

# MR-PATH: A Latent Mixture Model for Heterogeneous Causal Mechanisms in Mendelian Randomization

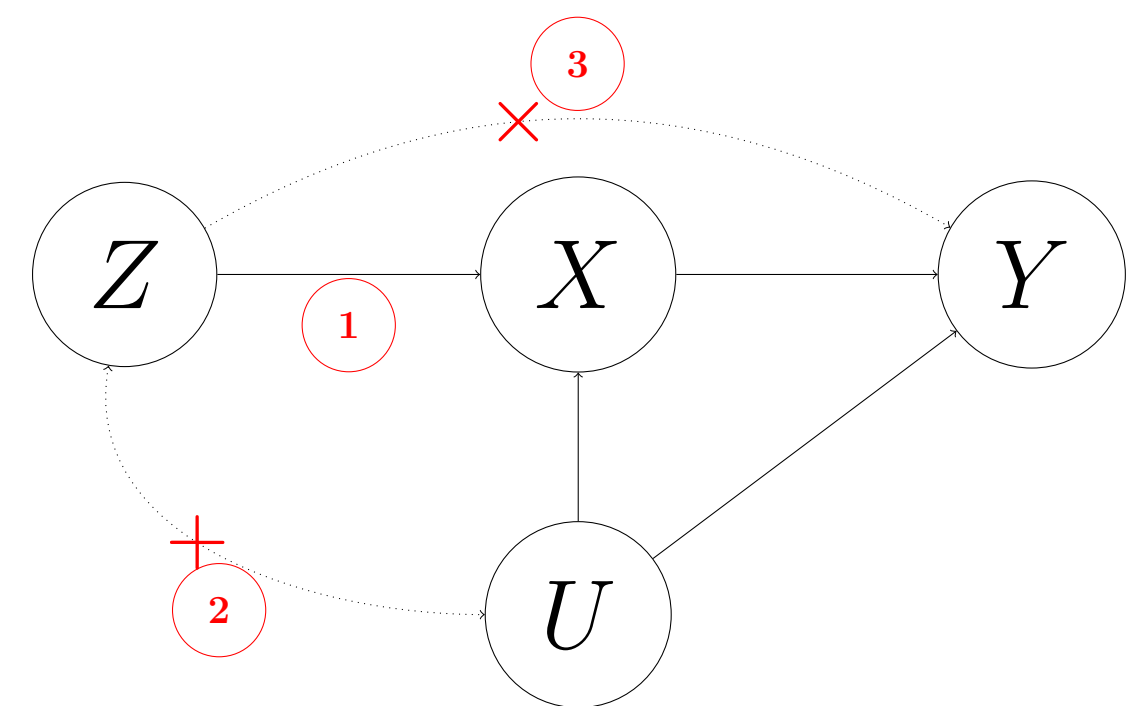


Daniel Long, Qingyuan Zhao, Yang Chen

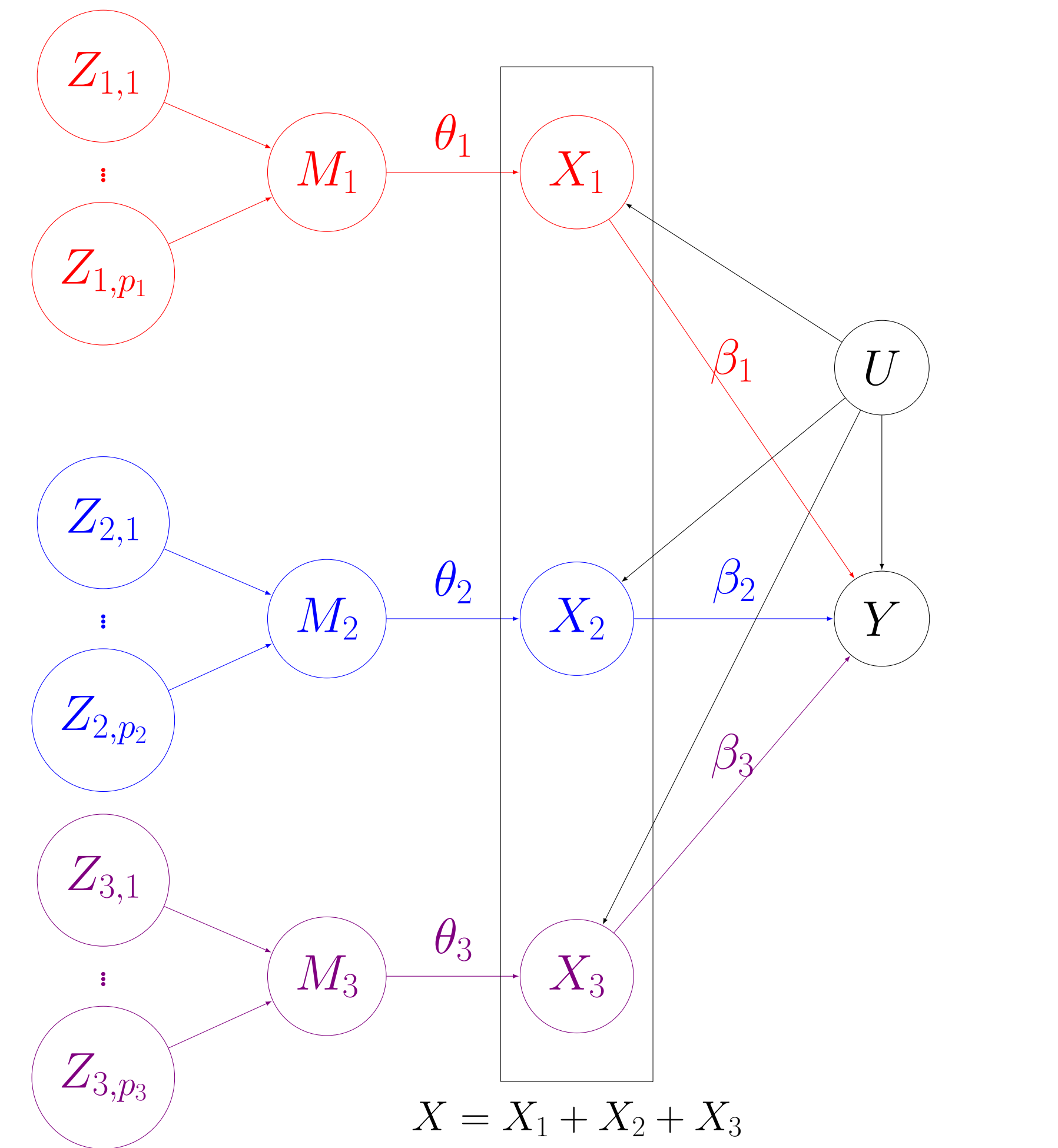
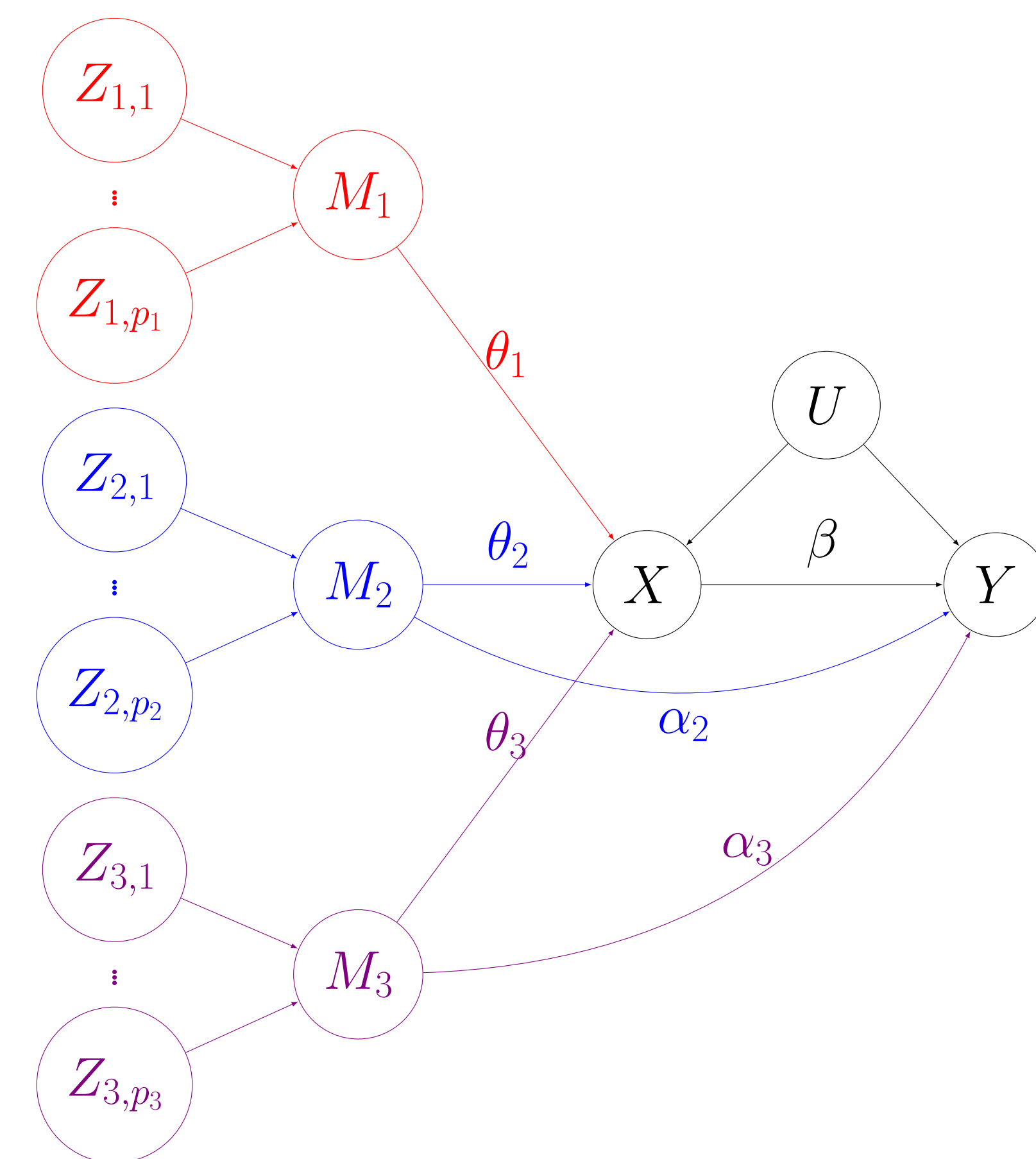


Department of Statistics, University of Michigan, Ann Arbor and Statistical Laboratory, University of Cambridge

## Background



## Mechanistic Heterogeneity in MR



## Application: Is HDL the good cholesterol?

- Observational studies have found a consistent **inverse association** between HDL cholesterol (*HDL-C*) and coronary heart disease (*CHD*)  
 $\Rightarrow$  **HDL hypothesis**: HDL protects from atherosclerosis. (*HDL is the "good" cholesterol.*)
- However, the HDL hypothesis has received scrutiny after
  - Several clinical trials raising HDL cholesterol showed, at best, modest cardiovascular benefit.
  - MR studies reached conflicting conclusions.
- Our hypothesis**: There are multiple causal mechanisms between HDL-C and CHD.

- Mendelian Randomization (MR)**: an instrumental variables (IV) method that uses genetic variants ( $Z$ ) as instruments to estimate the causal effect of a modifiable risk exposure ( $X$ ) on a disease outcome ( $Y$ ) in the presence of confounders ( $U$ ).
- Most robust MR methods rely on the **"effect homogeneity" assumption**: the risk exposure has the same causal effect for every individual. This assumption may be unrealistic when we use MR to study complex biological systems involving multiple mechanisms.

## Model

$$\begin{pmatrix} \hat{\theta}_{X_i} \\ \hat{\theta}_{Y_i} \end{pmatrix} \text{indep. } N\left(\begin{pmatrix} \theta_{X_i} \\ \beta_i \theta_{X_i} \end{pmatrix}, \begin{pmatrix} \sigma_{X_i}^2 & 0 \\ 0 & \sigma_{Y_i}^2 \end{pmatrix}\right) \quad i = 1, \dots, p.$$

$$Z_i \sim \text{Categorical}(\pi_1, \dots, \pi_K)$$

$$\beta_i | Z_i = k \sim N(\mu_k, \sigma_k^2), \quad k = 1, \dots, K.$$

### Observed Data

- $\hat{\theta}_{X_i}, \hat{\theta}_{Y_i}$ : Observed SNP-exposure/SNP-outcome effects
- $\sigma_{X_i}, \sigma_{Y_i}$ : Corresponding standard errors

### Latent variables

- $\theta_{X_i}$ : True SNP-exposure effects
- $\beta_i$ : SNP-specific causal effects

### Parameters

- $\pi_k$ : Proportion of SNPs in cluster  $K$
- $\mu_k$ : Average causal effect of cluster  $K$
- $\sigma_k^2$ : Variance of cluster  $K$

## Statistical Inference

### EM algorithm

- E-step is not tractable  $\Rightarrow$  approximate with importance sampling  $\Rightarrow$  **Monte-Carlo EM (MC-EM)**

### Approximate Confidence Intervals

- Compute observed information matrix using Louis (1982)  $\Rightarrow$  invert to get standard errors.

### Model Selection

- Modified BIC criterion from Ibrahim et al. (2008).

## Software

The **MRPATH** R package is available at <https://github.com/danieliong/MRPATH>

## Preprint

A preprint of this work is available at <https://arxiv.org/abs/2007.06476>

## Our contributions

- "Mechanistic heterogeneity"**: A novel concept we formalized to describe effect heterogeneity in MR due to multiple causal mechanisms.
- MR-PATH**: A mixture model for mechanistic heterogeneity in MR.
- Statistical inference for MR-PATH**: Monte-Carlo EM algorithm, approximate confidence intervals, modified BIC criterion

## Contact Information

Email: daniong@umich.edu  
Phone: (510) 816-8686

