshwara scattering experiment, two overtones: N=2; n=0, n=1 fixed; n=2 free

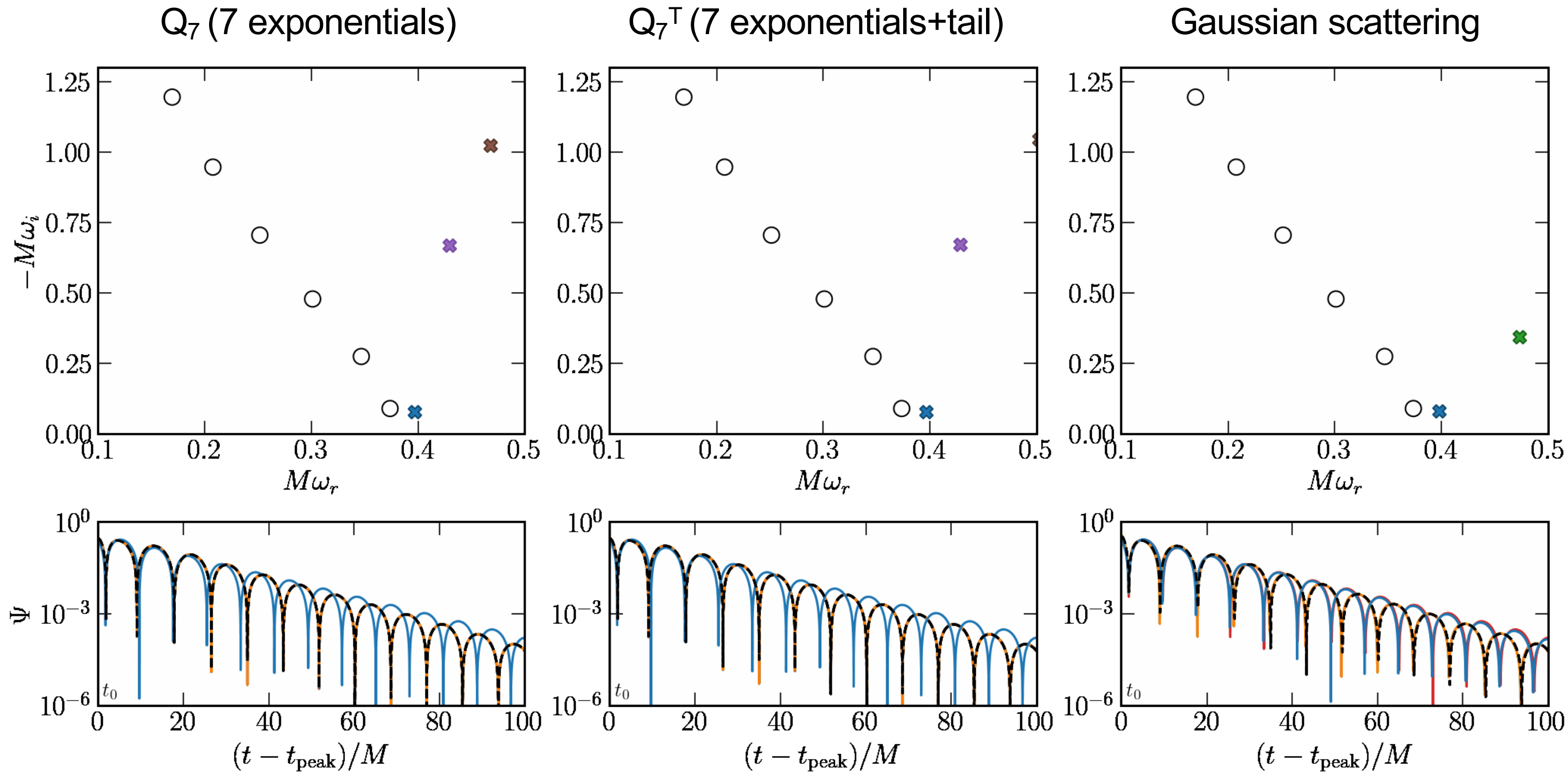




[Baibhav+, 2302.03050]

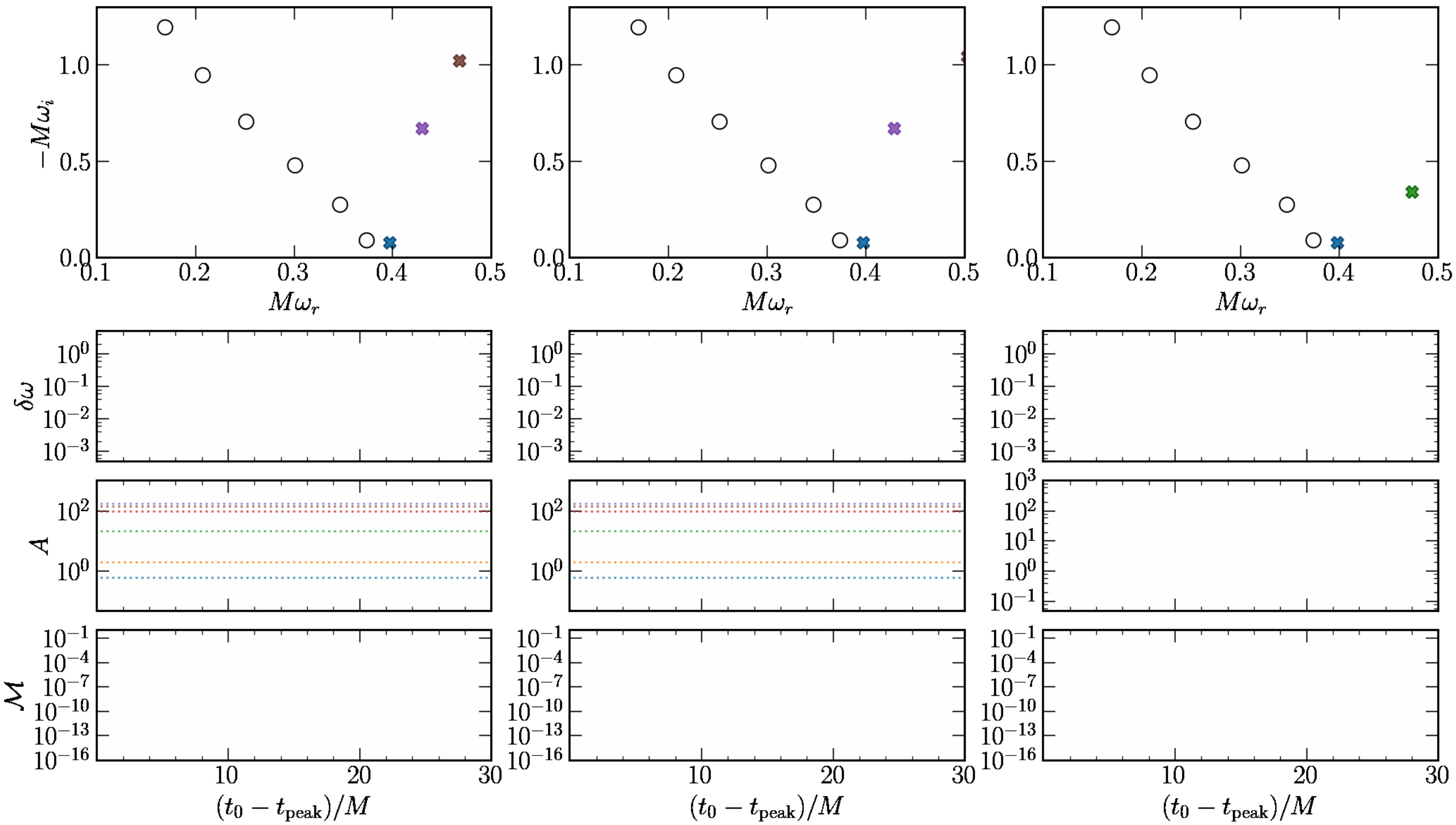
Why do free-frequency fits fail? QNM incompleteness? Tails? Prompt response?

Three increasingly realistic waveforms in the linear regime.
“Free-frequency” fitting here means: fix all previous modes and look for the last one.



[Baibhav+, 2302.03050]

Q$_7$ (7 exponentials) | Q$_7^{\mathrm{T}}$ (7 exponentials+tail) | Gaussian scattering

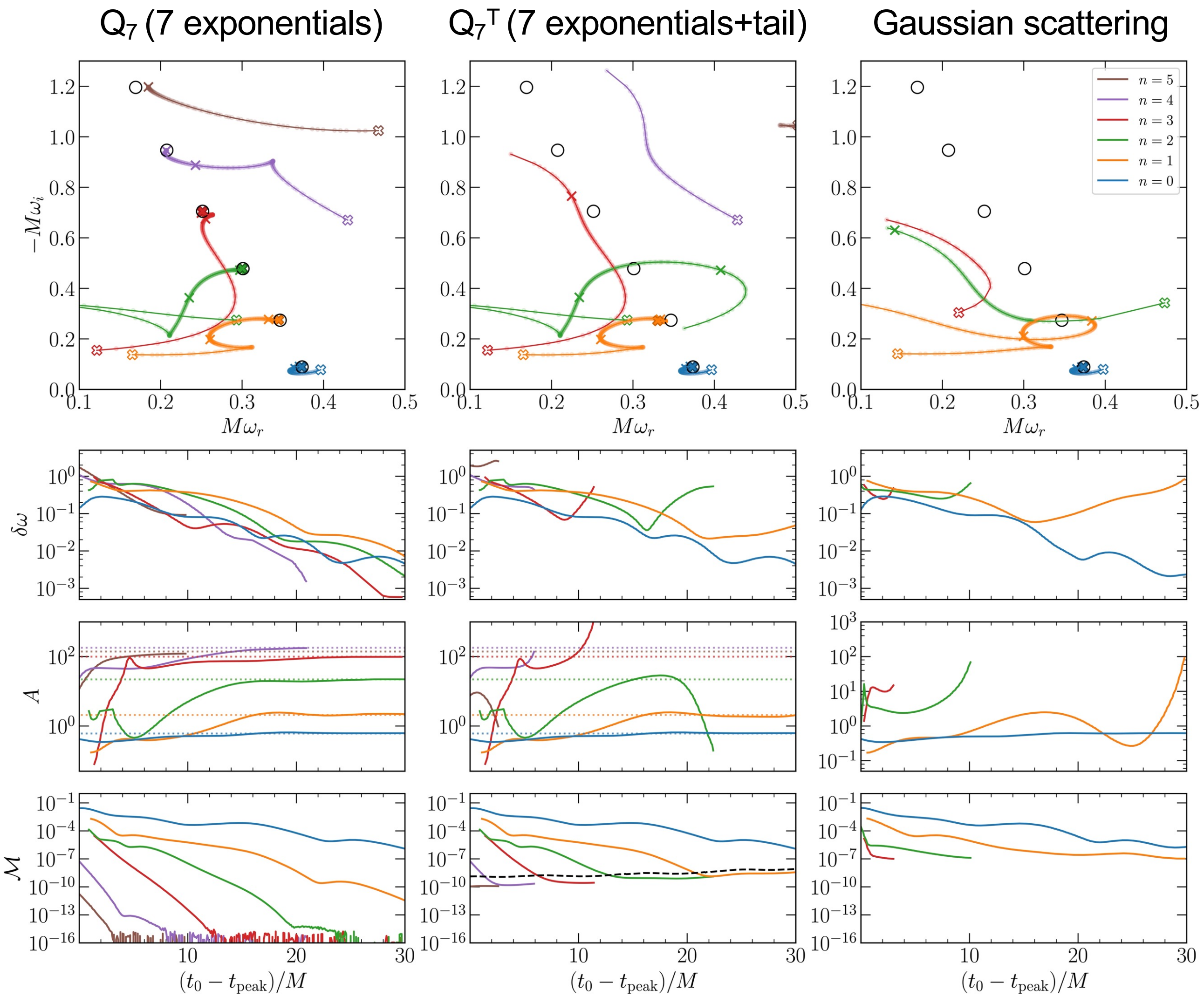


[Baibhav+, 2302.03050]

“Tail” model Q_7^T:
fits fail to find QNMs, but mismatch near the peak keeps decreasing

Overtones may still be there, but we fail to find them because of the extra physics (here just a tail)

Fits stop converging when the mismatch dives below the mismatch between Q_7 and Q_7^T (black dashed line)

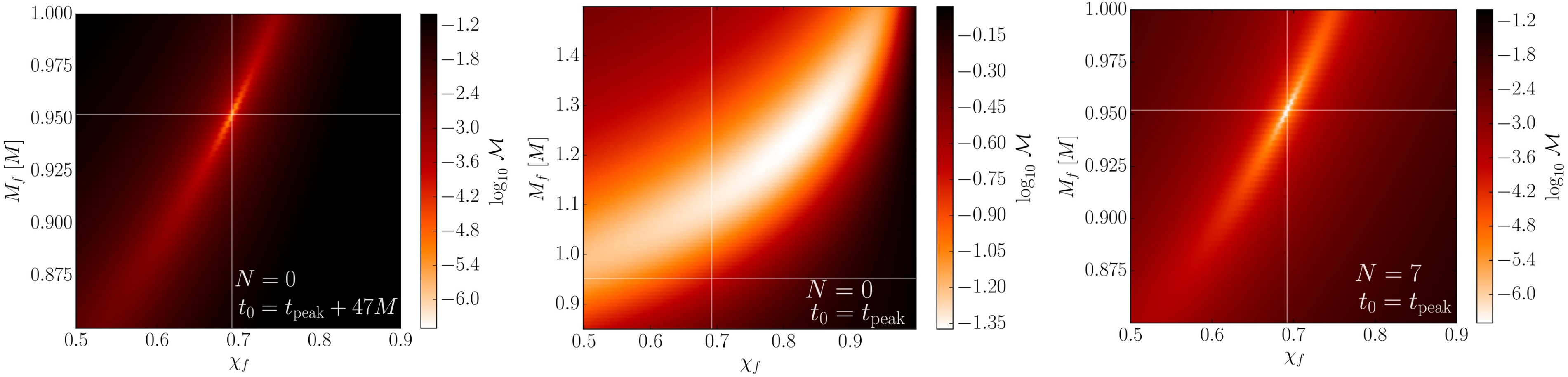


Inferred parameters of the final black hole

Fundamental mode at late times gives good estimate of mass and spin (as expected)

Non-agnostic approach: assume that frequencies are related to mass and spin as in GR
Then adding modes at the peak time improves estimate of mass and spin

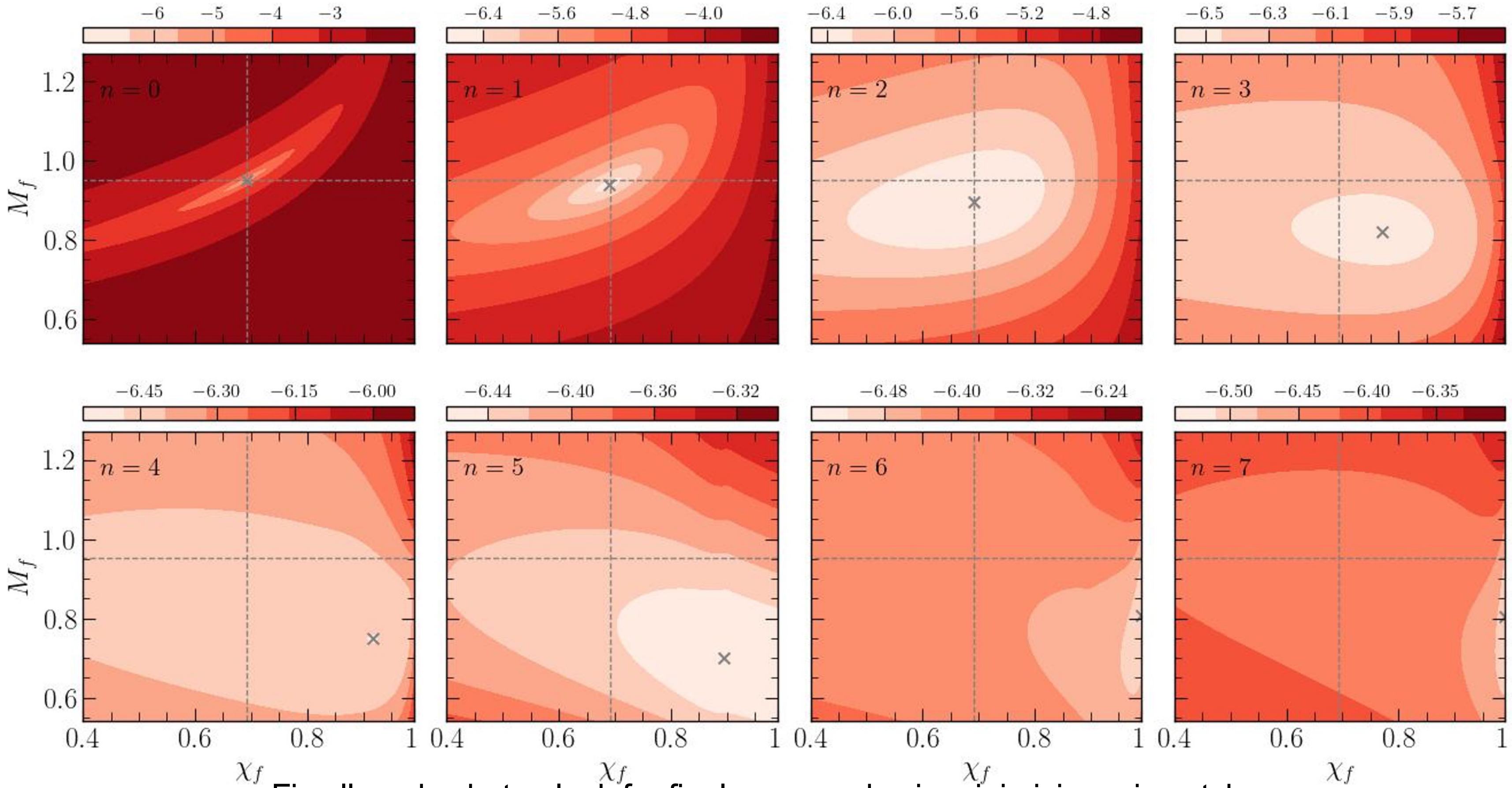
$$\mathcal{M} = 1 - \frac{\langle h_{22}^{\rm NR}, h_{22}^{N} \rangle}{\sqrt{\langle h_{22}^{\rm NR}, h_{22}^{\rm NR} \rangle \langle h_{22}^{N}, h_{22}^{N} \rangle}}$$



Do we really gain physical information by adding N=7 modes? Which ones do we really need?

[Giesler+, 1903.08284]

Inferred parameters of the final black hole are insensitive to higher overtones



Fix all modes but n, look for final mass and spin minimizing mismatch.

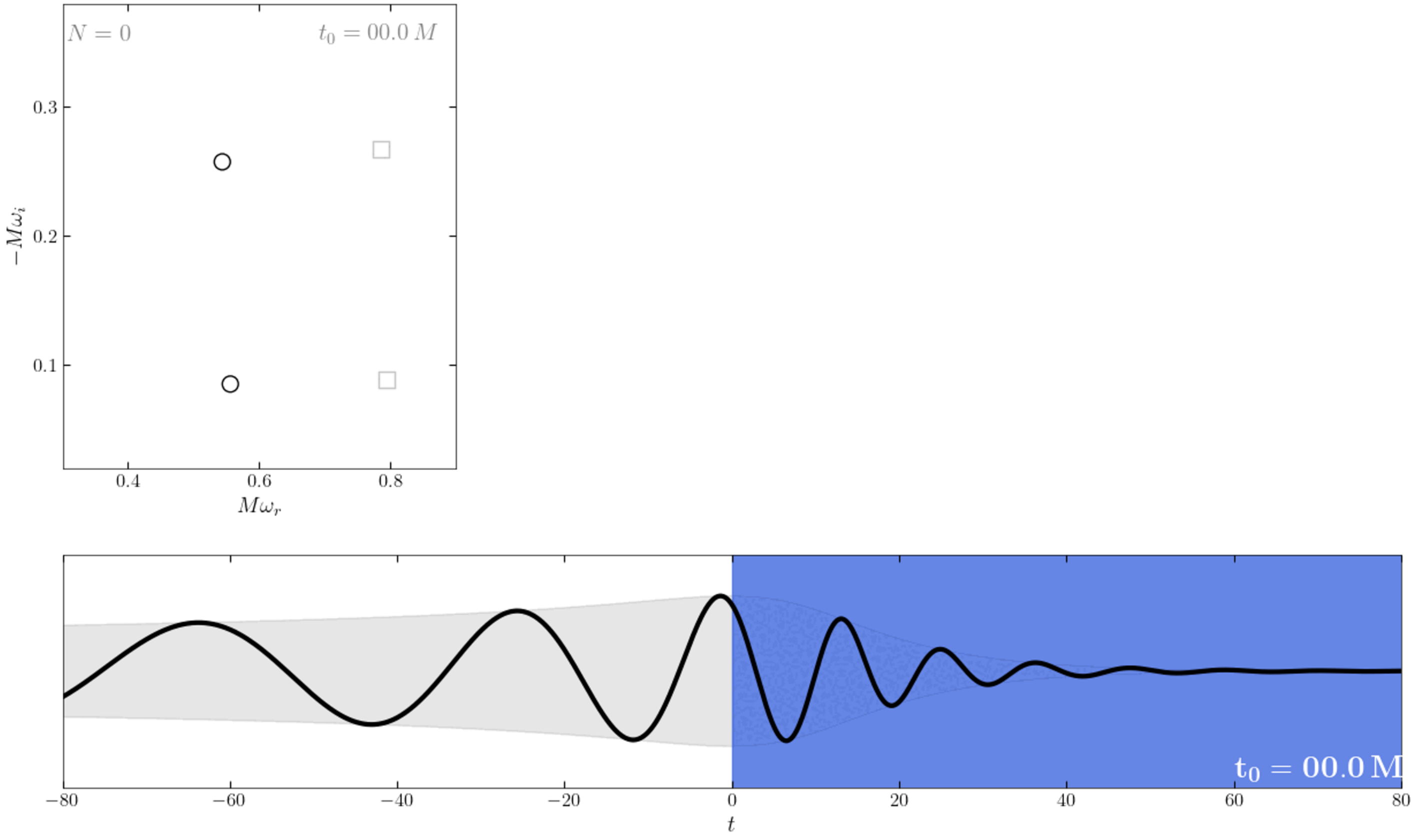
Note the very different scales!

[Baibhav+, 2302.03050]

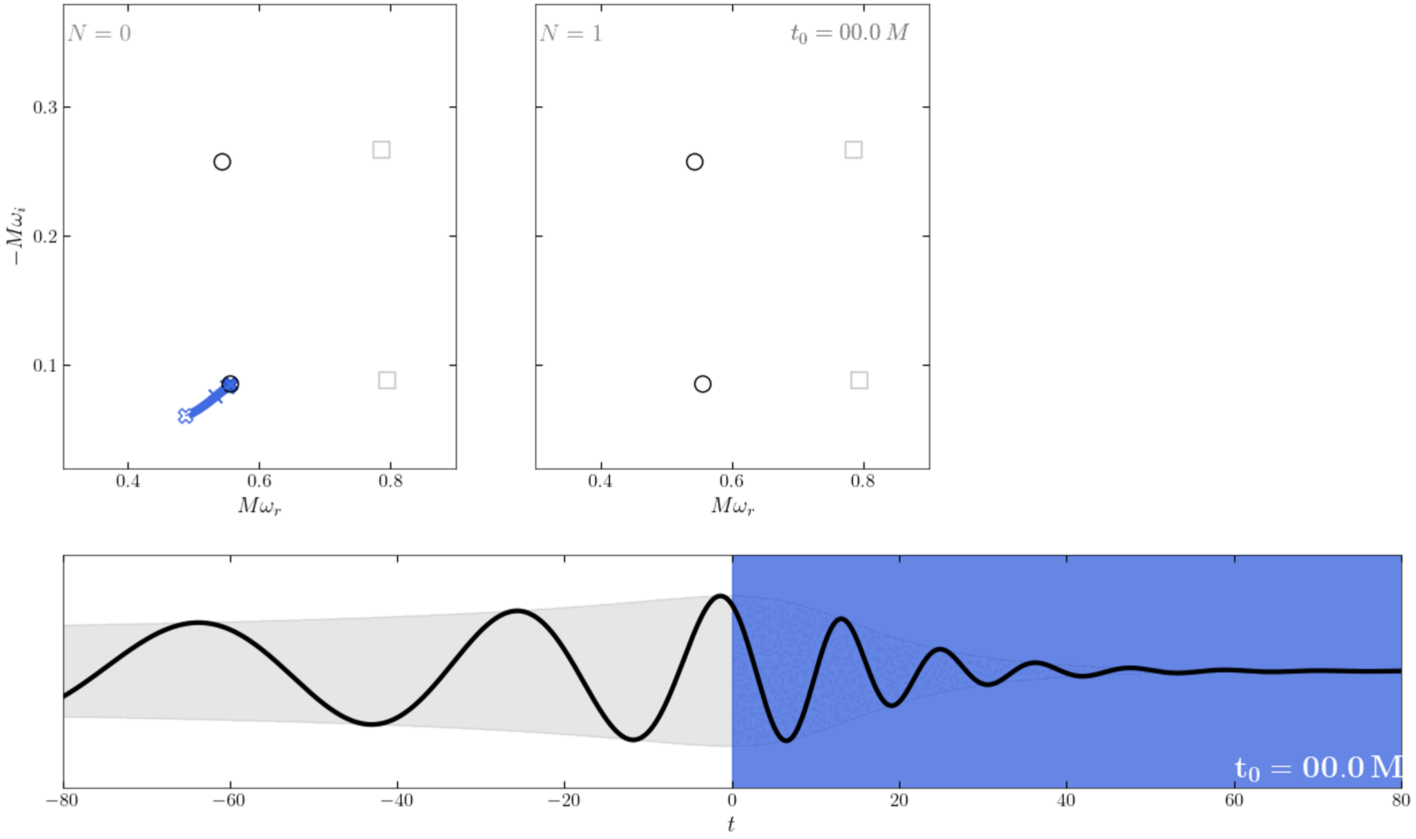
Agnostic spectroscopy

My mom always said life was like a box of chocolates.
You never know what you're gonna get.

Agnostic spectroscopy: fundamental mode from numerical simulations

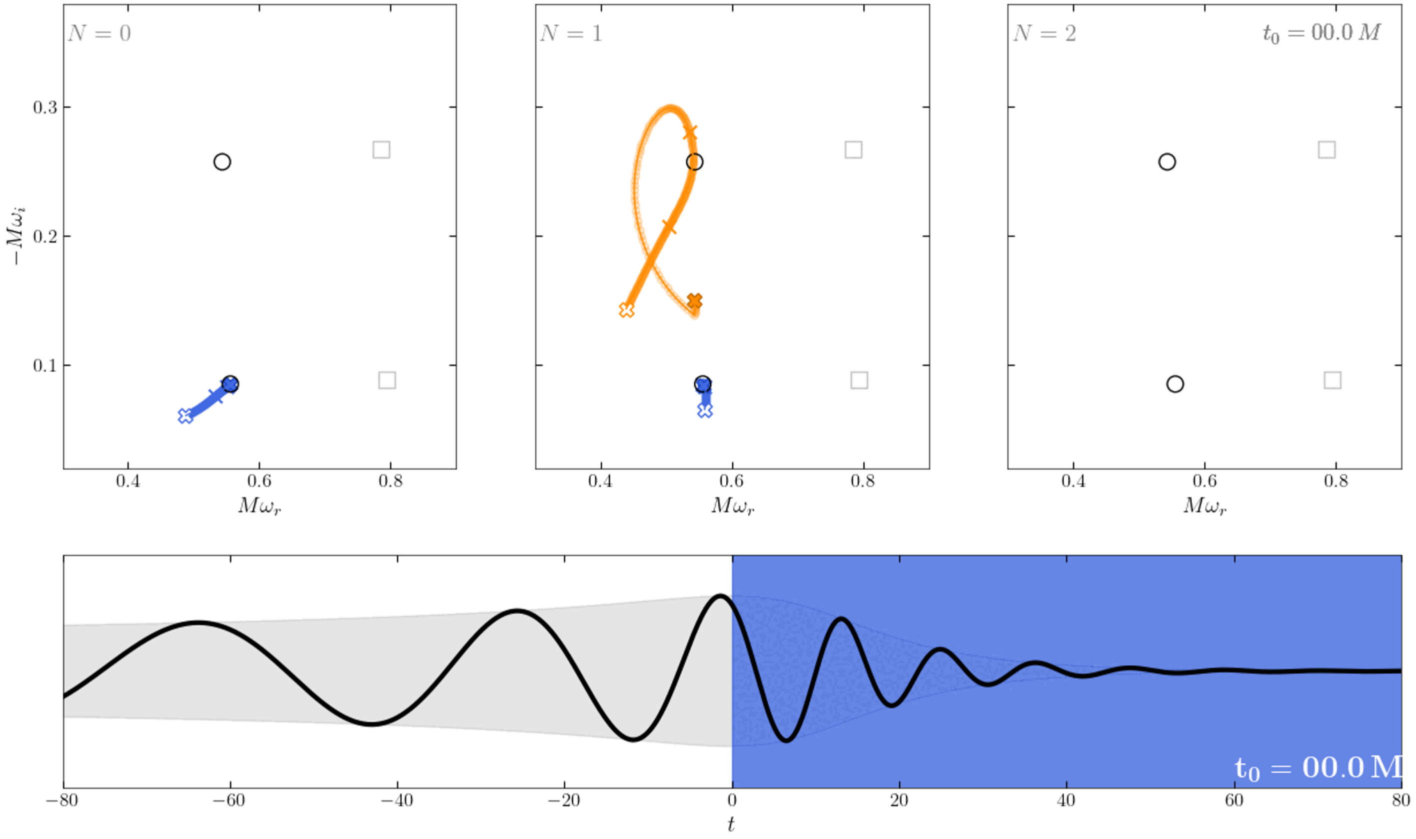


Agnostic spectroscopy: first overtone from numerical simulations

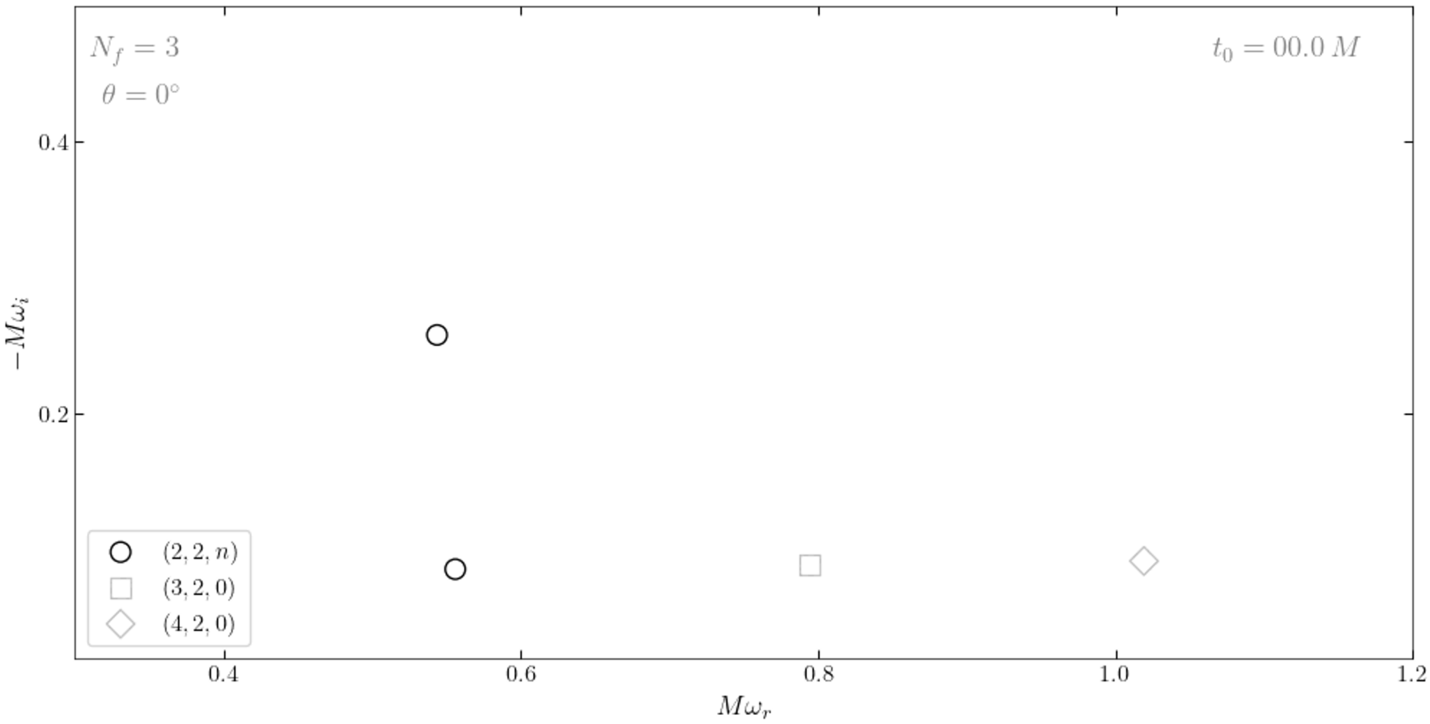


Why wrong? Spherical-spheroidal mode mixing

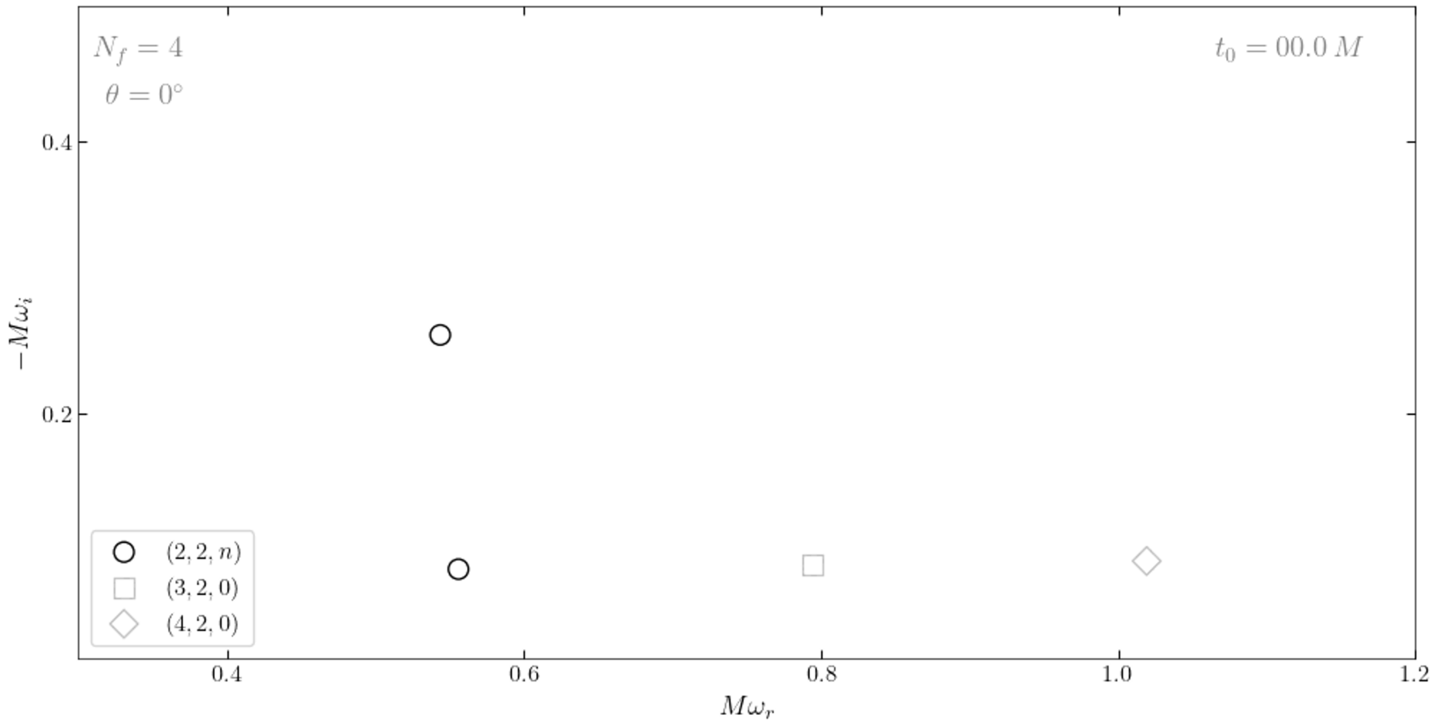
$$ {}_{-2}S_{lm} = {}_{-2}Y_{lm} + j_f\,\tilde{\omega}_{lmn} \sum_{l\neq l'} {}_{-2}Y_{l'm}\, c_{l'lm} $$



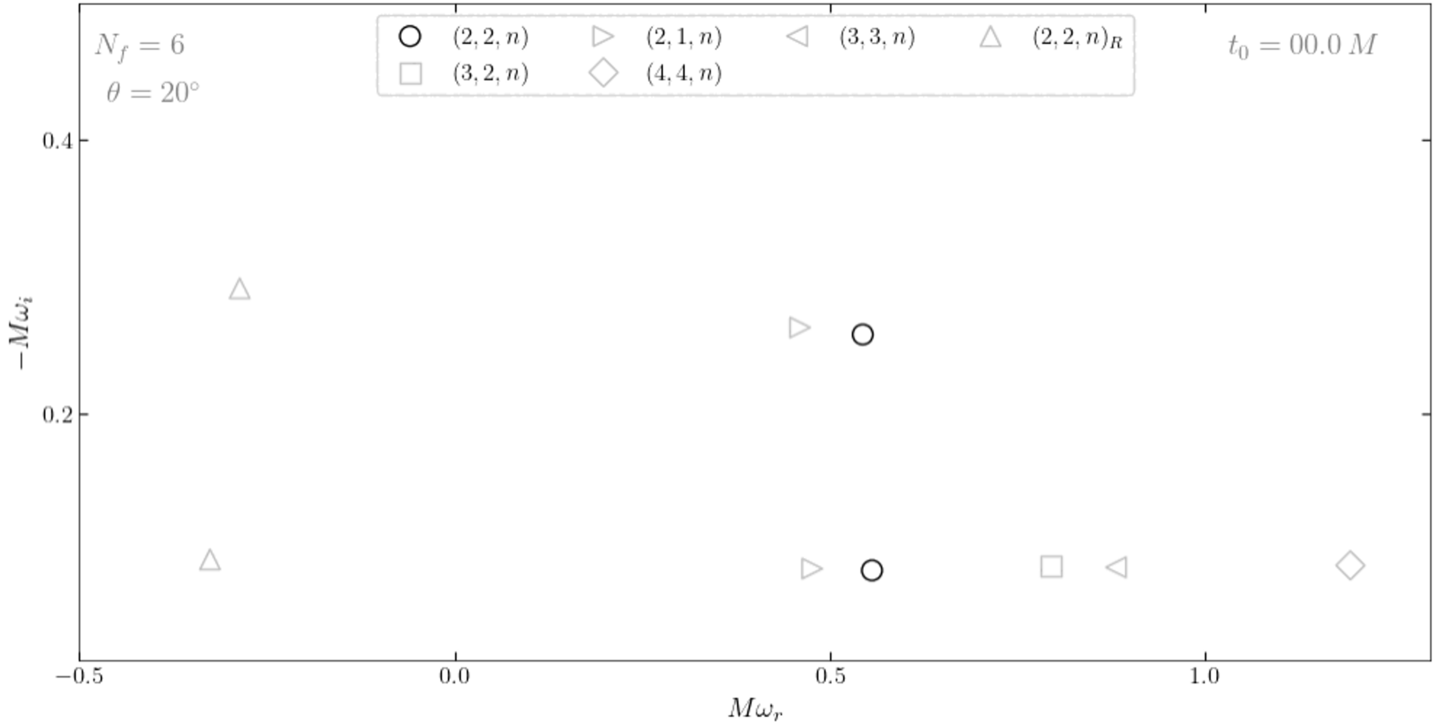
Agnostic spectroscopy: extracting modes from a face-on binary

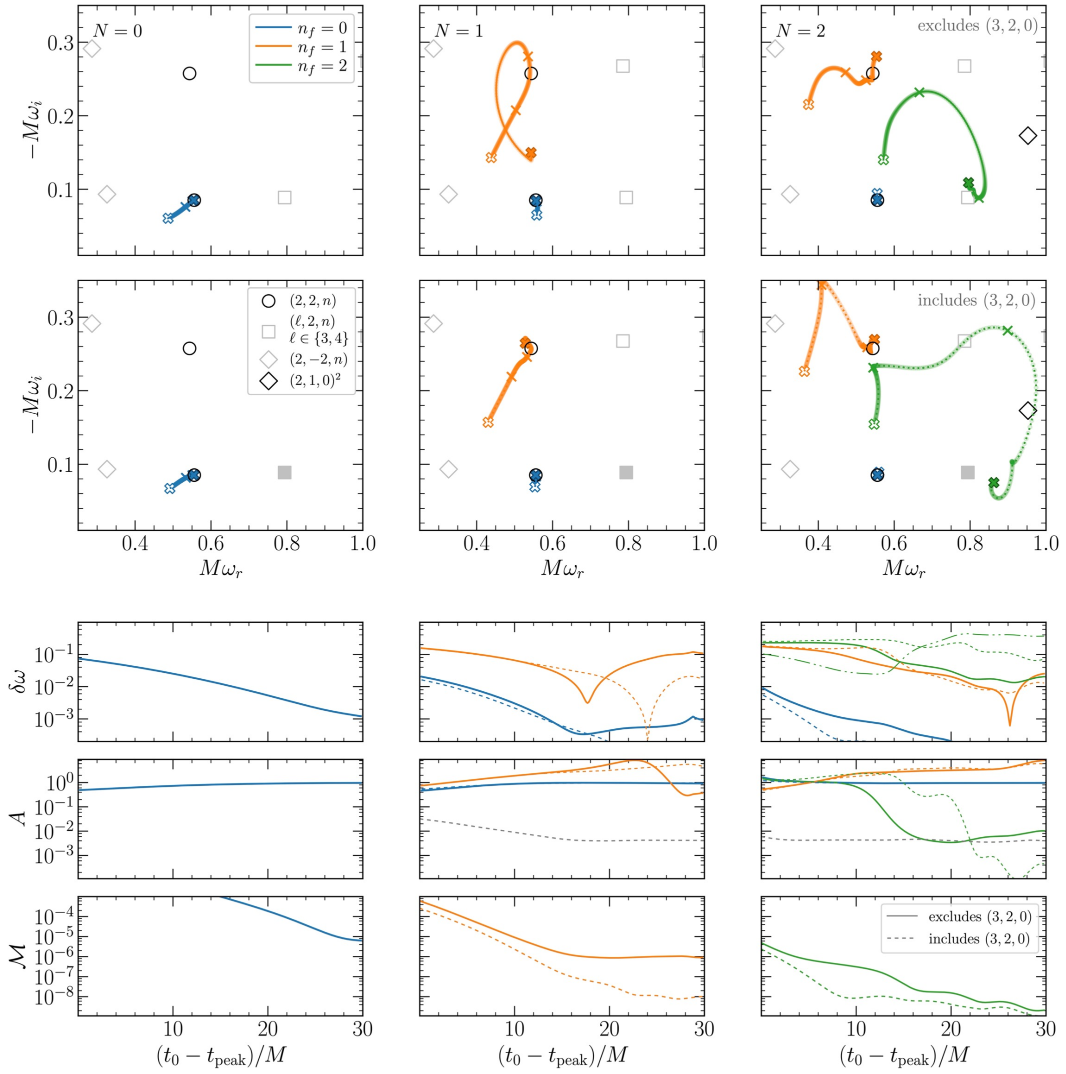


Agnostic spectroscopy: extracting modes from a face-on binary



Agnostic spectroscopy: extracting modes from a non-face-on binary





We should really do **agnostic tests**

Free-frequency fit of SXS0305:

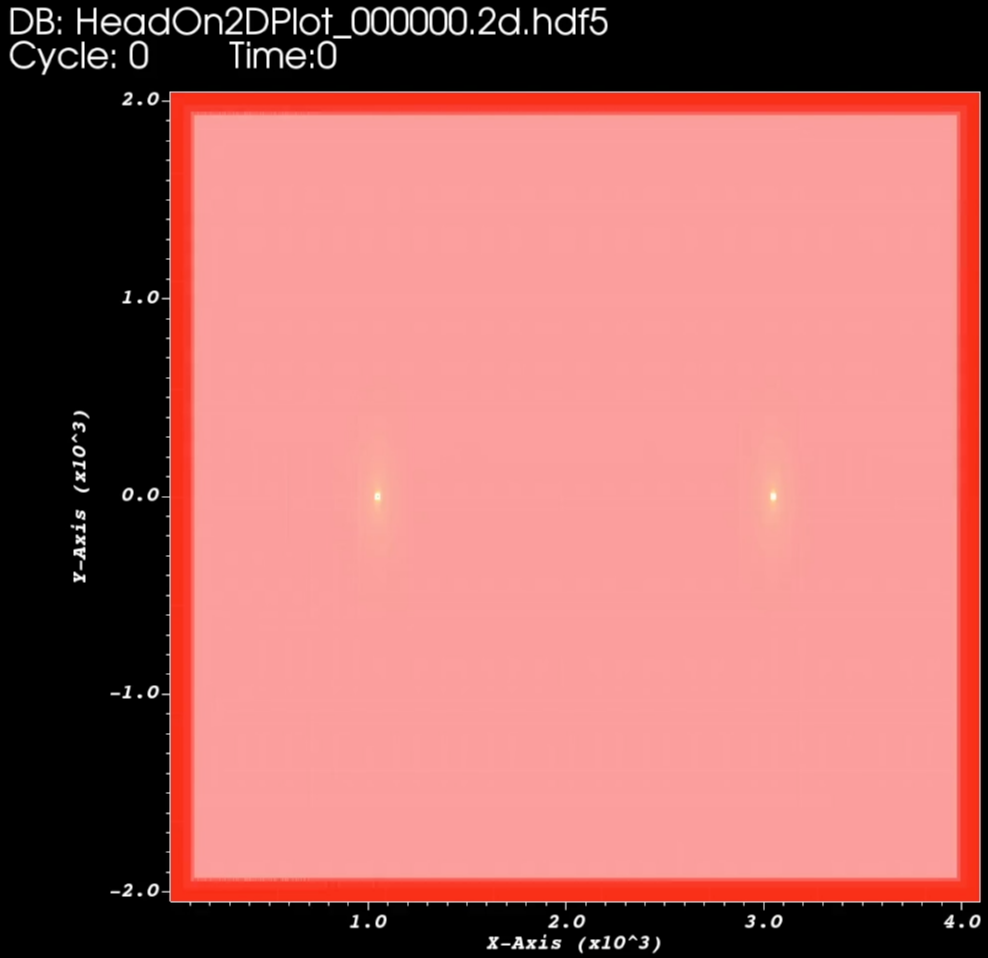
1) Including spherical-spheroidal mode mixing is crucial (grey square: 320 is fixed)

2) There is no strong evidence of modes beyond 220, 221 and 320

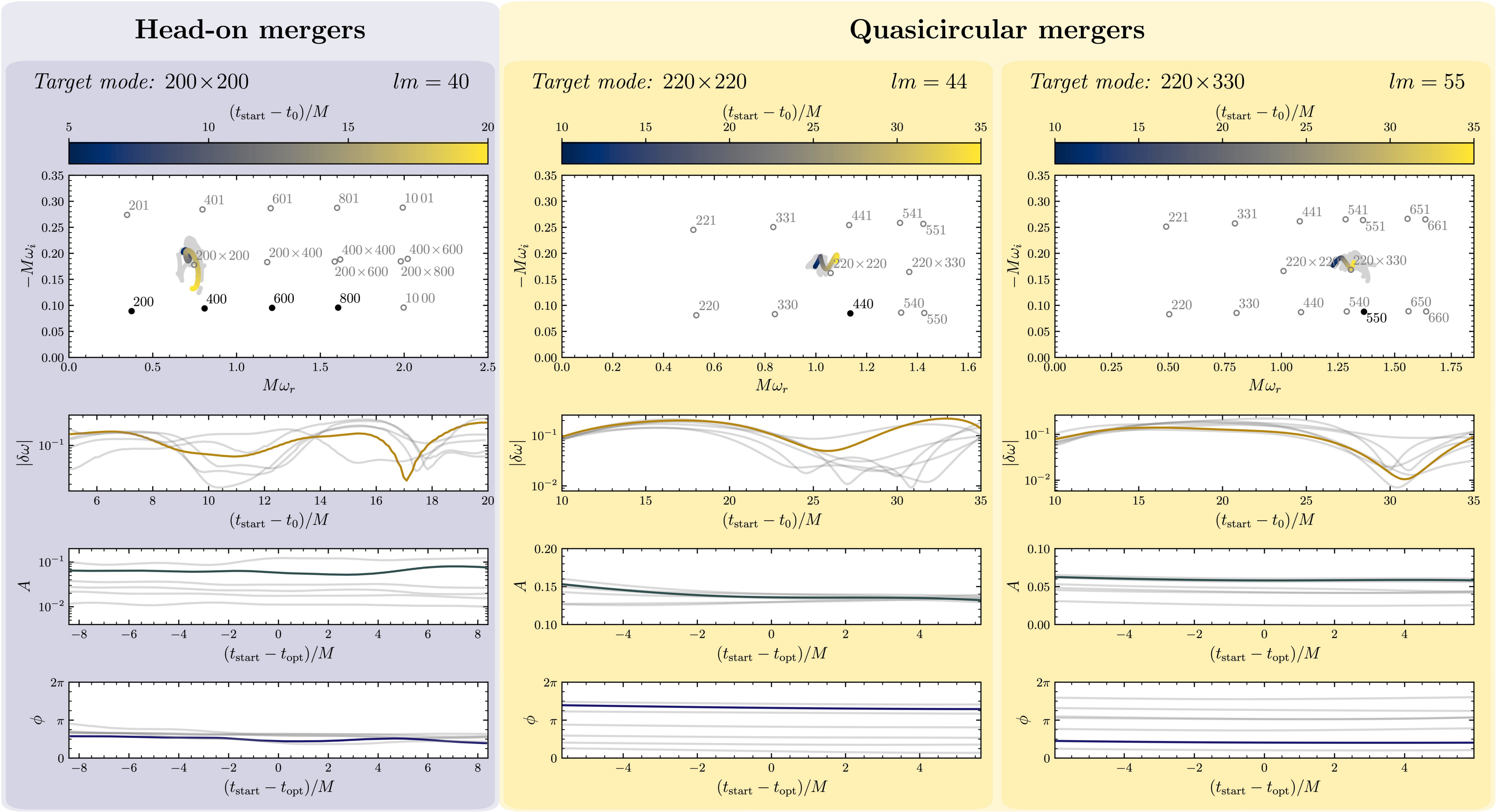
…but now we do know that **nonlinear modes** (first found by Lionel London, never confirmed) are there!

$$\psi = \epsilon\psi_{(1)} + \epsilon^2\psi_{(2)}$$
$$\mathcal{L}\psi_{(2)} \propto \psi_{(1)}^2 \sim A_1 A_2 e^{i(\phi_1+\phi_2)}$$

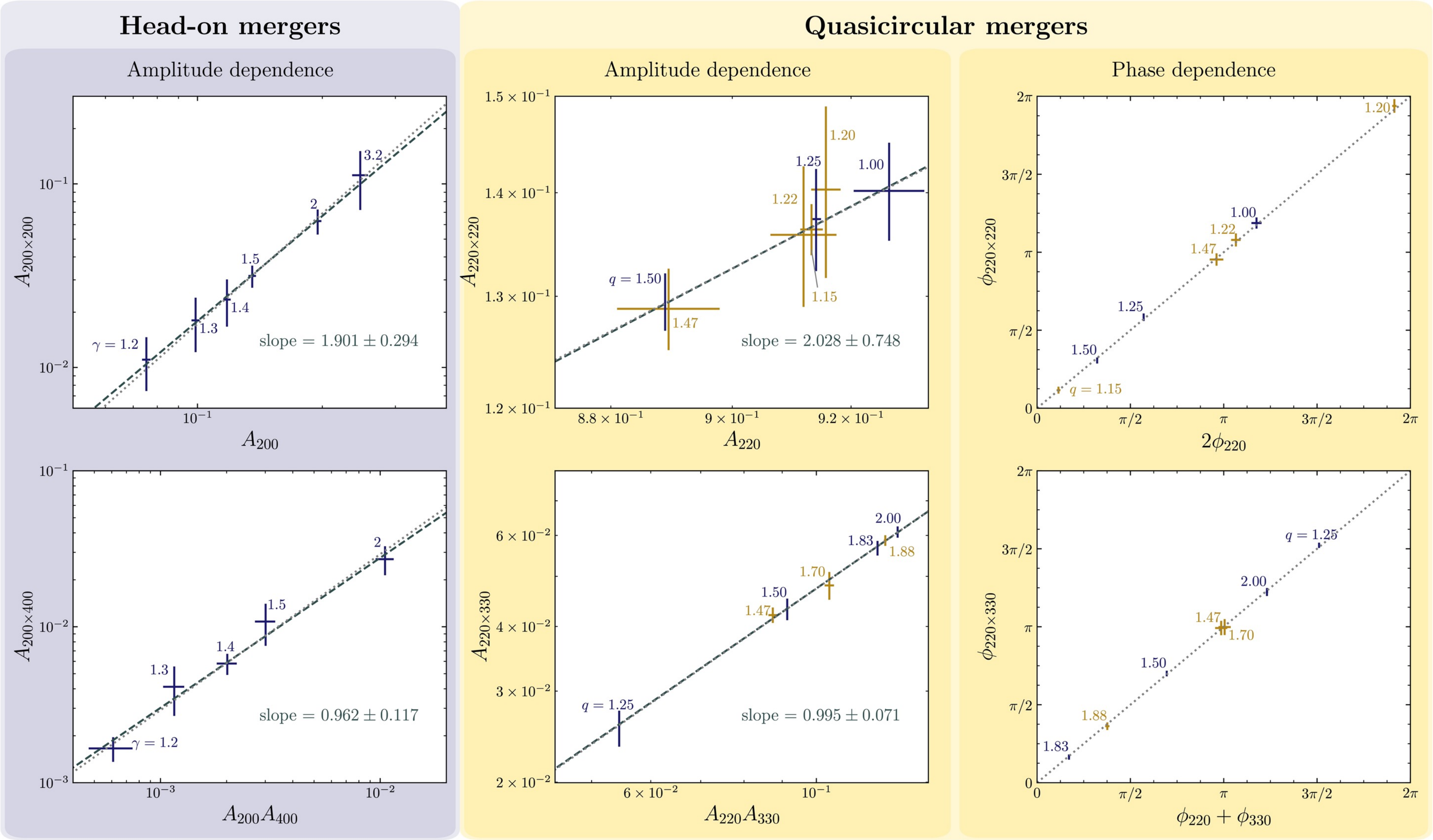
[London+, 1404.3197] [Ma+, 2207.10870] [Cheung+, 2208.07374] [Mitman+, 2208.07380]



[Cheung+, 2208.07374]



Head-on mergers
Quasicircular mergers
Target mode: 200×200
lm = 40
Target mode: 220×220
lm = 44
Target mode: 220×330
lm = 55
$(t_{\rm start}-t_0)/M$
$-M\omega_i$
$M\omega_r$
$|\delta\omega|$
A
ϕ
$(t_{\rm start}-t_{\rm opt})/M$


Head-on mergers
Quasicircular mergers
Amplitude dependence
Amplitude dependence
Phase dependence
$A_{200\times200}$
A_{200}
slope = 1.901 ± 0.294
$\gamma = 1.2$
1.3
1.4
1.5
2
3.2
$A_{200\times400}$
$A_{200}A_{400}$
slope = 0.962 ± 0.117
$A_{220\times220}$
A_{220}
slope = 2.028 ± 0.748
$q = 1.50$
1.47
1.22
1.25
1.15
1.20
1.00
$A_{220\times330}$
$A_{220}A_{330}$
slope = 0.995 ± 0.071
$q = 1.25$
1.47
1.50
1.70
1.83
1.88
2.00
$\phi_{220\times220}$
$2\phi_{220}$
$q = 1.15$
1.50
1.25
1.47
1.22
1.00
1.20
$\phi_{220\times330}$
$\phi_{220} + \phi_{330}$
1.83
1.88
1.50
1.47
1.70
2.00
$q = 1.25$

For all your quasinormal mode fitting needs…

https://mhycheung.github.io/jaxqualin/
pronounced “Jacqueline” (as in the Franz Ferdinand song)

[Cheung+, arXiv:2310.04489]

2,2,0 mode

Fit expressions

The hyperfit expressions should be used with caution, especially in regions of low amplitude or outside of the convex hull of the data points. The hyperfit function for the amplitude could go to negative values in these regions, which is unphysical. The phase data has been unwrapped before fitting to the best of our ability, but there may still be some jumps of 2π in the data, which could be seen in the error plot. Please consult the fit error plot on the bottom of this page before using the fits.

Amplitude

$$\begin{aligned} A_{2,2,0} = {} & 4.004 + 1.349\chi_+ + 0.333\chi_- - 1.325\eta^2 - 1.369\eta\chi_- + 2.622\chi_+\chi_- - 32.74\eta^2\chi_+ \\ & + 4.313\eta\chi_+^2 - 25.18\eta\chi_+\chi_- + 83.37\eta^3\chi_+ - 13.39\eta^2\chi_+^2 + 58.01\eta^2\chi_+\chi_- - 0.3837\eta\chi_+^3 \\ & - 0.2075\chi_+^4 \end{aligned}$$

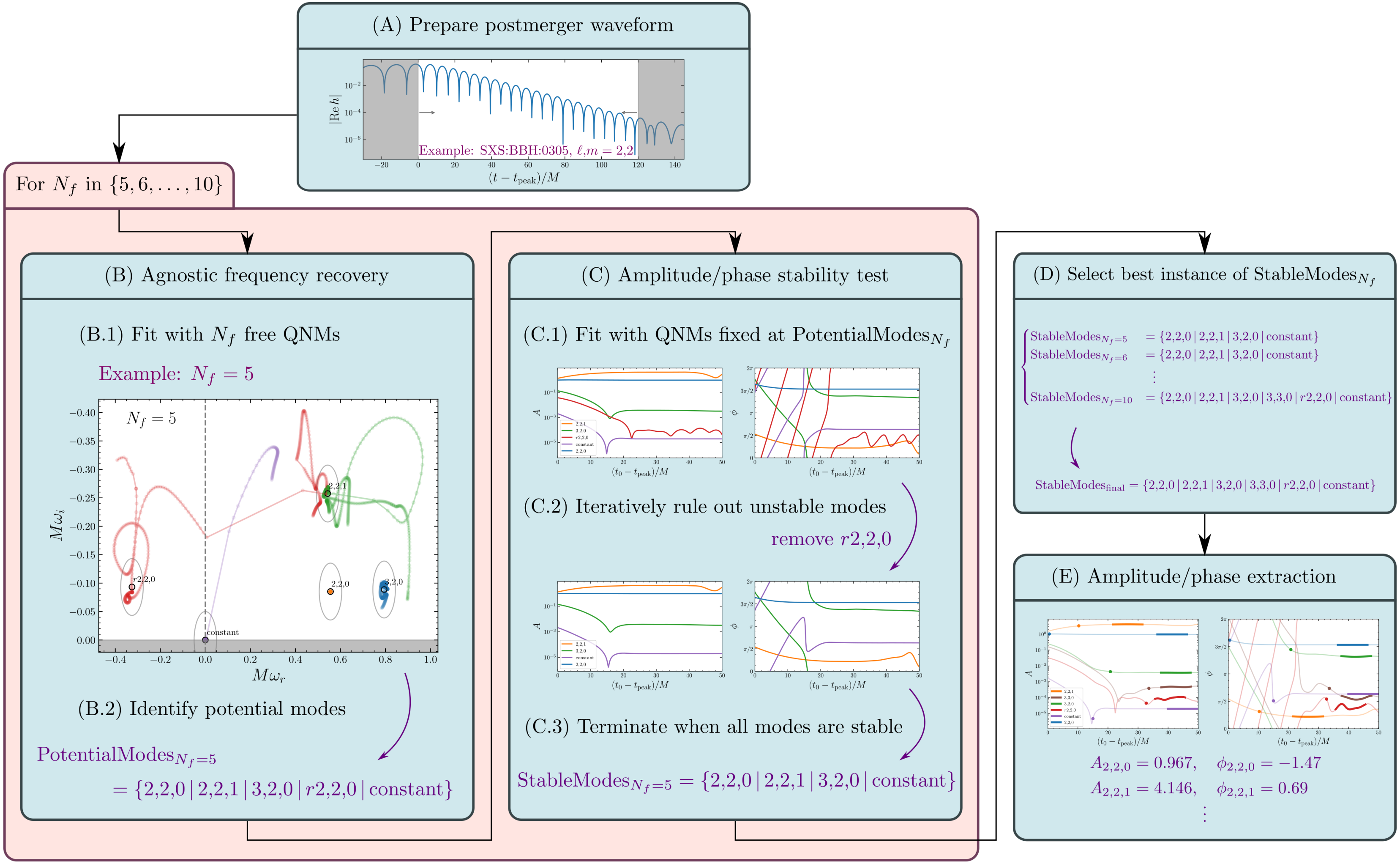
Phase

$$\phi_{2,2,0} = 0$$

Interactive plot

Click on the buttons below to switch between the amplitude, phase and starting time plots.

$A_{2,2,0}$ $\phi_{2,2,0}$ t_{start}

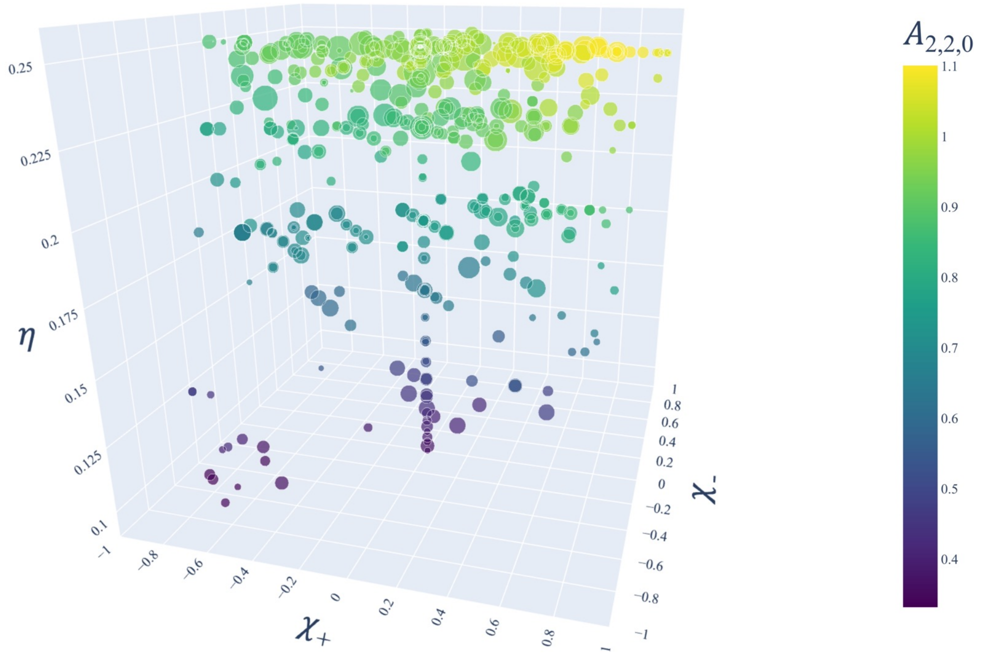


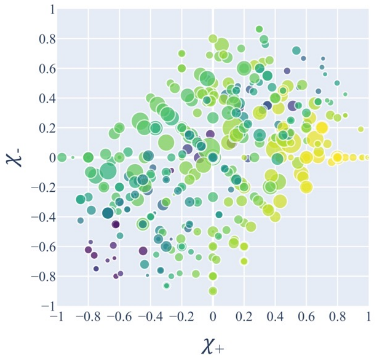
(A) Prepare postmerger waveform
Example: SXS:BBH:0305, $\ell,m = 2,2$
$(t - t_{\rm peak})/M$
For N_f in $\{5, 6, \ldots, 10\}$
(B) Agnostic frequency recovery
(B.1) Fit with N_f free QNMs
Example: $N_f = 5$
$N_f = 5$
$M\omega_r$
$M\omega_i$
(B.2) Identify potential modes
$\text{PotentialModes}_{N_f=5} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, r2{,}2{,}0 \,|\, \text{constant}\}$
(C) Amplitude/phase stability test
(C.1) Fit with QNMs fixed at $\text{PotentialModes}_{N_f}$
(C.2) Iteratively rule out unstable modes
remove $r2{,}2{,}0$
(C.3) Terminate when all modes are stable
$\text{StableModes}_{N_f=5} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, \text{constant}\}$
(D) Select best instance of StableModes_{N_f}
$\text{StableModes}_{N_f=5} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, \text{constant}\}$
$\text{StableModes}_{N_f=6} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, \text{constant}\}$
$\text{StableModes}_{N_f=10} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, 3{,}3{,}0 \,|\, r2{,}2{,}0 \,|\, \text{constant}\}$
$\text{StableModes}_{\rm final} = \{2{,}2{,}0 \,|\, 2{,}2{,}1 \,|\, 3{,}2{,}0 \,|\, 3{,}3{,}0 \,|\, r2{,}2{,}0 \,|\, \text{constant}\}$
(E) Amplitude/phase extraction
$A_{2,2,0} = 0.967, \quad \phi_{2,2,0} = -1.47$
$A_{2,2,1} = 4.146, \quad \phi_{2,2,1} = 0.69$

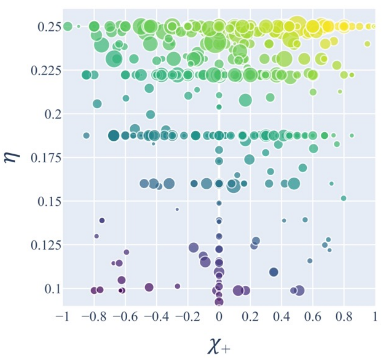
$$\eta = \frac{q}{(1+q)^2}$$

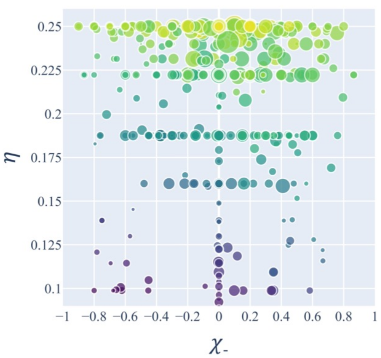
$$\chi_+ = \frac{q\chi_1 + \chi_2}{1+q}$$

$$\chi_- = \frac{q\chi_1 - \chi_2}{1+q}$$

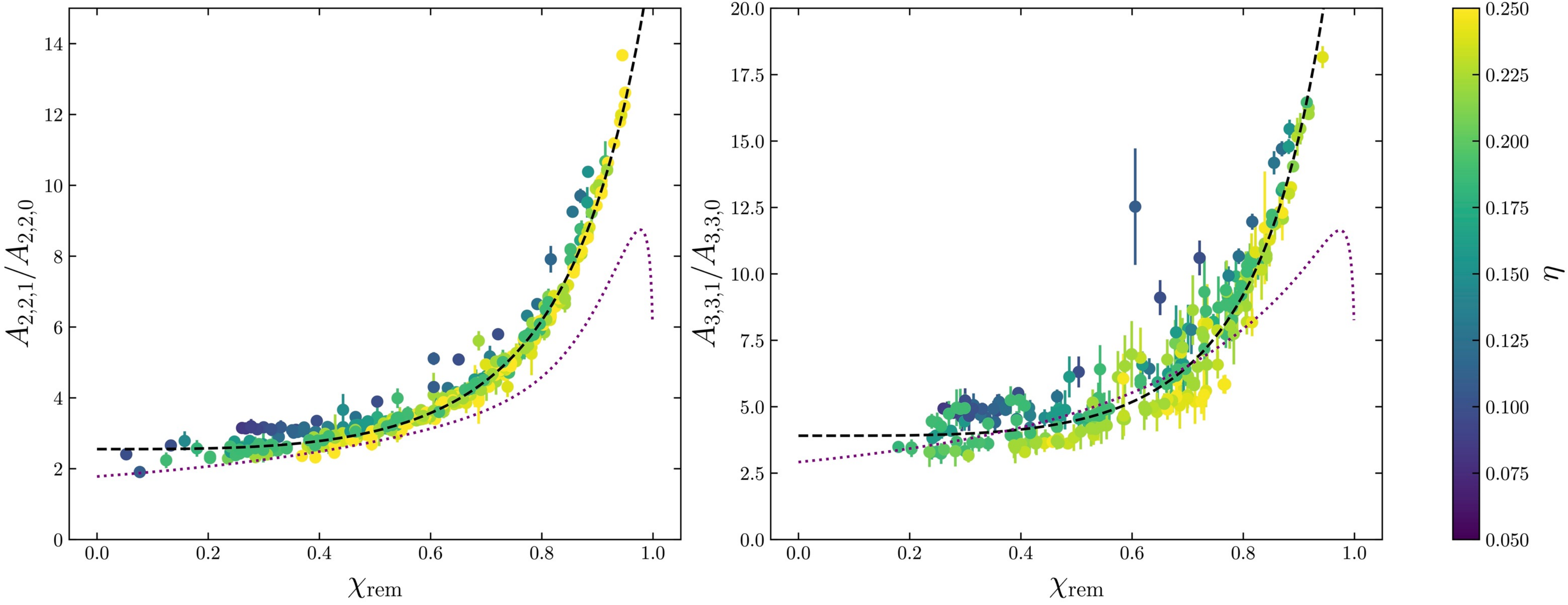






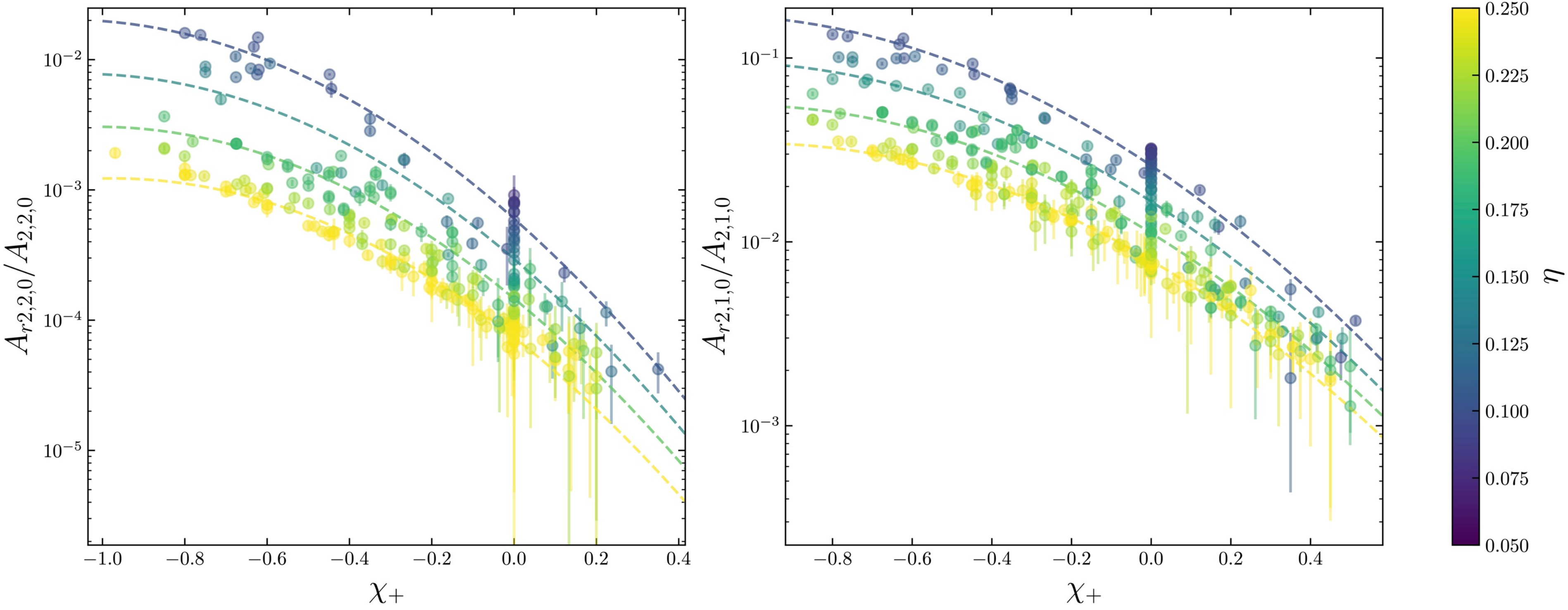


Ratio of first overtone to fundamental mode: excitation coeffs vs. excitation factors



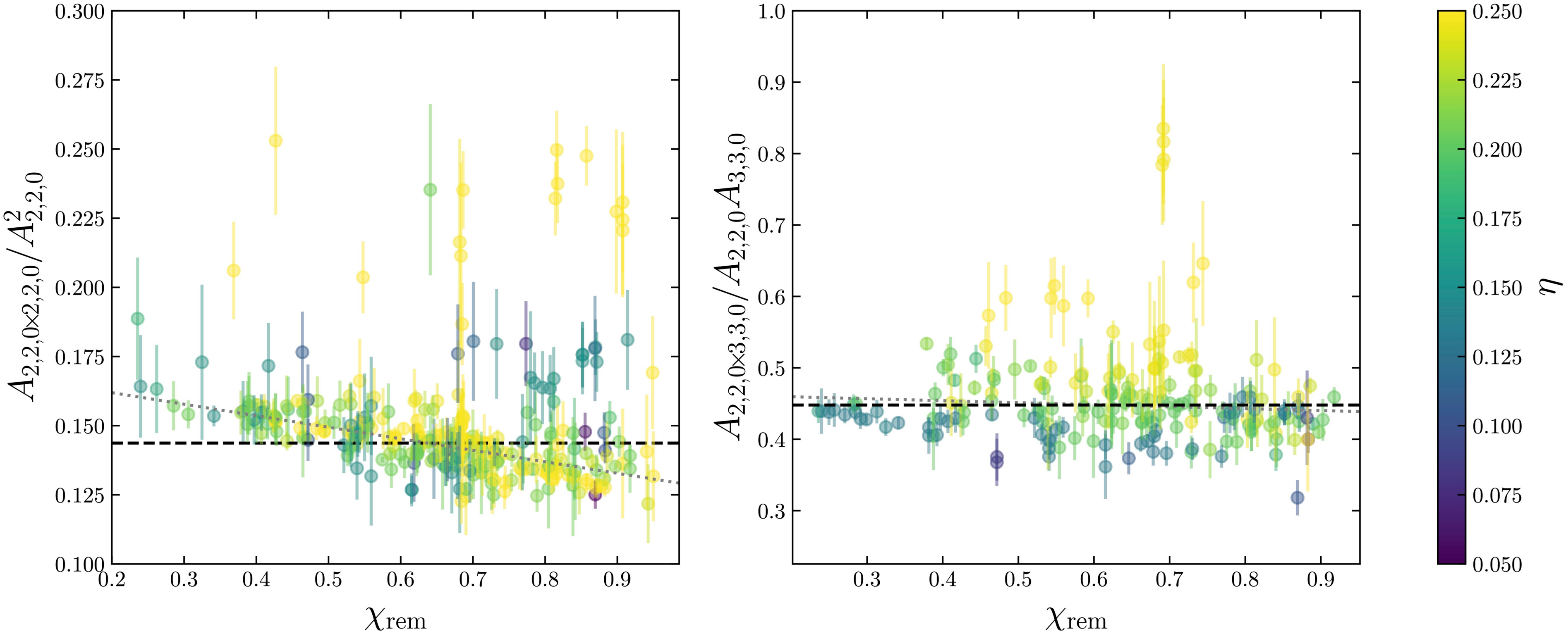
[Cheung+, arXiv:2310.04489]

Ratio of retrograde modes to prograde modes



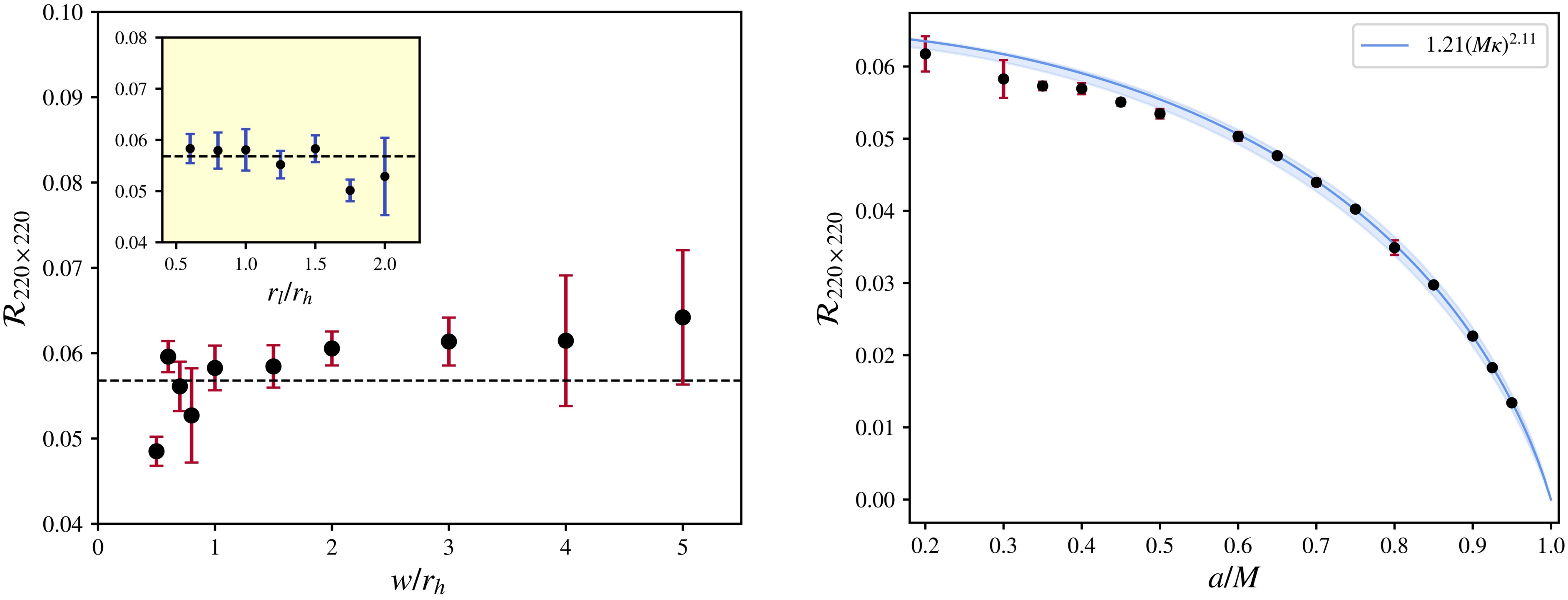
[Cheung+, arXiv:2310.04489]

Ratio of quadratic modes to linear modes: remnant spin and mass ratio dependence



[Cheung+, arXiv:2310.04489]

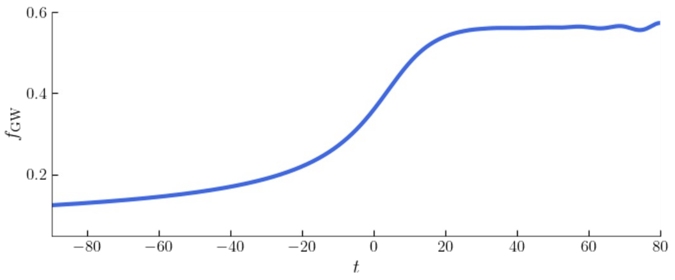
Gaussian scattering of second order perturbations: spin dependent!

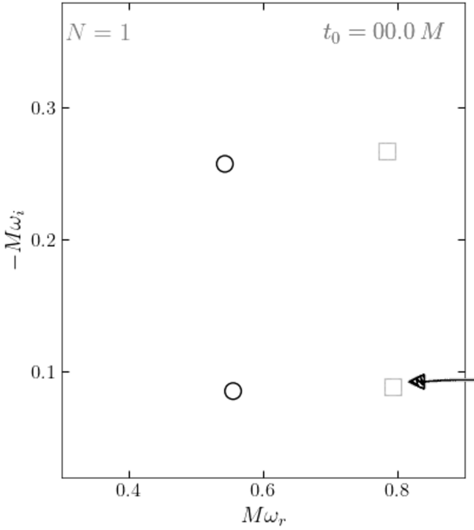


[Redondo-Yuste+, arXiv:2308.14796]

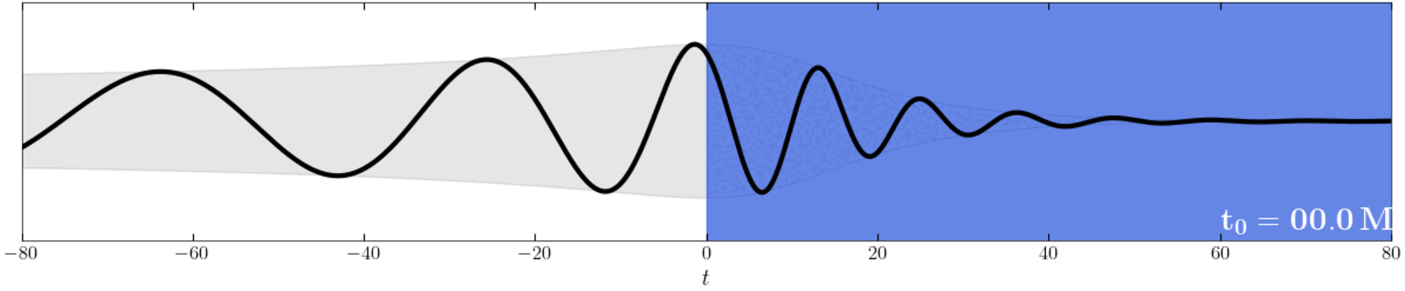
Have we observed overtones?

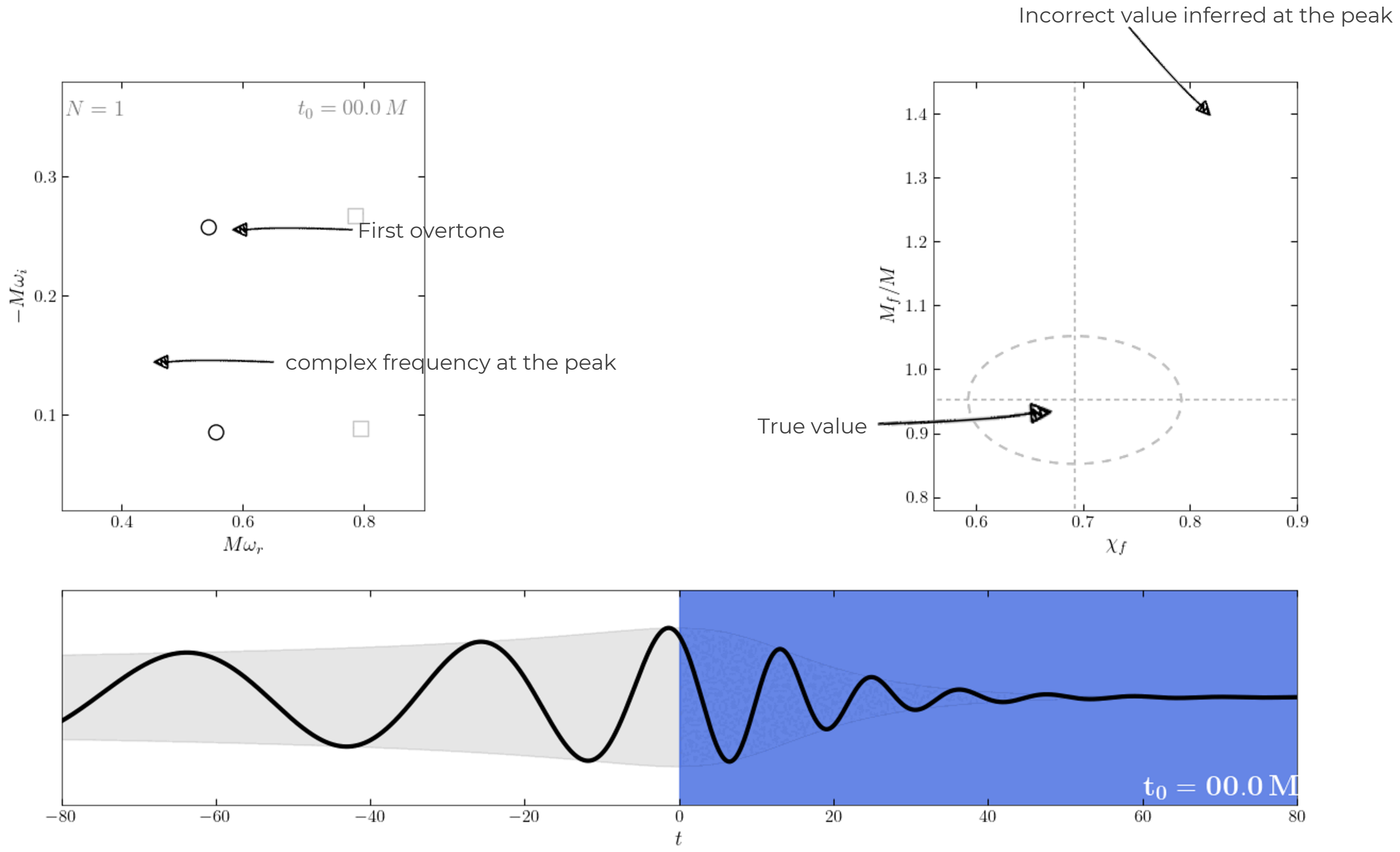
What would happen at infinite SNR? Extracting frequencies from overtones





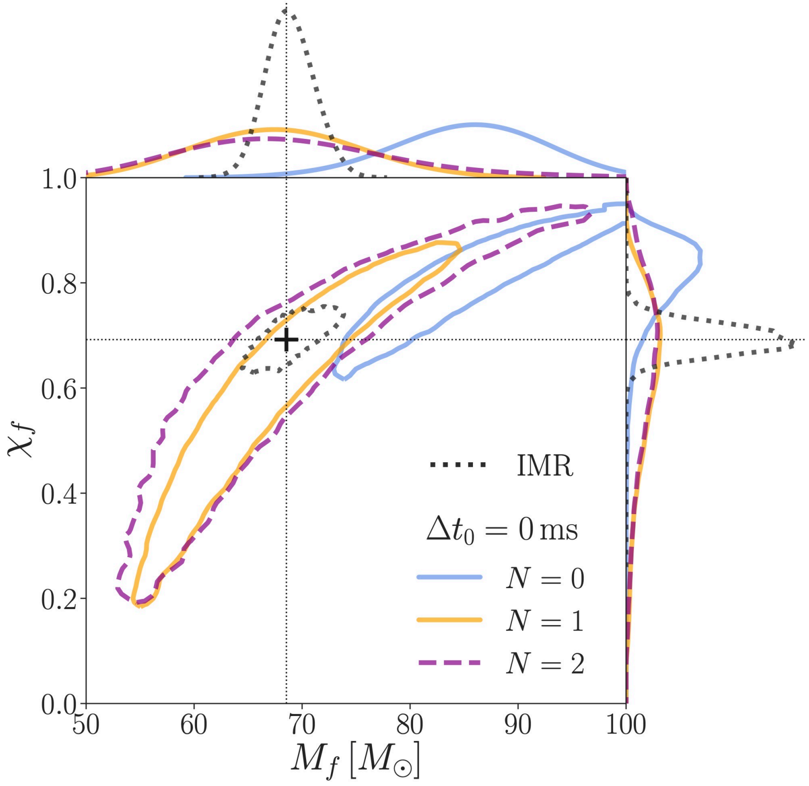
Fitting model: two free frequencies,
plus (3,2,0) mixing mode fixed at the right value



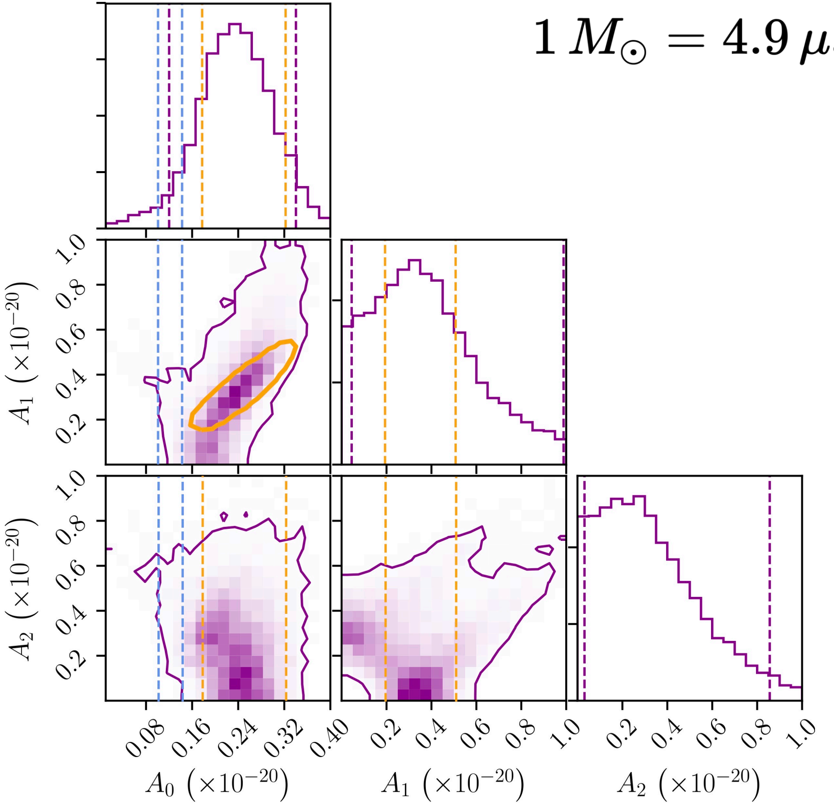

Incorrect value inferred at the peak
$N = 1$
$t_0 = 00.0\,M$
First overtone
complex frequency at the peak
$-M\omega_i$
$M\omega_r$
0.1
0.2
0.3
0.4
0.6
0.8
True value
M_f/M
χ_f
0.8
0.9
1.0
1.1
1.2
1.3
1.4
0.6
0.7
0.8
0.9
$t_0 = 00.0\,\mathrm{M}$
-80
-60
-40
-20
0
20
40
60
80
t

GW150914 tests of the no-hair theorem: the first overtone

Overtones improve quality of consistency tests for GW150914, but by the arguments we made, this is definitely not an overtone detection. Also, analysis assumes t_{start}=1126259462.423 s

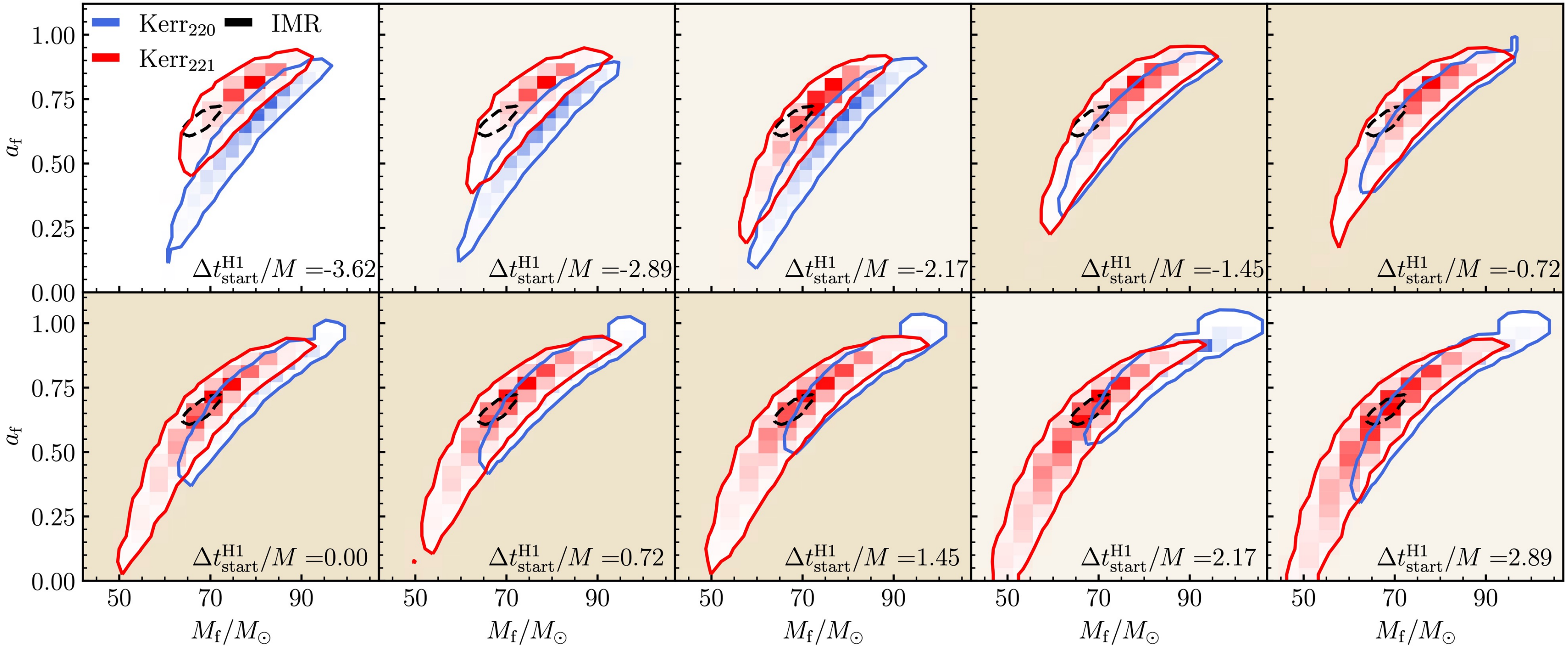


$$1\,M_\odot = 4.9\,\mu s$$



[Isi+, 1905.00869]

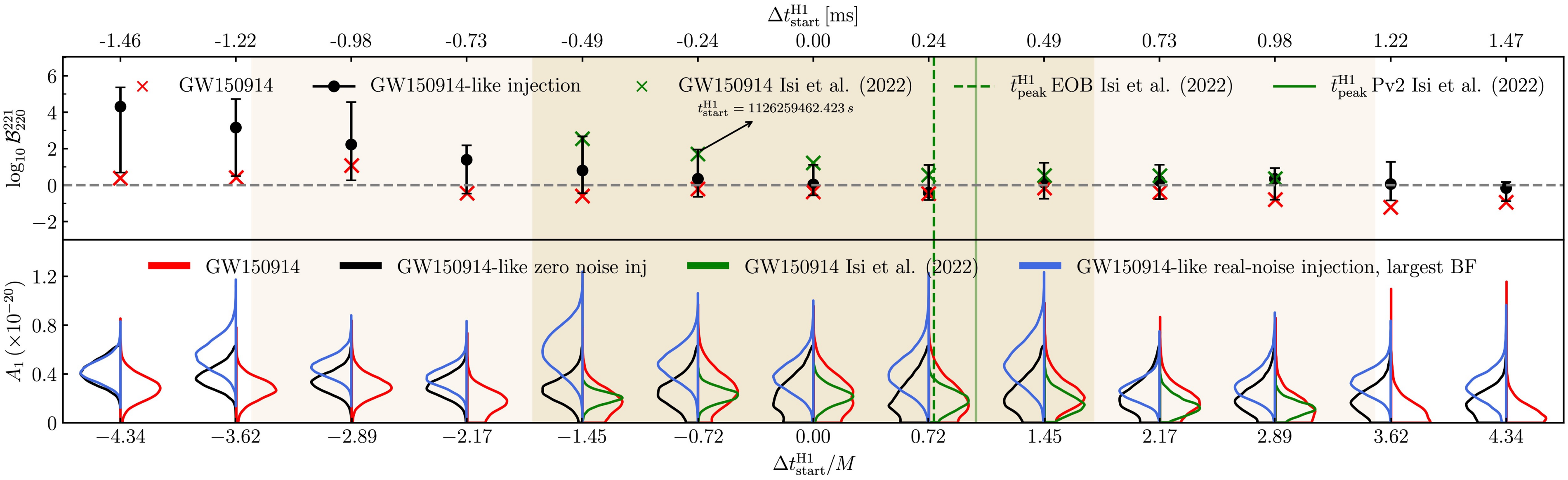
The first overtone: dependence on the starting time



$$M_{\rm GW150914} \simeq 62 M_\odot = 0.3\,{\rm ms}$$

[Cotesta+, 2201.00822]

The first overtone: amplitude and Bayes factor

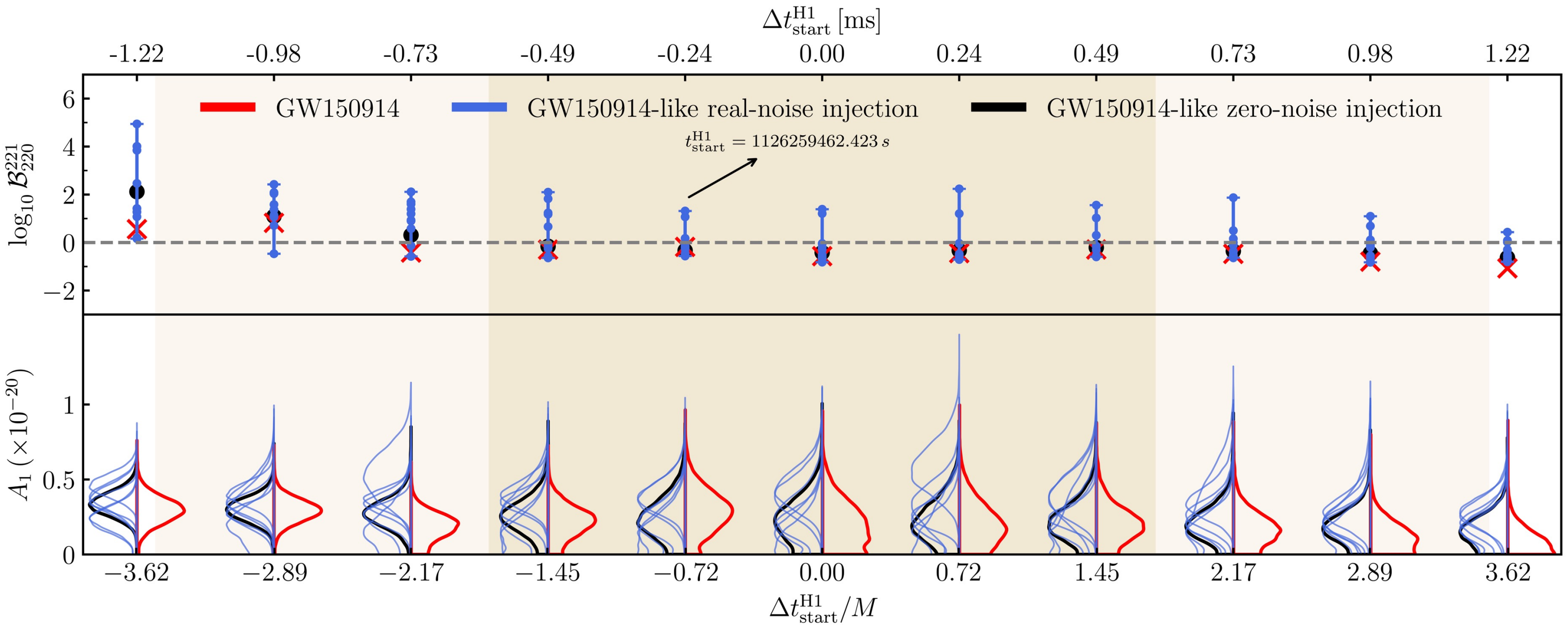


$$M_{\rm GW150914} \simeq 62 M_\odot = 0.3\,\rm ms$$

[Cotesta+, 2201.00822; green: Isi-Farr, 2202.02941]

The saga continues (PRL comment + reply)

1) Shift in pyRing discretized time axis: 0.06ms (compare to Δt_{peak} ~2.5ms)
2) Analysis segment T=0.2s instead of 0.1s slightly increases amplitudes (see below…)

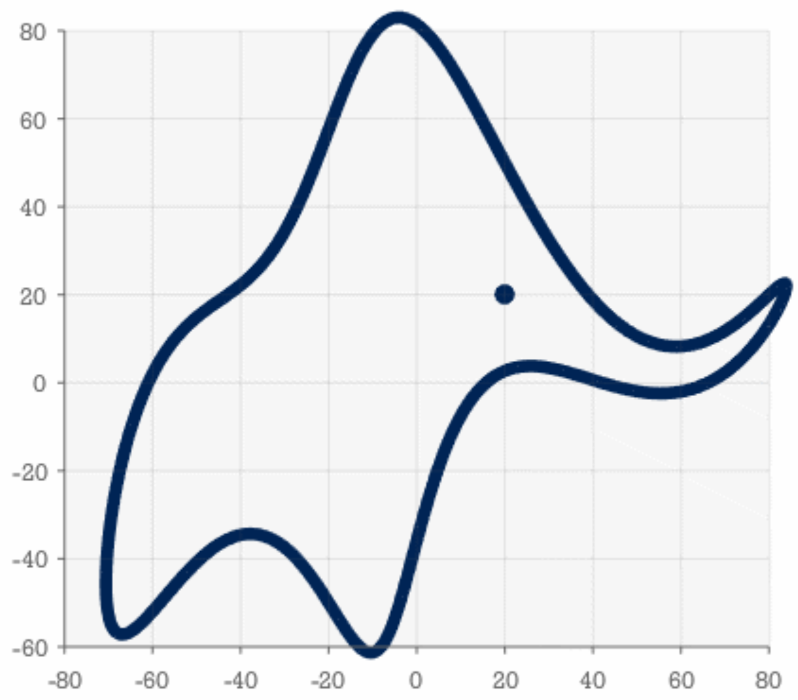


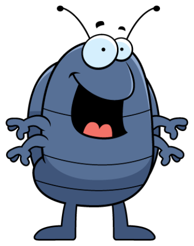
$$M_{\rm GW150914} \simeq 62 M_\odot = 0.3\,{\rm ms}$$

[Isi-Farr and Carullo+, to appear]

Pseudospectra and QNM instabilities

Is the overtone model stable? The elephant and the flea

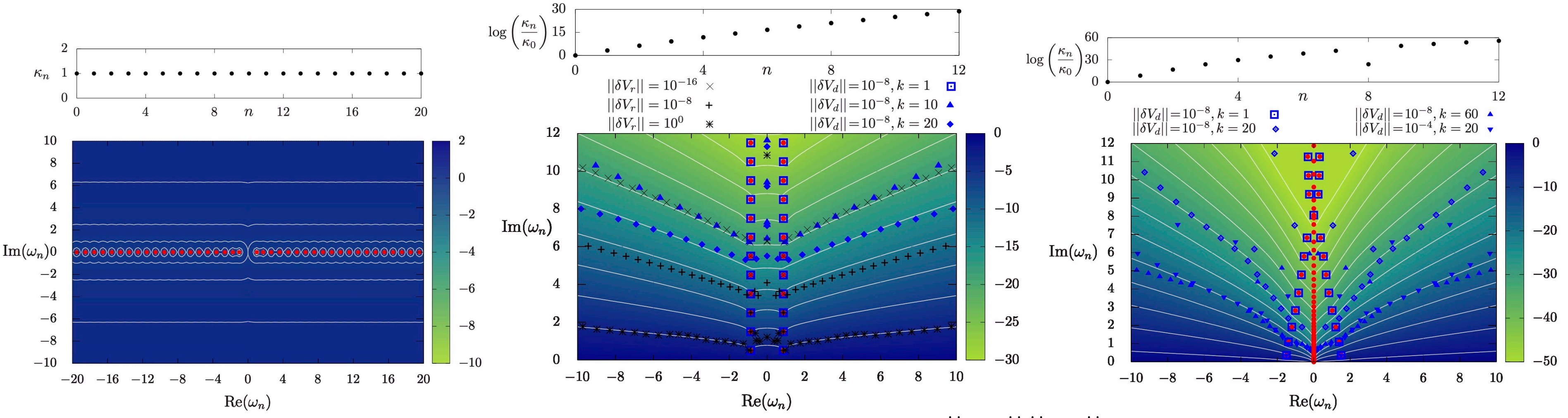




Flea (not to scale)

[Cheung+, 2111.05415]

Pseudospectra: is the spectrum itself stable?



$$Av_i = \lambda_i v_i$$
$$A(\epsilon) = A + \epsilon \delta A$$

$$|\lambda_i(\epsilon) - \lambda_i| \leq \epsilon \kappa_i, \quad \kappa_i = \frac{\|u_i\| \|v_i\|}{|\langle u_i, v_i \rangle|}$$

$$\sigma^\epsilon(A) = \left\{\lambda \in \mathbb{C} : \left\|(\lambda \mathrm{Id} - A)^{-1}\right\| > 1/\epsilon\right\}$$
$$= \left\{\lambda \in \mathbb{C}, \exists \delta A \in M_n(\mathbb{C}), \|\delta A\| < \epsilon : \lambda \in \sigma(A + \delta A)\right\}$$

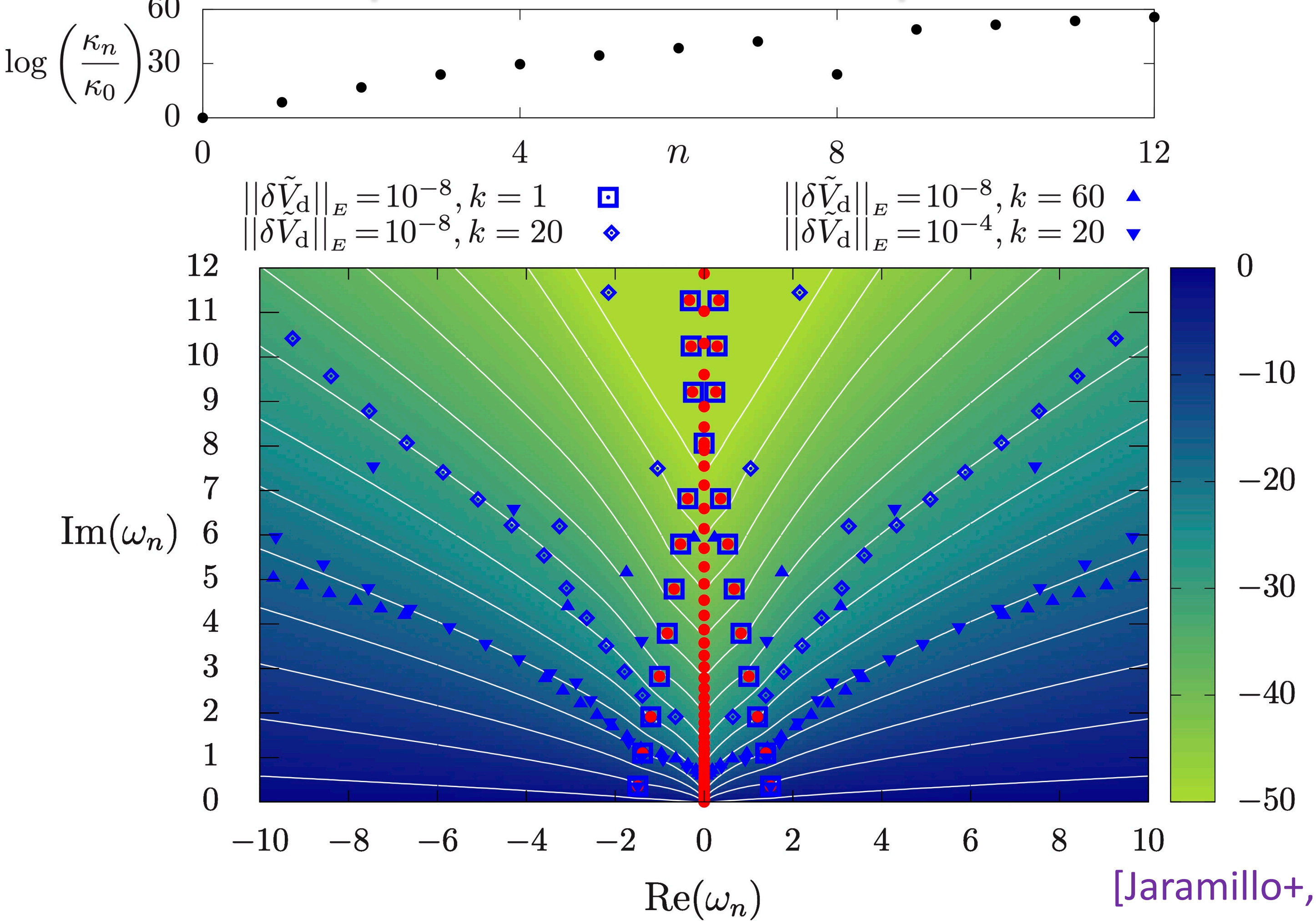
First definition: the resolvent can be very large far from the spectrum

Second definition: points in the pseudospectrum are eigenvalues of the perturbed operator

Under perturbations, the spectrum migrates out to the boundaries of the pseudospectrum

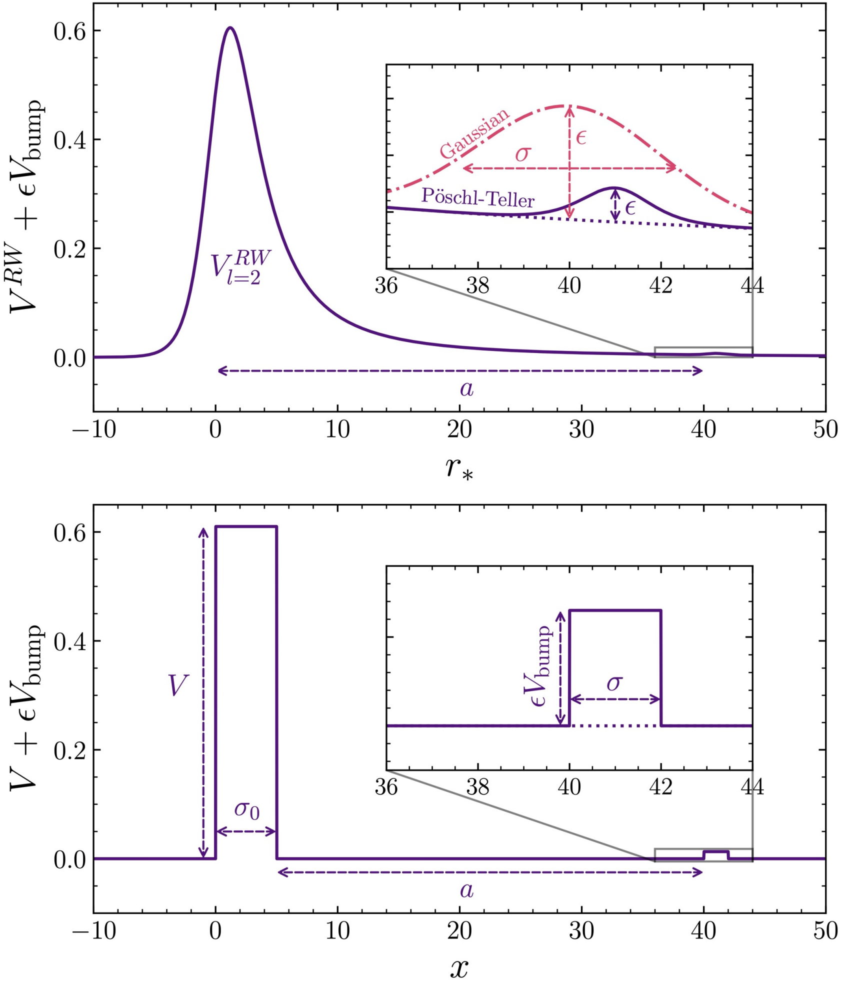
[Jaramillo+, 2004.06434]

Is the spectrum stable? Pseudospectrum: no!



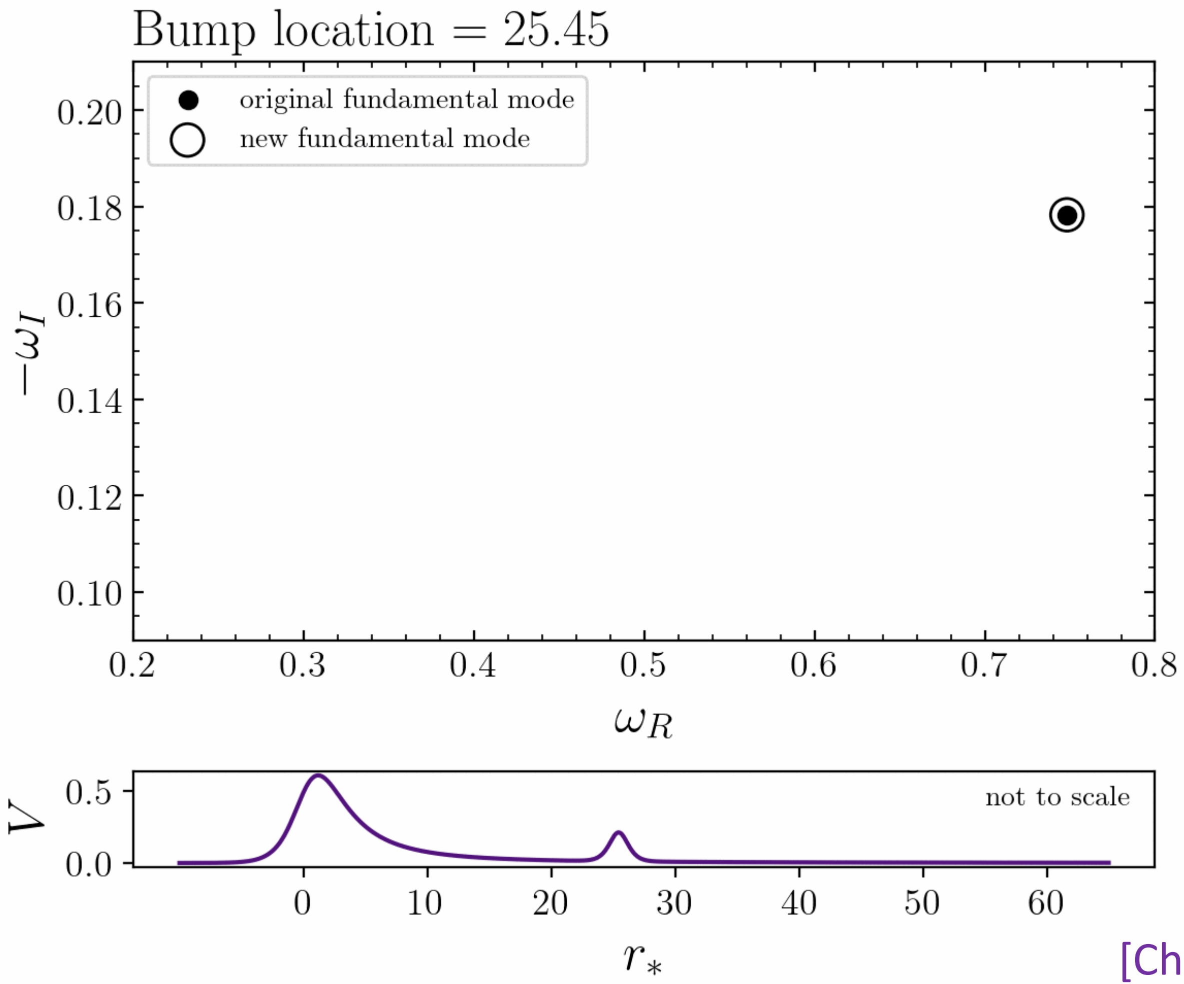
[Jaramillo+, 2004.06434]

Is the fundamental mode stable? The elephant and the flea



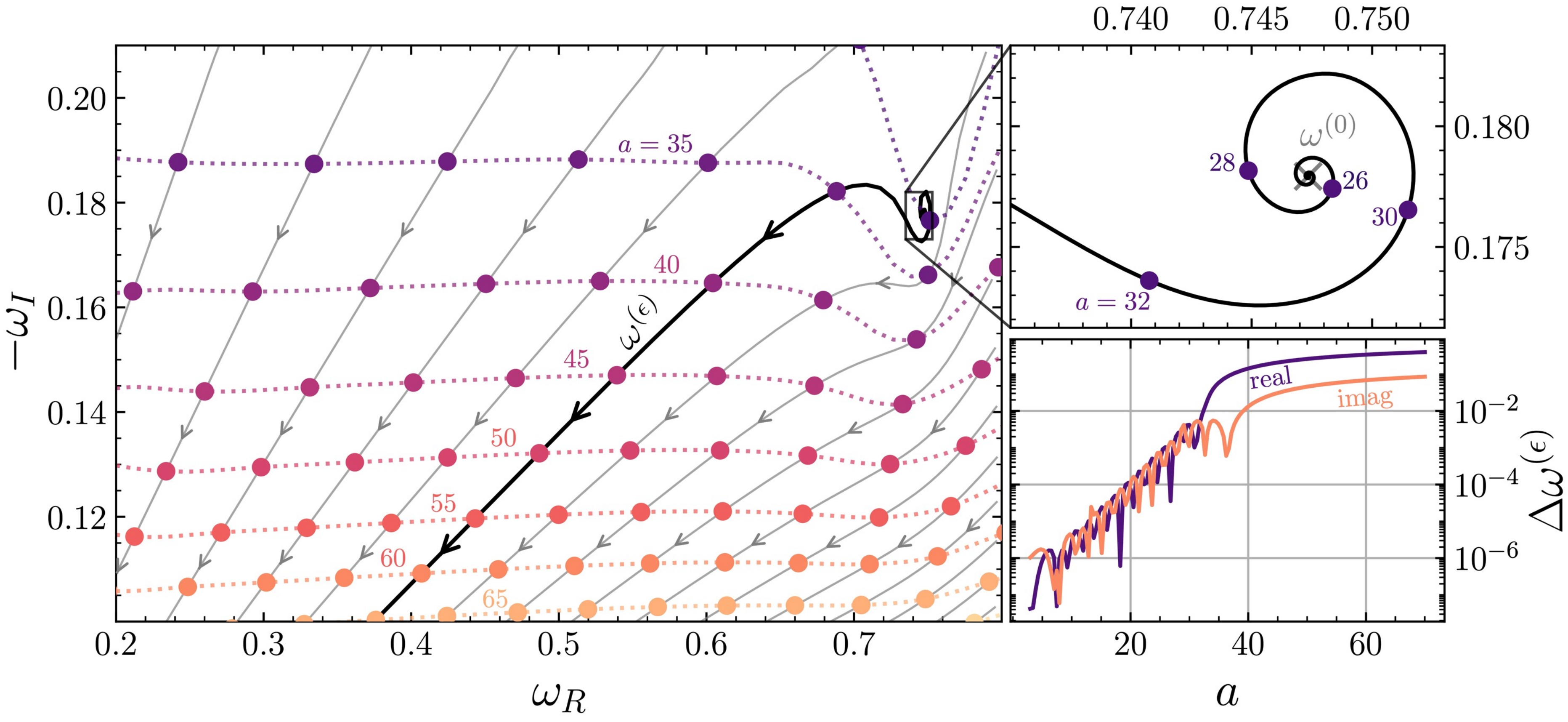
[Cheung+, 2111.05415]

Is the fundamental mode stable? The elephant and the flea



[Cheung+, 2111.05415]

Is the fundamental mode stable? No – but this does not affect spectroscopy



[Cheung+, 2111.05415; see also EB+, 2205.08547]

Summary

Addition of overtones long known to provide a better fit to:

point-particle waveforms, nonrotating (1970s) and rotating (1980s) collapse
head-on black hole collisions (1990s)
quasicircular mergers (circa 2005)

Can a linear superposition of overtones describe nonlinear mergers up to the peak? No

Free-frequency fitting shows that several other modes are easier to extract than the first (2,2) overtone
Need non-constant amplitudes, and high overtones do not add information to parameter estimation
Clear evidence (now from multiple groups) of nonlinear modes in numerical waveforms
jaxqualin: systematic extraction of linear and nonlinear modes from NR simulations

Have we observed overtones in GW150914? No

Strong evidence only **before** the peak, where the linear model is definitely not applicable
(even if you believe that the answer to the previous question is "yes"!)
Injections show that noise can induce artificial evidence for an overtone
My best bet for O4/O5: higher multipole observation in unequal-mass events

Do higher overtones make sense in a realistic astrophysical setting? Maybe

Pseudospectra suggest that higher overtones are prone to instabilities
(so is the fundamental mode, but this does not affect spectroscopy)
Need more work on Kerr to understand if these instabilities can be induced by perturbing "fleas":
e.g., astrophysical environments or nonlinearities