ERRATUM TO: HOMOLOGY OF THE MODULI SPACES AND MAPPING CLASS GROUPS OF FRAMED, r-SPIN AND PIN SURFACES

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Abstract. We correct a calculation in the appendix of the paper mentioned in the title. The stable first homology of the Pin^- mapping class group is $(\mathbb{Z}/2)^2$, rather than $(\mathbb{Z}/2)^3$. We also clarify some properties of the function q_ξ.

N. Friedrich has pointed out an error in the proof of Theorem A.5 of [RW14] which invalidates that calculation. In fact, the error takes place before the beginning of the proof, where the structure of $H^*(\mathbf{C};\mathbb{F}_2)$ as a module over the Steenrod algebra is described. The total Stiefel–Whitney class of $-\gamma_2^{\mathrm{Pin}^-} \oplus \gamma_1^{\pm} \to B\mathrm{Pin}^-(2)$ is actually

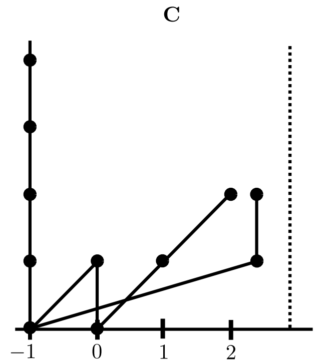
$$(1+w_1+w_1^2)^{-1}\cdot(1+w_1) = 1+w_1^2 \in H^*(B\mathrm{Pin}^-(2);\mathbb{F}_2) \cong \mathbb{F}_2[w_1,x_4]/(w_1^3)$$

rather than $(1+w_1+w_1^2)\cdot(1+w_1)=1$ as claimed.

The effect of this is that the cohomology of the spectrum $\mathbf{C}$ as a module over the Steenrod algebra up to degree 2 should instead be described as follows: it has basis u_{-1} in degree -1, $w_1 \cdot u_{-1}$ in degree 0, $w_1^2 \cdot u_{-1}$ in degree 1, and is zero in degree 2, and the only nontrivial Steenrod operations in this range are

$$\mathrm{Sq}^2(u_{-1}) = w_1^2 \cdot u_{-1}$$
$$\mathrm{Sq}^1(w_1 \cdot u_{-1}) = w_1^2 \cdot u_{-1}.$$

The corrected chart for the Adams E_2-page for $\mathbf{C}$ is as shown.



Theorem A.5′. $\pi_1(\mathbf{MTPin}^-(2)) = (\mathbb{Z}/2)^2$.

Proof. Proceed as in the published proof, but use the corrected chart for the Adams E_2-page for $\mathbf{C}$. In particular, we see that $\pi_1(\mathbf{C}) = \mathbb{Z}/2$, as there can be no differentials in this range. Consider the portion of the long exact sequence on homotopy groups

$$\pi_1(\mathbf{MTSpin}(2)) \longrightarrow \pi_1(\mathbf{MTPin}^-(2)) \longrightarrow \pi_1(\mathbf{C}) = \mathbb{Z}/2.$$

The chart for $\mathbf{MTSpin}(2)$ shows that $\pi_1(\mathbf{MTSpin}(2))$ is a quotient of $\mathbb{Z}/8$ so cyclic, so we find that $\pi_1(\mathbf{MTPin}^-(2))$ can be generated by 2 elements. The chart for $\mathbf{MTPin}^-(2)$ shows that this group is either $(\mathbb{Z}/2)^3$ or $(\mathbb{Z}/2)^2$, so it follows that it must be $(\mathbb{Z}/2)^2$. □

In a different direction, A. Gilles has pointed out that the observations about q_ξ on p. 162 leading up to equation (2.3) only hold for $r=2$, as the arguments of Johnson we referred to neglect signs in an essential way. This does not affect the overall argument, however, as these observations are used only in the proof of Proposition 2.8, which concerns the case $r=2$.

References

[RW14] Oscar Randal-Williams, *Homology of the moduli spaces and mapping class groups of framed, r-Spin and Pin surfaces*, J. Topol. **7** (2014), no. 1, 155–186.

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