ERRATUM TO: MONOIDS OF MODULI SPACES OF MANIFOLDS

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Abstract. We correct a technical error in the statement and proof of Lemma 2.8 of the paper in the title. The statement of Theorem 2.9 is unaffected.

We are grateful to Elmar Vogt for pointing out that equation (2-2) in the proof of Lemma 2.8 of [GRW10] may not hold as asserted there. Here (after first giving a technical lemma) we will prove Lemma 2.8′, which replaces Lemma 2.8 of the published paper. Theorem 2.9 may then be deduced from Lemma 2.8′ with only a slight modification to the proof given in the published paper.

Lemma A. *Given a neighbourhood A of zero in the topological vector space $\Gamma_c(NM)$ and functions $\varphi_1, \dots, \varphi_r \in C_c^\infty(M)$, there is a smaller neighbourhood $B = B(A, \varphi_1, \dots, \varphi_r)$ of zero such that if $s_1, \dots, s_r \in B$ then $\sum \varphi_i \cdot s_i \in A$.*

Proof. This is the statement that the linear function $(s_1, \dots, s_r) \mapsto \sum \varphi_i \cdot s_i$ is continuous. By our definition of the topology on $\Gamma_c(NM)$, it is enough to show that the topological vector space $C_c^\infty(M)$ has the same property, and since addition is continuous in any topological vector space it suffices to prove that pointwise multiplication by any $\varphi \in C_c^\infty(M)$ defines a continuous function $C_c^\infty(M) \to C_c^\infty(M)$. This follows from the formula

$$X_1 \cdots X_r(\varphi \cdot f) = \sum (X_{i_1} \cdots X_{i_k} \varphi) \cdot (X_{j_1} \cdots X_{j_{r-k}} f),$$

where $X_1, \dots, X_r$ are vector fields on M and the sum is taken over the 2^r partitions of $\{1, \dots, r\}$ into two disjoint subsets $\{i_1, \dots, i_k\}$ and $\{j_1, \dots, j_{r-k}\}$. $\square$

Lemma 2.8′. *Let $K_i \subseteq U$ be compact, $i = 1, \dots, r$, and let $K \subseteq \cup_i \mathrm{int}(K_i)$ be another compact set. Then the diagonal map $\delta : \Psi(U)^K \to \prod \Psi(U)^{K_i}$ is an open map onto its image.*

Proof. Let $\Delta = \delta(\Psi(U)^K)$. We need to see that $\delta : \Psi(U)^K \to \Delta$ is open. For each open set $A \subseteq \Psi(U)^{cs}$ and each $M \in A$, we shall show that there is a smaller open neighbourhood $M \in B \subseteq A \subseteq \Psi(U)^{cs}$ such that

$$\delta(\pi_K^{-1}\pi_K(A)) \supseteq \left(\prod \pi_{K_i}^{-1}\pi_{K_i}(B)\right) \cap \Delta. \tag{0.1}$$

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That will imply that $\delta(\pi_K^{-1}\pi_K(A))$ is open in Δ. By Lemma 2.5, the sets $\pi_K^{-1}\pi_K(A)$ for open $A \subseteq \Psi(U)^{cs}$ form a basis for the topology of $\Psi(U)^K$, so we will have shown that δ is an open map.

Let $\varepsilon : M \to (0,\infty)$ be a smooth function, small enough that the standard map $NM \to \mathbb{R}^n$ restricts to an embedding of the open subset of NM of vectors of length less than ε with image in U. Write $\nu \subseteq NM$ for this subset, and $p : \nu \to M$ for the restriction of the bundle projection. The sets $V_i = M \setminus p(\nu \setminus K_i)$, which are closed in M, will play an important role. They have the property that $p^{-1}(V_i) \subseteq K_i$, and also that $s(V) \subseteq K_i$ for any $s \in \Gamma_c(\nu)$.

The subset $\Gamma_c(\nu) \subseteq \Gamma_c(NM)$ gives an open neighbourhood $M \in c_M(\Gamma_c(\nu)) \subseteq \Psi(U)^{cs}$, and we may suppose without loss of generality that $A \subseteq c_M(\Gamma_c(\nu))$. An element $N \in \Psi(U)^K$ then has $N \in \pi_K^{-1}\pi_K(A)$ precisely when there exists an $s \in A$ such that N and $s(M)$ agree near K. Similarly, if $B \subseteq A$ and $N \in \pi_{K_i}^{-1}\pi_{K_i}(B)$ for all $i = 1,\ldots,r$, then there exists an $s_i \in B$ with that N and $s_i(M)$ agree near K_i. Such a section s_i is likely not unique, but on the closed subset $V_i \subseteq M$ it must agree with the inverse of $p|_{N\cap\nu}$. In particular s_i and s_j must agree near $V_i \cap V_j$.

We now construct the smaller neighbourhood $B \subseteq A$ as follows. First, after possibly shrinking the function ε, we may assume that the closure of $p(\nu \cap K)$ is contained in $\cup_i \mathrm{int}(V_i)$. Then we may choose smooth functions $\varphi_i : M \to [0,1]$ such that $\mathrm{supp}(\varphi_i) \subseteq \mathrm{int}(V_i)$ and $p(\nu \cap K) \subseteq (\sum \varphi_i)^{-1}(1)$ and choose $B = B(A, \varphi_1, \ldots, \varphi_r) \subseteq A$ as in Lemma A. We claim that the containment (0.1) holds with this choice of B.

To see this, let $\delta(N)$ be an element of the right-hand side of (0.1), i.e. $N \in \pi_{K_i}^{-1}\pi_{K_i}(B)$ for all $i = 1,\ldots,r$. Then there exist elements $s_i \in B \subseteq \Gamma_c(\nu)$ such that N and $s_i(M)$ agree near K_i. Our choice of B guarantees that the linear combination $s = \sum \varphi_i s_i$ has $s \in A$. This section s must agree with s_i on the subset $V_i \cap p(\nu \cap K) \subseteq M$, since $\sum \varphi_j = 1$ there and s_i agrees with s_j whenever $\varphi_j > 0$. It follows that the subsets $s_i(M)$ and $s(M)$ agree inside

$$p^{-1}(V_i \cap p(\nu \cap K)), \tag{0.2}$$

which is contained in $p^{-1}(V_i) \subseteq K_i$, and hence N and $s(M)$ agree inside the subset (0.2). This holds for all i and hence N and $s(M)$ agree inside the union

$$p^{-1}(\big(\cup_{i=1}^r V_i\big) \cap p(\nu \cap K)) = p^{-1}\big(\cup_{i=1}^r V_i\big),$$

which we have arranged to contain the closure of $K \cap \nu$ in its interior. Since N and $s(M)$ are both disjoint from $K \setminus \nu$, we see that N and $s(M)$ actually agree near all of K, and hence $N \in \pi_K^{-1}\pi_K(A)$, as desired. □

Proof of Theorem 2.9. As in the published paper, but pick $K_i \subseteq U_i$ compact with $K \subseteq \cup_i \mathrm{int}(K_i)$. □

References

[GRW10] S. Galatius, O. Randal-Williams, *Monoids of moduli spaces of manifolds.* Geom. Topol. 14 (2010), no. 3, 1243-1302.

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