SISCER Module 15
Lecture 6: Negative Controls and Difference-in-Differences

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Plan[1]

Review: causal inference in observational studies

Negative controls

Difference-in-Differences

Key references for this lecture

- Shi et al. (2020) for negative controls
- Wing et al. (2018) and Roth et al. (2022) for difference-in-differences

[1]Acknowledgement: This lecture is built in part upon lecture notes from Xu Shi (UMich) and Linbo Wang (U Toronto).

Casual inference in observational studies

- Methods under the no unmeasured confounders assumption
  1. Matching (Lecture 2)
  2. Outcome regression, IPW, AIPW, and entropy balancing weight (Lecture 3)
- Methods to address unmeasured confounding
  1. Sensitivity analysis (Lecture 3)
  2. Natural experiment: instrumental variable (Lecture 5), regression discontinuity design[2]
  3. Causal exclusion (this lecture): negative control exposure/outcome, difference-in-differences, placebo sample[3]

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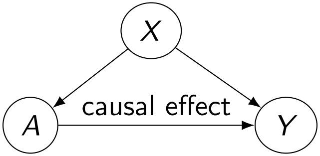
[2] See https://en.wikipedia.org/wiki/Regression_discontinuity_design. Biggs et al. (2017) applied the regression discontinuity design to compare those who received abortions and those were denied abortion in the near-limit group.

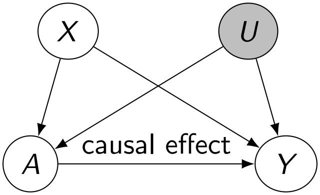
[3] Ye et al. (2022)

The "randomized" scenario in causal inference



- Estimand: the average treatment effect $\text{ATE} = E[Y(1)] - E[Y(0)]$ and many others
- Key assumption: All confounders are measured
  - "Randomized" within each stratum of X
  - Not empirically verifiable
  - Sensitivity analysis quantifies how robust the study conclusion is

Unmeasured confounding is a threat to causal inference



- Unmeasured confounders U
  - Cannot create a "randomized" scenario within stratum of X
  - The observed association might be an artifact of confounding bias
- For ease of presentation, X will be omitted from the graph.

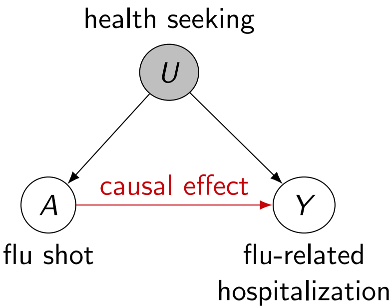
Association (prediction) $\neq$ Causality

- Dog obesity is associated with (predictive of) human obesity.
- Intervention on dog does not reduce owner's weight. (no causal effect)

(Unmeasured confounders)

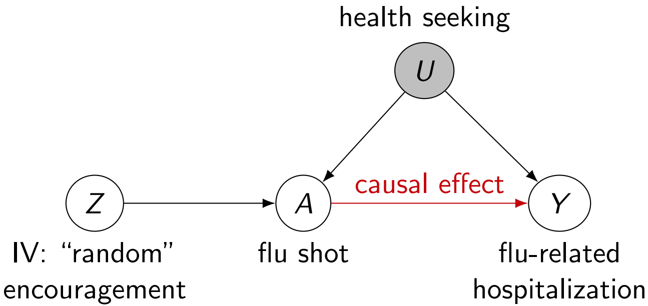
$$
\begin{tikzcd}
 & \text{Life style } (U) \arrow[dl] \arrow[dr] & \\
\text{Dog's weight } (A) \arrow[rr, "0"] & & \text{Owner's weight } (Y)
\end{tikzcd}
$$

Association = Causation + Confounding bias

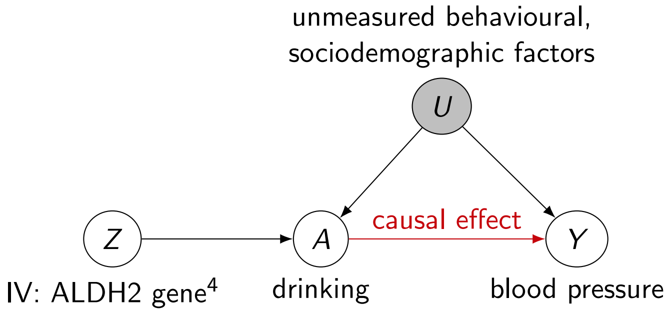


- Unmeasured confounding by health seeking behavior
- How to generate more reliable evidence?

Instrumental variable

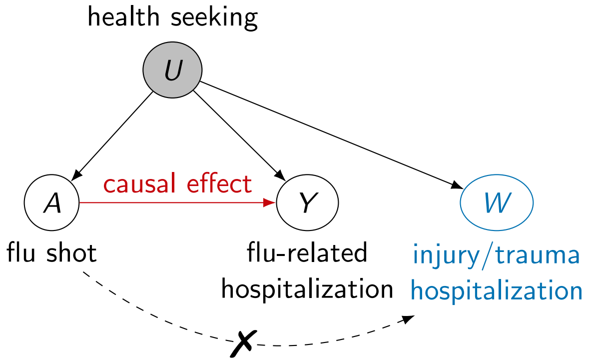


Mendelian randomization: using genetic variants as IVs



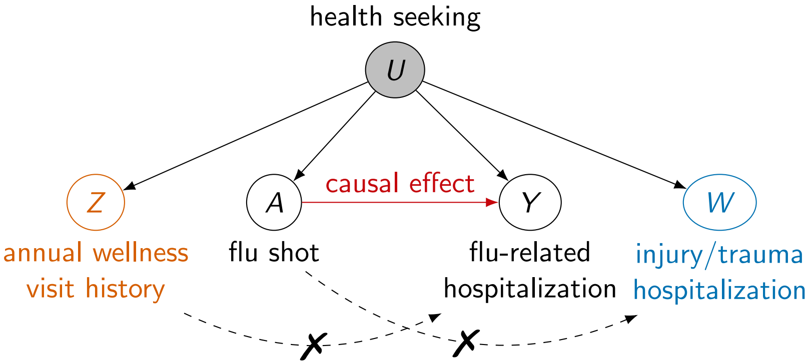
[4] Chen et al. (2008)

Negative control outcome (NCO)



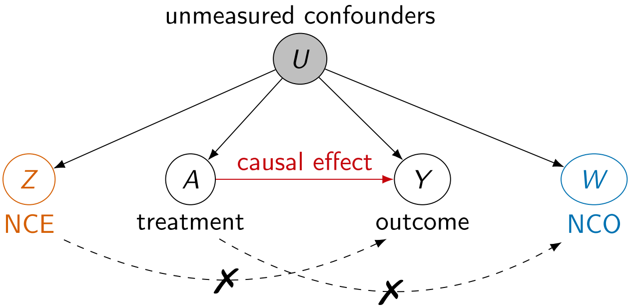
- Find a proxy of health-seeking: injury/trauma hospitalization
- Key knowledge: flu shot does not prevent injury/trauma hospitalization
- Repeat the analysis using the NCO
- Unexpected association indicates unmeasured confounding bias

Negative control exposure (NCE)



- Find another proxy of health-seeking: annual wellness visit history
- Key knowledge: wellness visit history does not prevent flu-related hospitalization
- Repeat the analysis using the NCE
- Unexpected association indicates unmeasured confounding bias

Negative control exposure (NCE) and outcome (NCO)

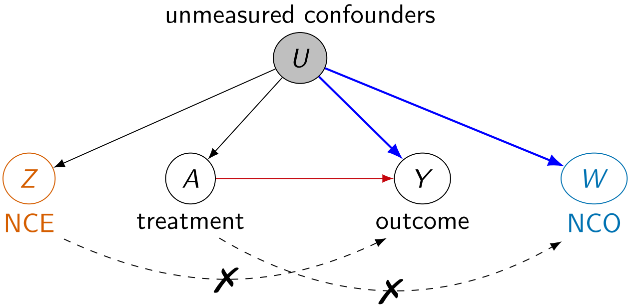


- Z is NCE if
  1. it does not causally affect Y, W
  2. it is associated with U
- W is NCO if
  1. it is not causally affected by A, Z
  2. it is associated with U

Identification assumptions

- (Proxy variables) $(Z, A) \perp (Y(a), W) \mid (U, X)$
- (Positivity) $0 < P(A = 1 \mid U, X) < 1$
- (Full rank/Completeness) Z, W should have enough variability relative to the variability of U

Double negative control: intuition for bias adjustment (Shi et al., 2020)



- Confounding bias is a product of U-A and U-Y association
  - Effect of A on W is a product of U-A and U-W association
  - Problem solved if U has the same effect on Y and W (the strategy taken by DID)
- Otherwise: effect of Z on Y and W can recover the difference
  - Effect of Z on W is a product of U-Z and U-W association
  - Effect of Z on Y is a product of U-Z and U-Y association

Nonparametric identification of ATE using double negative control

For binary U, Z, W,

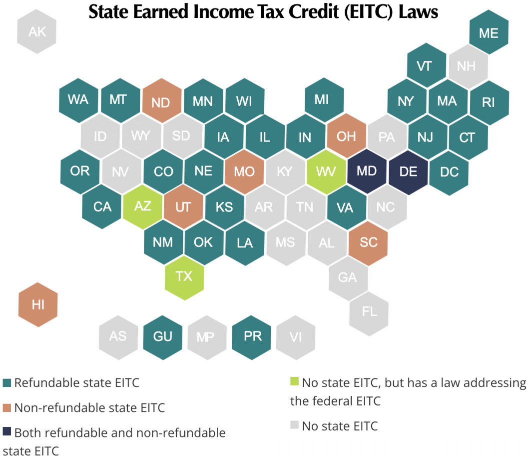
$$\text{ATE} = \Delta_{\text{naive}} - \Delta_{\text{bias}}$$

$$\Delta_{\text{naive}} = E[\delta_A^Y(Z, X)], \quad \Delta_{\text{bias}} = E[\frac{\delta_Z^Y(1-A, X)}{\delta_Z^W(1-A, X)} \delta_A^W(Z, X)]$$

where $\delta_*^\star(\cdot)$ is the effect of $*$ on $\star$ conditional on all other observed variables.

- NCO recovers the bias via $\delta_A^W(\cdot)$ up to a scale; NCE recovers the scale
- $\delta_A^W(Z, X) = E[W|A=1, Z, X] - E[W|A=0, Z, X]$
- $\delta_Z^Y(A, X) = E[Y|A, Z=1, X] - E[Y|A, Z=0, X]$
- $\delta_Z^W(A, X) = E[W|A, Z=1, X] - E[W|A, Z=0, X]$

Example: do earned income tax credits (EITC) reduce deaths of despair?

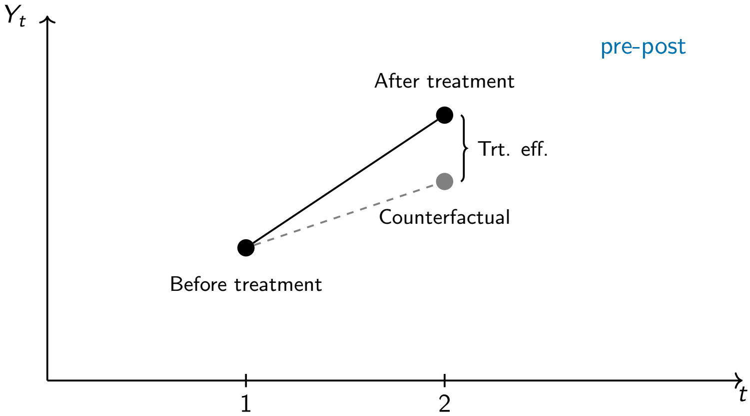


(National Conference of State Legislatures)

Difference-in-Differences (DID) for causal effect

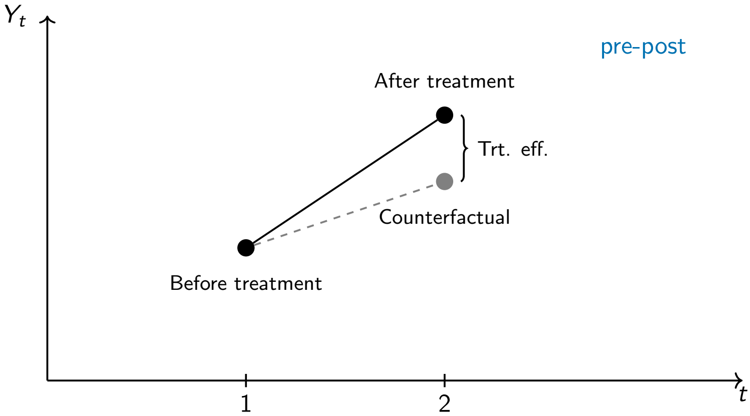
- Challenges from unmeasured confounding: states with EITC laws differ from states without them in other ways that may be related to deaths of despair
- DID is commonly used for estimating causal effects with panel data
- Prototypical DID application: how do changes in state policies affect individual
  - Did Missouri's handgun purchaser licensing law affects firearm homicide rates?
  - Did minimum wage laws change employment levels?
  - Motivating application: do EITC reduce deaths of despair?

DID for Causal Effects



DID for Causal Effects

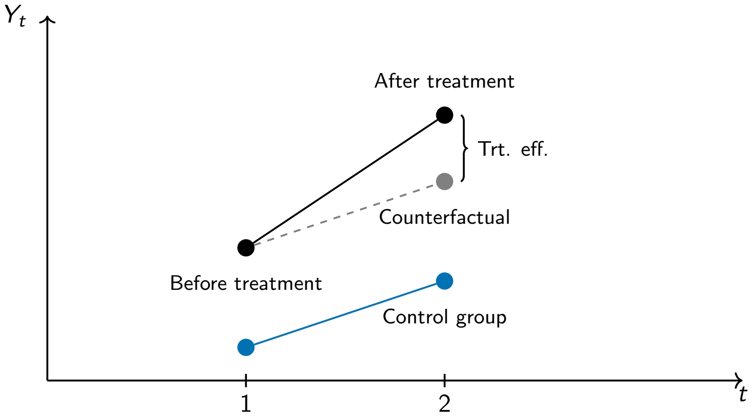
Identify the counterfactual $\Leftrightarrow$ Identify the treatment effect



DID for Causal Effects

Identify the counterfactual $\Leftrightarrow$ Identify the treatment effect

Parallel Trends: Absent treatment, treated and control would evolve over time in the same way. (functional-form dependent)



DID for Causal Effects

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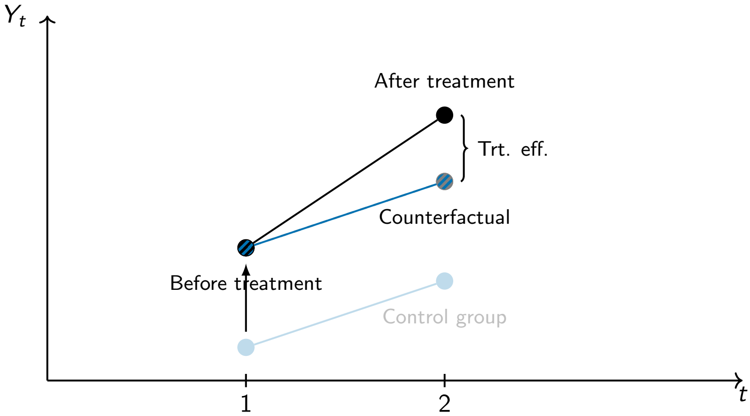
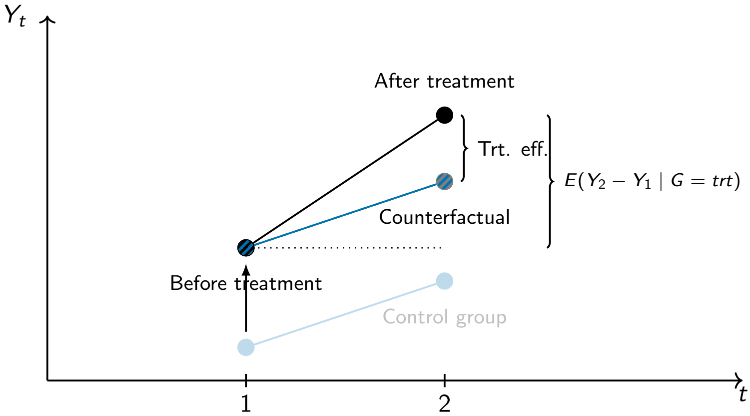


DID for Causal Effects

Identify the counterfactual ⇔ Identify the treatment effect

Parallel Trends: Absent treatment, treated and control would evolve over time in the same way. (functional-form dependent)



DID for Causal Effects

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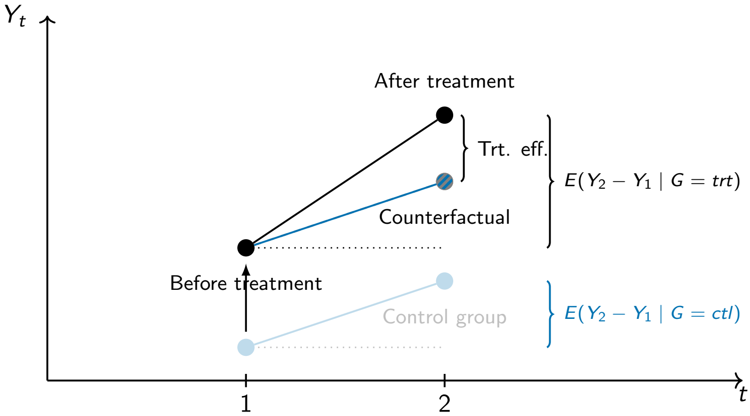

DID for Causal Effects

Identify the counterfactual $\Leftrightarrow$ Identify the treatment effect

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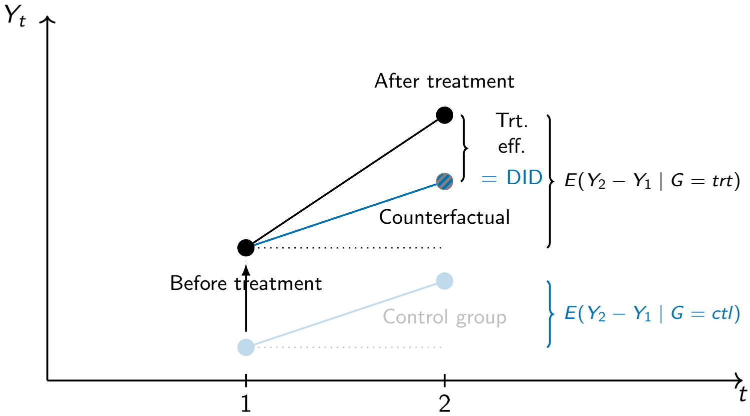
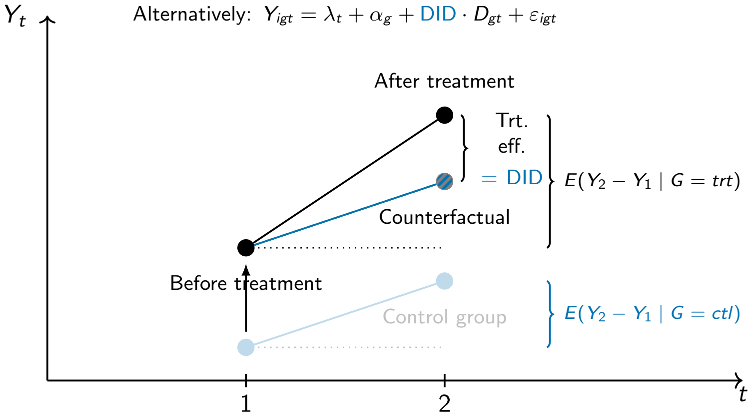


DID for Causal Effects

Identify the counterfactual $\Leftrightarrow$ Identify the treatment effect

Parallel Trends: Absent treatment, treated and control would evolve over time in the same way. (functional-form dependent)

Alternatively: $Y_{igt} = \lambda_t + \alpha_g + \text{DID} \cdot D_{gt} + \varepsilon_{igt}$



Statistical methods (Roth et al., 2022)

- **If all treated units adopt the treatment at the same time:**
  - Static Two-way fixed effects (TWFE) model,

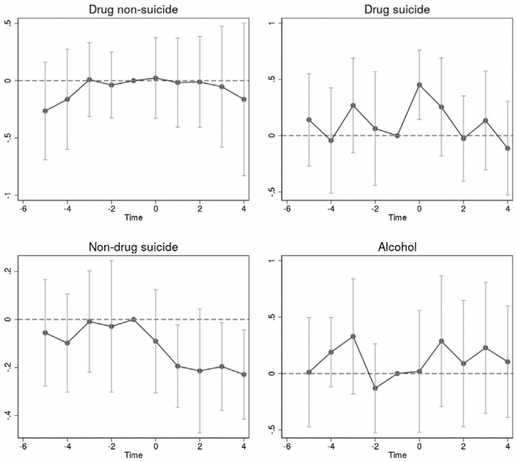
$$Y_{it} = \beta D_{it} + \gamma^T X_{it} + \alpha_i + f_t + \epsilon_{it}$$

  - Dynamic TWFE model, E_i is when unit i initiates the treatment ($E_i = \infty$ if unit i is never treated)

$$Y_{it} = \sum_{-\underline{k} \leq \ell \leq \bar{k}} \beta_\ell I(t - E_i = \ell) + \gamma^T X_{it} + \alpha_i + f_t + \epsilon_{it}$$

- **If treated units adopt the treatment at different time (staggered adoption):**
  - Use the static TWFE model only if confident in treatment effect homogeneity
  - Use the dynamic TWFE model only if confident that there is heterogeneity only in time since treatment
  - Otherwise, consider using a "heterogeneity-robust" estimator, e.g., Callaway and Sant'Anna (2021)

Back to the EITC example (Dow et al., 2020)



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