Part III Differential Geometry, Michaelmas 2015 **P.M.H. Wilson**

A tentative syllabus is as follows.

Smooth manifolds and vector fields. Definition and examples of manifolds, smooth maps. Tangent vectors and vector fields, tangent bundle. Geometric consequences of the implicit function theorem, submanifolds. Lie Groups. [8]

Forms and tensors on manifolds. Differential 1-forms, cotangent bundle. Vector bundles; example of Hopf bundle. Bundle morphisms and automorphisms. Exterior algebra of differential forms. Tensors. Orientability of manifolds. Partitions of unity and integration on manifolds, Stokes Theorem; de Rham cohomology. Lie derivative of tensors. [6]

Connections on vector bundles. Linear connections on vector bundles and horizontal sections: covariant exterior derivative, curvature. General Bianchi identity, orthogonal connections. [4]

Connections on the tangent bundle. Koszul connections. Covariant derivative of tensor along a curve. Torsion free connections. Bianchi's identities for torsion free connections. [3]

Riemannian manifolds. Riemannian metrics, Levi–Civita connection, Christoffel symbols, geodesics. Riemannian curvature tensor and its symmetries, second Bianchi identity, sectional curvatures. Ricci tensor and Einstein metrics. Ricci and scalar curvatures. Schur's theorem. [3]

The main references for this course are the books listed below and some printed notes on the lecturer's home page: https://www.dpmms.cam.ac.uk/∼pmhw/DG2007.pdf. These printed notes were expanded by the auditor and so cover some topics not in lectures.

Pre-requisites An essential pre-requisite is a working knowledge of linear algebra (including multilinear forms) and multivariate calculus. Exposure to some of the ideas of classical differential geometry would be found useful (but not essential).

Literature

D. Barden, C. Thomas, An introduction to differentiable manifolds. Imperial College Press, 2003.

R.W.R. Darling, Differential forms and connections. CUP, 1994.

M. Spivak, Differential Geometry, Volume 2. Publish or Perish, 1999.

F.W. Warner, Foundations of differentiable manifolds and Lie groups, Springer-Verlag, 1983.