70 Years of Percolation
Schedule & Abstracts

	Monday	Tuesday	Wednesday	Thursday	Friday
9:20-10:00	Balint Toth (Bristol & Renyi Inst.)	Jeff Steif (Chalmers)	Frank den Hollander (Leiden)	Jean-François Le Gall (Paris-Sarclay)	Gordon Slade (UBC)
10:00-10:40	Svante Janson (Uppsala)	Tony Guttmann (Melbourne)*	Mark Holmes (Melbourne)	Asaf Nachmias (Tel Aviv)	Stanislav Volkov (Lund)
10:40-11:20	Coffee	Coffee	Coffee	Coffee	Coffee
11:20-12:00	Tom Hutchcroft (Caltech)	Zhongyang Li (Connecticut)	Cristina Toninelli (Paris-Dauphine)	Omer Angel (UBC)	Barbara Dembin (ETH Zurich)
12:00-12:40	Rob van den Berg (Amsterdam)	Laurent Saloff-Coste (Cornell)	Mikhail Menshikov (Durham)	Vincent Tassion (ETH Zurich)	Wendelin Werner (Cambridge)
12:40-14:30	Lunch at CMS	Lunch at Churchill College	Lunch at CMS	Lunch at Churchill College	Lunch at CMS
14:30-15:10	Alison Etheridge (Oxford)	Robin Pemantle (Penn)	Chuck Newman (NYU)	Christina Goldschmidt (Oxford)	
15:10-15:50	Jakob Björnberg (Gothenburg)	David Aldous (Berkeley and UW)	Michael Aizenman (Princeton)	Poster session	
15:50-16:30	Coffee	Coffee	Coffee	Coffee	
16:30-17:10	Béatrice de Tilière (Paris-Dauphine)		Hugo Duminil-Copin (IHES & Geneva)	Russ Lyons (Indiana)*	
17:30	Wine reception at CMS				
19:00			Conference dinner at Downing College		*Zoom

Monday

Balint Toth (Bristol and Renyi Institute Budapest)

(Towards an) invariance principle for the random Lorentz gas under weak coupling limit beyond kinetic time scale

Kesten-Papanicolaou (1980) proved that in the weak coupling limit the random Lorentz-gas process with soft scatterers converges to the Spherical Langevin Process. Under a second, diffusive limit the spatial component of the Spherical Langevin Process converges to Brownian motion. Komorowski-Ryzhik (2006) proved that combining the weak coupling and diffusive limits, the Brownian motion is obtained, at least for a time horizon slightly beyond the kinetic time-scale. We attempt to extend this last result robustly for time scales way beyond the kinetic one. (Work in progress.)

Svante Janson (Uppsala)

The sum of powers of subtrees sizes for random trees.

For a rooted tree T and a complex number α, consider the functional $F_\alpha(T) = \sum_v |T_v|^\alpha$ of powers of the sizes $|T_v|$ of the (fringe) subtrees T_v OF T over all vertices v in T. We study the asymptotics of this functional when T is a random conditioned Galton-Watson tree; the offspring distribution is assumed to be critical and with finite variance.

The use of complex powers instead of just real ones is important for two reasons: 1. It is useful in proofs (also for real powers) since powerful methods of analytic functions can be used. 2. It gives us new problems to study. In particular, how do the phasetransitions look in the complex plane?

The main results show phase transitions at the lines $Re\alpha = 0$ and $1/2$, with limit distributions that are normal but non-universal for $Re\alpha < 0$ and non-normal but universal for $\Re\alpha > 0$; here, "universal" means that they depend on the offspring distribution only through the offspring variance as a scale factor. We consider also the behaviour at the phase transitions.

(Based on joint works with Jim Fill and Stephan Wagner.)

Tom Hutchcroft (Caltech)

Locality of the percolation critical probability

Around 2008, Schramm conjectured that the critical probability p_c of a transitive graph is entirely determined by the local geometry of the graph, subject to the global constraint that $p_c < 1$. In other words, if G_n is a sequence of transitive graphs with $p_c(G_n) < 1$ for all n converging locally to a transitive graph G then $p_c(G_n)$ converges to $p_c(G)$. Previous works had verified the conjecture in various special cases, including nonamenable graphs of high girth (Benjamini, Nachmias and Peres 2012); Cayley graphs of abelian groups (Martineau and Tassion 2013); nonunimodular graphs (H. 2017 and 2018); graphs of uniform exponential growth (H. 2018); and graphs of (automatically uniform) polynomial growth (Contreras, Martineau and Tassion 2022). In this talk I will describe our complete resolution of the conjecture in forthcoming joint work with Easo.

Rob van den Berg (CWI Amsterdam)

A version of the OSSS inequality for uniformly drawn subsets of fixed size

The OSSS inequality (O'Donnell, Saks, Schramm and Servedio, 2005) gives an upper bound for the variance of a function of independent 0-1 valued random variables in terms of the influences of these random variables and the computational complexity of a (randomised) algorithm for determining the value of the function.

Duminil-Copin, Raoufi and Tassion, 2019, obtained a generalization to monotonic measures and used it to prove new results for Potts models and random-cluster models. That generalization naturally triggers the question if there are still other measures for which a version of the OSSS inequality holds.

In this talk I will present a version of the OSSS inequality for a family of measures that are clearly not monotonic, namely the k-out-of-n measures (these measures correspond with drawing k elements from a set of size n uniformly). An illustration of its use will be given by studying the event that there is an occupied horizontal crossing of an $R \times R$ box in the site percolation model where exactly half of the vertices in the box are occupied.

This talk is closely related to my paper https://arxiv.org/pdf/2210.16100.pdf

Alison Ethridge (Oxford)

Forwards and backwards in spatially heterogeneous populations

We introduce a broad class of mechanistic spatial models that might describe how spatially heterogeneous populations live, die, and reproduce. Questions that we (start to) address include: how does the population density change across space and time? And how does genetic ancestry spread across geography when looking back through space and time in these populations?

Jakob Björnberg (Gothenburg)

Dimerization in mirror models and quantum spin chains

We consider two models of random loops where we prove breaking of translational symmetry. The first is a mirror model, where the loops are formed by light rays bouncing in a labyrinth of randomly oriented mirrors. The second is a probabilistic representation of a quantum spin chain, and can be obtained as a limit of the first, for inhomogeneous mirror weights. In the terminology of quantum spins, this symmetry-breaking is called “dimerization”. Based on joint works with K. Ryan as well as with P. Muehlbacher, B. Nachtergaele and D. Ueltschi.

Béatrice de Tilière (Paris-Dauphine)

The dimer model on minimal graphs: the elliptic case and beyond

The dimer model represents the adsorption of diatomic molecules on the surface of a crystal. It is modeled through perfect matchings of a planar graph chosen with respect to the Boltzmann measure. When the graph is periodic, Kenyon, Okounkov and Sheffield show that the phase diagram is given by the spectral curve, which has the remarkable property of being Harnack. Another important result is the local expression obtained by Kenyon for one Gibbs measure when the underlying graph is isoradial and the model is critical. In a series of works with Cédric Boutillier (Sorbonne University) and David Cimasoni (University of Geneva), we extend these results in a unified framework. We consider the model on minimal graphs and prove an explicit correspondence with the set of Harnack curves; we also prove local formulas for the two parameter family of Gibbs measures.

Tuesday

Jeff Steif (Chalmers)

Where to stand when playing darts?

We study a simple model for playing darts where we have a dart (which is a random vector in R^n) and a payoff function (a map from R^n to R) and we aim (translate the random dart) in order to maximize the expected payoff.

Question: If we are allowed to move closer to the target (which corresponds to scaling the distribution), is it always better to do so?

While this is true for certain families of darts including stable distributions and what are called selfdecomposable distributions which had already been studied by Paul Levy in the 1940's, it is not true for other distributions, such as (1) the uniform distribution and more generally compactly supported distributions and (2) distributions whose Fourier transform has a zero in the upper half plane. This yields an interesting difference between the folded Cauchy distribution and the folded normal distribution.

This is joint work with Björn Franzen and Johan Wästlund.

Tony Guttmann (Melbourne)

Walks in a square and the gerrymander sequence

We give an improved algorithm for the enumeration of self-avoiding walks and polygons within an N x N square as well as for SAWs crossing a square. We present some proofs of the expected asymptotic behaviour as the size N of the square grows, and then show how one can numerically estimate the parameters in the asymptotic expression. We then show how the improved algorithm can be adapted to count gerrymander sequences (OEIS A348456), and prove that the asymptotics of the gerrymander sequence is similar to that of SAWs crossing a square. This work has been done in collaboration with Iwan Jensen.

Zhongyang Li (Connecticut)

Planar Site Percolation via Tree Embeddings

I will show that for any infinite, connected, planar graph G properly embedded into the plane with minimal vertex degree at least 7, the i.i.d. Bernoulli(p) site percolation on G a.s. has infinitely many infinite 1-clusters for any $p \in (p_c^{site},1-p_c^{site})$. Moreover $p_c^{site}<1/2$, so the above interval is non-empty. This confirms a conjecture of Benjamini and Schramm in 1996.

The proof is based on a novel construction of embedded trees on such graphs.

Laurent Saloff-Coste (Cornell)

TBC

Robin Pemantle (University of Pennsylvania)

Negative association and the strong Rayleigh property

I will start by reviewing the theory of negative dependence of binary random variables, beginning with origins in mathematical statistics and statistical mechanics. The high point is the Borcea-Branden-Liggett theory connecting negative dependence to the geometry of zero sets of polynomials, and providing the long sought checkable condition (strong Rayleigh = "SR") with strong consequences such as negative association (NA). I will end with some more recent (last ten years) work of various people, including concentration inequalities, the notion of Lorentzian measures, examples of SR measures that are not NA, and a CLT based on the geometry of zeros.

David Aldous (U.C. Berkeley and University of Washington)

The Critical Beta-splitting Random Tree

In the critical beta-splitting model of a random n-leaf rooted tree, clades (subtrees) are recursively split into sub-clades, and a clade of m leaves is split into sub-clades containing i and $m-i$ leaves with probabilities $\propto 1/(i(m-i))$. This model turns out to have interesting properties. There is a canonical embedding into a continuous-time model (CTCS(n)).

There is an inductive construction of CTCS(n) as n increases, analogous to the stick-breaking constructions of the uniform random tree and its limit continuum random tree. We study the heights of leaves and the limit {\em fringe distribution} relative to a random leaf. In addition to familiar probabilistic methods, there are analytic methods (developed by co-author Boris Pittel) based on explicit recurrences which often give more precise results. So this model provides an interesting concrete setting in which to compare and contrast these methods.

Preprints at https://arxiv.org/abs/2302.05066 and https://arxiv.org/abs/2303.02529

Wednesday

Frank den Hollander (Leiden University)

Large deviations for the capacity of the Wiener sausage

The Wiener sausage is the 1-environment of Brownian motion. It is an important mathematical object because it is one of the simplest non-Markovian functionals of Brownian motion. Its asymptotic behaviour has been the subject of intensive study since the 1970's. In this talk we focus on C_t, the capacity of the Wiener sausage up to time t. We show that C_t satisfies a downward large deviation principle with a rate that depends on d. We identify the rate function in terms of a variational formula, and analyse its scaling properties. The main technique that is used to derive the large deviation principle is a `skeleton approach', where the path of the Brownian motion is coarse-grained and large deviations of the resulting skeleton are transferred to large deviations of the Wiener sausage.

Joint work with Michiel van den Berg (Bristol) and Erwin Bolthausen (Zurich).

Mark Holmes (Melbourne)

Limit theorems for the half orthant model

The half-orthant model is a model of a random medium that arises in the study of random walks in i.i.d. random environments such that from each site either (with probability p) steps in positive directions are available, or (with probability 1-p) steps in all directions are possible. We will discuss various facts about this model, including phase transitions and shape theorems. We will also mention a few of the many open problems for this model and other so-called degenerate random environments.

Cristina Toninelli (Paris-Dauphine)

Bootstrap percolation and kinetically constrained models: universality results

Recent years have seen a great deal of progress in the field of bootstrap percolation models (BP), a large class of monotone cellular automata. On Z^d there is now a quite complete understanding of their evolution starting from a random initial condition, with a universality picture for their critical behavior.

In this seminar we will focus on their non-monotone stochastic counterpart, namely kinetically constrained models (KCM). In KCM each vertex is either infected or healthy and, iff it is infectable according to the BP rules, its state is resampled (independently) at rate one to infected with probability p, and healthy with probability 1-p.

I will discuss a series of results which establish the full universality picture of KCM in two dimensions. We will see that, compared to those of BP, the universality classes of KCM are richer and the critical time scales diverge faster due to the dominant contribution of energy barriers.

If time remains, we will discuss the relevance of KCM as toy models for the liquid/glass transition, whose understanding is one of the major open problems in condensed matter physics.

Mikhail Menshikov (Durham)

Dynamics of finite inhomogeneous particle systems with exclusion interaction

(Joint work with V. Malyshev; S. Popov; A. Wade)

We study finite particle systems on the one-dimensional integer lattice, where each particle performs a continuous-time nearest-neighbour random walk, with jump rates intrinsic to each particle, subject to an exclusion interaction which suppresses jumps that would lead to more than one particle occupying any site. We show that the particles jumps rates determine explicitly a unique partition of the system into maximal stable sub-systems, and that this partition can be obtained by a linear-time algorithm of elementary steps, as well as by solving a finite non-linear system. The internal configuration of each stable sub-system possesses an explicit product-geometric limiting distribution, and each stable sub-system obeys a strong law of large numbers with an explicit speed; the characteristic parameters of each stable sub-system are simple functions of the rate parameters for the corresponding particles. For the case where the entire system is stable, we prove a central limit theorem describing the fluctuations around the law of large numbers. Our approach draws on ramifications, in the exclusion context, of classical work of Goodman and Massey on unstable Jackson queueing networks.

Chuck Newman (NYU)

Some Open Percolation Problems

I discuss three open problems, chosen to involve relatively little technical background but with important consequences if they could be resolved. They all concern bond percolation on the nearest neighbor graph in $d dimensions. The first is about invasion percolation. The second and third concern FK random cluster percolation with two replicas of the bond variables; these are relevant for \pm J spin glasses.

Michael Aizenman (Princeton University)

Percolation meets Quantum ... and non-perturbative results follow

A random cluster system that is an extension of a continuous-time percolation model is naturally expressed in terms of a 1+1 dimensional loop-soup measure, whose loops form the inner or outer boundaries of the model’s connected clusters. The same random loop system shows up in a stochastic geometric representation of two distinct quantum spin models. Combining insights based on by the three different “projections” of the common loop system one arrives at a structural explanation of threshold parameter values for three very different phenomena:

i) the discontinuity of the phase transition in a 1+1 dimensional classical random cluster model at Q > 4,

ii) dimerization of the ground states of a flattened version of the quantum Heisenberg model at S > 1/2,

iii) Nèel order in the ground state of the a-symmetric HXXZ quantum spin chain at ∆ > 0.

(Talk based on a joint 2020 paper with H. Duminil-Copin and S. Warzel, with input from previous works by G. Ray and Y. Spinka (2020) on Q state Potts models, and Aizenman and Nachtergaele (1994) on quantum spin chains.)

Hugo Duminil-Copin (IHES and Geneva)

Critical planar percolation: rotation invariance and beyond?

A few years back, Grimmett and Manolescu introduced a novel technique to prove universality of critical exponents for planar bond Bernoulli percolation models. This strategy, based on the star-triangle transformation, was later used in the more general context of random cluster models, in particular to prove rotation invariance of the model. In this talk, we will discuss some prospecting on how to use rotation invariance to obtain new information on the critical random cluster models on Z^2.

Thursday

Jean-François Le Gall (Université Paris-Saclay)

An SDE for local times of tree-indexed Brownian motion

We prove that the pair consisting of the local time of Brownian motion indexed by the Brownian tree and its derivative satisfies a stochastic differential equation whose drift term involves the Airy function. The same property holds for local times of super-Brownian motion. This complements an earlier work giving the Markov property of the same pair of processes. This talk is based on a joint work with Ed Perkins (UBC).

Asaf Nachmias (Tel Aviv)

The scaling limit of critical hypercube percolation

We study the large connected components in critical percolation on the Hamming hypercube {0,1}^m. We show that their sizes, properly rescaled, converge in distribution, and that, considered as rescaled metric measure spaces, they converge in distribution with respect to the Gromov--Hausdorff--Prokhorov topology. The corresponding limits are as in critical Erdos-Renyi graphs.

Joint work with Arthur Blanc-Renaudie and Nicolas Broutin

Omer Angel (UBC)

TBC

Vincent Tassion (ETH Zurich)

Robust noise sensitivity of percolation

Consider critical site Bernoulli percolation on the triangular lattice, where each vertex is colored black or white with probability 1/2, independently of the other vertices. In 1999, Benjamini, Kalai and Schramm proved that crossing probabilities are noise sensitive: resampling a small proportion of the vertices lead to an independent percolation picture. Ten years later, Garban, Pete and Schramm obtained a sharp quantitative version of this result. These works rely on Fourier analysis, and are restricted to Bernoulli percolation (i.e. product measure) and the independent resampling Dynamics.

In this talk, we will discuss noise sensitivity for more general percolation models, and more general dynamics. Based on a recent approach to noise sensitivity with Hugo Vanneuville (that relies on geometrical arguments and not on spectral methods), we show noise sensitivity of crossing probabilities for high temperature Ising under Glauber dynamics.

Based on a joint work with Hugo Vanneuville.

Christina Goldschmidt (Oxford)

Random tree encodings and snakes

There are several functional encodings of random trees which are commonly used to prove (among other things) scaling limit results. We consider two of these, the height process and Lukasiewicz path, in the classical setting of a branching process tree with critical offspring distribution of finite variance, conditioned to have n vertices. These processes converge jointly in distribution after rescaling by n^{-1/2} to constant multiples of the same standard Brownian excursion, as n goes to infinity. Their difference (taken with the appropriate constants), however, is a nice example of a discrete snake whose displacements are deterministic given the vertex degrees; to quote Marckert, it may be thought of as a “measure of internal complexity of the tree”. We prove that this discrete snake converges on rescaling by n^{-1/4} to the Brownian snake. We believe that our methods should also extend to prove convergence of a broad family of other “globally centred” discrete snakes which seem not to be susceptible to the methods of proof employed in earlier works of Marckert and Janson.

This is joint work with Louigi Addario-Berry, Serte Donderwinkel and Rivka Mitchell.

Russ Lyons (Indiana)

Voronoi Tessellations without Nuclei

Given a discrete set of points in a metric space, called nuclei, one associates to each such nucleus its Voronoi cell, which consists of all points closer to it than to other nuclei. This construction is widely used in mathematics, science, and engineering; it is even used in baking. In Euclidean space, one commonly uses a homogeneous Poisson point process to assign the locations of the nuclei. As the intensity of the point process tends to 0, the nuclei spread out and disappear in the limit, with each pair of points eventually belonging to the same cell. Surprisingly, this does not happen in other settings such as hyperbolic space; instead, one obtains a Voronoi tessellation without nuclei! We describe properties of such a limiting tessellation, as well as analogous behavior on Cayley graphs of finitely generated groups. We will illustrate results with many pictures and several animations. The talk is based on work of Sandeep Bhupatiraju and joint work in progress with Matteo d'Achille, Nicolas Curien, Nathanael Enriquez, and Meltem Unel. We will not assume knowledge of hyperbolic space.

Friday

Gordon Slade (UBC)

Boundary conditions and finite-size scaling in high dimensions

Above the upper critical dimension, boundary conditions play a dramatic role in finite-size scaling for percolation and related models, as has been widely discussed in the physics literature. We present recent work (joint with Emmanuel Michta and Jiwoon Park) which provides a thorough and precise account of the effect of free vs periodic boundary conditions on the finite-size scaling of the weakly coupled hierarchical $|\varphi|^4$ spin system in dimensions 4 and higher, and offers precise conjectures for other spin systems and self-avoiding walk in high dimensions.

Stas Volkov (Lund)

Is coexistence possible for mutual predators?

Once upon a time, on a remote island nestled in the heart of a tropical paradise, there existed a peculiar coexistence between humans and alligators. Every day either an alligator or a human was chosen, randomly and uniformly from the total population of both. If it was an alligator, then $A>0$ new alligators appeared on the island. Sadly, it also meant that $B>=0$ humans were eaten. On the other hand, if it was a human, then A new humans appeared on the island; which also means that $C>=0$ alligators were eaten. If one of the species was eaten completely, it would become extinct (no immigration of either people or reptiles was allowed on the island).

Could both populations sustainably coexist forever, or one of them is bound to become extinct? How does the answer depend on A,B,C? What if there are more than two species, with some intricate relations between them? The answer to these questions was obtained in a joint work by British, Portuguese, and Swedish mathematicians, and I will describe what it is.

Barbara Dembin (ETH Zurich)

Upper tail large deviations for chemical distance in supercritical percolation

We consider supercritical bond percolation on Z^d and study the chemical distance, i.e., the graph distance on the infinite cluster. It is well-known that there exists a deterministic constant $\mu(x)$ such that the chemical distance $D(0,nx)$ between two connected points 0 and nx grows like $n\mu(x)$. We prove the existence of the rate function for the upper tail large deviation event $\{D(0,nx)>n\mu(x)(1+\epsilon),0<->nx\}$ for $d>=3$.

Joint work with Shuta Nakajima.

Wendelin Werner (Cambridge)

On percolation of continuous loops.

I will discuss results about clusters of Brownian loops in the plane (related to the so-called Bosonic Free Field) and their decomposition into individual Brownian loops. We will in particular see that given the closure C of a cluster of loops, some points of C have a conditional probability 1/2 to be actually part of a loop, and conditional probability 1/2 to be in no loop at all. This type of phenomenon only appears in the continuum picture and is not present in the discrete (or cable-graph) settings. This talk will be partially based on ongoing work with Matthis Lehmkuehler and Wei Qian.