

Oscar Randal-Williams

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RESEARCH INTERESTS Algebraic and Geometric Topology: diffeomorphism groups, moduli spaces, cobordism categories, spaces of manifolds, surgery theory, configuration spaces, characteristic classes, K-theory, applications of homotopy theory to geometry.

POSITIONS

- ◇ **King's College, University of Cambridge**, (October 2016 –)
Ordinary fellow.
- ◇ **University of Cambridge**, (October 2013 –)
University Lecturer.
- ◇ **King's College, University of Cambridge**, (October 2012 – September 2016)
Research fellow.
- ◇ **University of Cambridge**, (October 2012 – October 2013)
Herchel Smith postdoctoral research fellow.
- ◇ **University of Copenhagen**, (September 2010 – September 2012)
Postdoctoral position and member of the Centre for Symmetry and Deformation.
- ◇ **Mathematical Institute, University of Oxford**, (January – August 2010)
EPSRC PhD Plus postdoctoral position.
- ◇ **Jesus College, University of Oxford**, (October – December 2009)
Retained lecturer in Mathematics.

EDUCATION

- ◇ **Jesus College, University of Oxford**, (2006 – 2009)
DPhil Mathematics, 2009.
Thesis title: *Stable moduli spaces of manifolds*.
Supervisor: U. Tillmann.
- ◇ **Jesus College, University of Oxford**, (2002 – 2006)
MMath Mathematics, First Class Honours.

PUBLICATIONS

S. Galatius and O. Randal-Williams, *Homological stability for moduli spaces of high dimensional manifolds. I*, 43 pages.
Journal of the American Mathematical Society, to appear.

O. Randal-Williams, *Cohomology of automorphism groups of free groups with twisted coefficients*, 20 pages.
Selecta Mathematica, to appear.

S. Galatius and O. Randal-Williams, *Homological stability for moduli spaces of high dimensional manifolds. II*, 59 pages.
Annals of Mathematics, to appear.

B. Botvinnik, J. Ebert, and O. Randal-Williams, *Infinite loop spaces and positive scalar curvature*, 67 pages.
Inventiones mathematicae, to appear.

O. Randal-Williams, *An upper bound for the pseudoisotopy stable range*, 11 pages.
Mathematische Annalen, to appear.

- S. Galatius, I. Grigoriev, and O. Randal-Williams, *Tautological rings for high dimensional manifolds*.
Compositio Mathematica, **153** (4) (2017) 851-866.
- F. Cantero and O. Randal-Williams, *Homological stability for spaces of surfaces*.
Geometry & Topology **21** (2017) 1387-1467.
- S. Galatius and O. Randal-Williams, *Abelian quotients of mapping class groups of highly connected manifolds*.
Mathematische Annalen **365** (1) (2016) 857-879.
- O. Randal-Williams, *Resolutions of moduli spaces and homological stability*.
Journal of the European Mathematical Society **18** (2016) 1-81.
- J. Ebert and O. Randal-Williams, *Torelli spaces of high-dimensional manifolds*.
Journal of Topology **8** (1) (2015) 38-64.
- S. Galatius and O. Randal-Williams, *Stable moduli spaces of high dimensional manifolds*.
Acta Mathematica **212** (2014), no. 2, 257-377.
- S. Galatius and O. Randal-Williams, *Detecting and realising characteristic classes of manifold bundles*.
Algebraic Topology: Applications and New Directions (Stanford, CA, 2012), Contemp. Math.
- J. Ebert and O. Randal-Williams, *Generalised Miller-Morita-Mumford classes for block bundles and topological bundles*.
Algebraic & Geometric Topology **14** (2014) 1181-1204.
- O. Randal-Williams, *“Topological chiral homology” and configuration spaces of spheres*.
Morfismos (MIMS proceedings issue), **17** (2) (2013) 57-70.
- O. Randal-Williams, *Homology of the moduli spaces and mapping class groups of framed, r -Spin and Pin surfaces*.
Journal of Topology **7** (1) (2014) 155-186.
- O. Randal-Williams, *“Group-Completion”, local coefficient systems, and perfection*.
Quarterly Journal of Mathematics **64** (3) (2013) 795-803 .
- O. Randal-Williams, *The space of immersed surfaces in a manifold*.
Mathematical Proceedings of the Cambridge Philosophical Society **154** (3) (2013) 419-438.
- O. Randal-Williams, *Relations among tautological classes revisited*.
Advances in Mathematics, **231**(3-4) (2012) 1773-1785.
- O. Randal-Williams, *The Picard group of the moduli space of r -Spin Riemann surfaces*.
Advances in Mathematics, **231**(1) (2012) 482-515.
- J. Ebert and O. Randal-Williams, *Stable cohomology of the universal Picard varieties and the extended mapping class group*.
Documenta Mathematica, **17** (2012) 417-450.
- O. Randal-Williams, *Homological stability for unordered configuration spaces*.
Quarterly Journal of Mathematics, **64**(1) (2013) 303-326.
- O. Randal-Williams, *Embedded cobordism categories and spaces of submanifolds*.
International Mathematics Research Notices **3** (2011) 572-608.
- S. Galatius and O. Randal-Williams, *Monoids of moduli spaces of manifolds*.
Geometry & Topology **14**(3) (2010) 1243-1302.
- O. Randal-Williams, *The homology of the stable non-orientable mapping class group*.
Algebraic & Geometric Topology **8** (2008) 1811-1832.
- J. Ebert and O. Randal-Williams, *On the divisibility of characteristic classes of non-orientable surface bundles*.
Topology and its Applications **156** (2008) 246-250.

- PREPRINTS J. Ebert and O. Randal-Williams, *Semi-simplicial spaces*, 32 pages. arXiv:1705.03774.
O. Randal-Williams and N. Wahl, *Homological stability for automorphism groups*, 65 pages. arXiv:1409.3541.
O. Randal-Williams, *The stable cohomology of automorphisms of free groups with coefficients in the homology representation*, 9 pages. arXiv:1012.1433.
- GRANTS ◇ EPSRC First Grant, EP/M027783/1, £90,898.
 ◇ LMS Scheme 1 grant, £5,000 (with J. Giansiracusa).
- TEACHING ◇ **Lecturer** (at Cambridge)
EXPERIENCE · Part IB Groups, Rings, and Modules, Lent 2016, again 2017.
 · Part II Algebraic Topology, Michaelmas 2013, again 2014.
 · Part III Homotopy Theory, Lent 2015.
 · Part III Algebraic Topology, Michaelmas 2016.
- ◇ **Part III essay advisor** (at Cambridge), with titles *Complex cobordism and formal group laws*, *Algebraic K-theory of finite fields*, *Homological stability*, *Finite H-spaces*, *Representation stability*, *Topics in geometric topology*, *Vassiliev theory*, *Algebraic K-theory*, *Exotic spheres*.
- ◇ **Supervisor** (at Cambridge)
 · IA Numbers & Sets revision, Easter 2014.
 · IB Linear Algebra, Michaelmas 2014.
 · IB Geometry, Lent 2013.
- ◇ **Lecturer** (at Copenhagen)
 · Topology, Autumn 2011.
 · Algebraic Topology 1.5, Spring 2011.
- ◇ **Graduate supervisor**
 · Supervisor of Cambridge PhD theses of Nina Friedrich 2014-, Nils Prigge 2016-
 Informal co-supervisor of Copenhagen PhD thesis of Mauricio Esteban Gómez López. Co-supervisor of Barcelona PhD thesis of Federico Cantero Morán.
 · Advisor for two MS theses, Paolo Masulli (2011) and Mauricio Esteban Gómez López (2011), at Copenhagen.
 · Advisor for two BSc theses, at Copenhagen.
- ◇ **Graduate lectures**
 · Co-organiser of lecture series “Moduli spaces of manifolds”, February–June 2012.
 · Organiser of seminar series “Surgery theory”, 2011.
 · Organiser of lecture series “Surgery theory”, 2010.
 · Lecture course on “Resolutions of moduli spaces and homological stability”, 2010.
 · Organiser of seminar series “Classification of Topological Quantum Field Theories, after J. Lurie”, 2009.
 · Organiser of seminar series “Nilpotence and periodicity in stable homotopy theory”, 2008.
- ◇ **Teaching Assistant**
 · Homological algebra, Winter 2010.
 · Part C Lie Groups (4th year undergraduate course), Michaelmas 2007.
 · Part C Algebraic Topology (4th year undergraduate course), Hilary 2007.

- Part C Topology & Groups (4th year undergraduate course), Michaelmas 2006.
- Part B Geometry of Surfaces (3rd year undergraduate course), Michaelmas 2006.
- ◇ **Class Tutor** (at Oxford)
 - Part C Lie Groups (4th year undergraduate course), Michaelmas 2009.
 - Part C Algebraic Topology (4th year undergraduate course), Hilary 2009.
 - Part C Topology & Groups (4th year undergraduate course), Michaelmas 2008.
- ◇ **Tutor** (at Oxford)
 - Introduction to complex numbers, Analysis I, Probability I (1st year undergraduate courses), Michaelmas 2009.
 - Linear Algebra (2nd year undergraduate course), Hilary 2007.

CONFERENCE
TALKS AND
SEMINARS

- E_n -cells and applications* – Moduli and Traces, Copenhagen, 21st April 2017.
- Metastable cohomology of moduli spaces of Riemann surfaces* – British Mathematical Colloquium morning talk, Durham, 4th April 2017.
- Stable twisted cohomology via scanning* – OAC-manifolds, Oxford, 13th March 2016.
- Stable moduli spaces of manifolds* – Topology Seminar, QMUL, 11th March 2016.
- Lecture course on *Cohomology of moduli spaces of manifolds* – European Autumn School in Topology, Utrecht, September 19-23 2016.
- Cohomology of $\text{Aut}(F_n)$ with twisted coefficients* – Topologie, Oberwolfach, 18th July 2016.
- Lecture course on *Moduli Spaces of Manifolds* – Young Topologists' Meeting, Copenhagen, 4th July 2016.
- E_2 -cells and homological stability* – Topology of manifolds, Lisbon, 27th June 2016.
- Realising characteristic numbers of fibre bundles* – Scottish Topology Seminar, Edinburgh, 11th March 2016.
- Characteristic classes of fibre bundles* – OAC-manifolds, Oxford, 22nd February 2016.
- Tautological rings for high-dimensional manifolds* – Warwick, 18th February 2016.
- Infinite loop spaces and positive scalar curvature* – Aspects of Homotopy Theory, Southampton, 15th December 2015.
- Moduli spaces of manifolds* – Colloquium, Columbia University, New York, 15th October 2015.
- Tautological rings for high-dimensional manifolds* – Stanford University, 6th October 2015.
- Tautological rings for high-dimensional manifolds* – Manifolds Unlocking Higher Structures, Clay Mathematical Institute Workshop, Oxford, 28th September-2nd October 2015.
- Moduli spaces of high-dimensional manifolds* – Moduli spaces and their applications, Liverpool, 21st September 2015.
- Moduli spaces of high-dimensional manifolds* – PIMS Symposium on the Geometry and Topology of Manifolds, Vancouver.
- Infinite loop spaces and positive scalar curvature* – PIMS Symposium on the Geometry and Topology of Manifolds, Vancouver.
- Lecture course on *Moduli spaces of manifolds*, Hausdorff Institute for Mathematics, Bonn, 4th-8th May 2015.
- Tautological rings for high-dimensional manifolds* – Stockholm, 15th April 2015.

- Infinite loop spaces and positive scalar curvature* – Stockholm, 14th April 2015.
- Homological stability and abelianness* – Teichmüller Modular Groups: A Celebration of Nikolai Ivanov's 60th Birthday, Chicago, 21st March 2015.
- Cohomology of some diffeomorphism groups* – Southampton, 23th February 2015.
- Infinite loop spaces and positive scalar curvature* – Oxford, 19th January 2015.
- Cohomology of some diffeomorphism groups* – Aberdeen, 15th December 2014.
- Stable moduli spaces of high-dimensional manifolds* – Topologie; Oberwolfach, 15th September 2014.
- Stable moduli spaces of high-dimensional manifolds: applications* – ICM Satellite Conference on Algebraic Topology, Dalian, China, 10th August 2014.
- Cohomology of diffeomorphism groups* – Workshop on Algebraic Topology and its Applications, Dalian, China, 7th August 2014.
- Infinite loop spaces and positive scalar curvature* – Analysis and Topology in Interaction, Cortona, 16th June 2014.
- Lecture course on *Moduli spaces of high-dimensional manifolds and applications to positive scalar curvature metrics* – Westfälischen Wilhelms-Universität Münster, 9th-13th June 2014.
- Moduli spaces of high-dimensional manifolds* – British Mathematical Colloquium, Queen Mary, London, 9th April 2014.
- Moduli spaces of high-dimensional manifolds* – Algebraic Topology, MSRI, 27th February 2014.
- Abelianisations of the mapping class groups of some highly-connected manifolds* – Rice, Houston, 17th February 2014.
- The tautological class κ_{-1}* – Topology seminar, Stanford, 11th February 2014.
- Homological stability of moduli spaces of manifolds* – Algebraic Topology, MSRI, 31st January 2014.
- Infinite loop spaces and positive scalar curvature* – Geometry seminar, UCL, 14th January 2014.
- Infinite loop spaces and positive scalar curvature* – Glasgow, 2nd December 2013.
- Stable moduli spaces of high-dimensional manifolds* – Manchester, 18th November 2013.
- The tautological class κ_{-1}* – Cohomology of the moduli space of curves, Zürich, 11th October 2013.
- Infinite loop spaces and positive scalar curvature* – British Topology Meeting, Aberdeen, 9th September 2013.
- Homological stability and moduli spaces* – Homology Stability Workshop, Copenhagen, 26th August 2013.
- The Picard groups of moduli spaces of r -Spin Riemann surfaces* – Séminaire de géométrie algébrique; Jussieu, Paris, 16th May 2013.
- Infinite loop spaces and positive scalar curvature* – Sheffield, 2nd May 2013.
- Infinite loop spaces and positive scalar curvature* – Stanford, 9th April 2013.
- Stable moduli spaces of high-dimensional manifolds* – Nottingham, 7th December 2012.
- Cohomology of the universal Picard varieties, and tautological relations* – Algebraic geometry seminar; Humboldt-Universität zu Berlin, 5th December 2012.
- The “Mumford conjecture” package, with applications* – SFB colloquium; Humboldt-Universität zu Berlin, 4th December 2012.

The “Mumford conjecture” package, with applications – GQT colloquium; Universiteit van Amsterdam, 16th November 2012.

Stable moduli spaces of high-dimensional manifolds – Oxford, 29th October 2012.

El espacio de superficies en una variedad – XIX Encuentro de Topología, La Coruña, 19th October 2012.

Homological stability for moduli spaces of manifolds – Topologie; Oberwolfach, 19th September 2012.

Homological stability for moduli spaces of manifolds – 27th British Topology Meeting; Cambridge, 6th September 2012.

Stable moduli spaces of high-dimensional manifolds – Fourth Arolla Conference on Algebraic Topology; Arolla, 23rd August 2012.

Homological stability for moduli spaces of manifolds – Algebraic Topology: Applications and New Directions; Stanford, 27th July 2012.

Homological stability for moduli spaces of manifolds – 17th NRW meeting; Münster, 4th May 2012.

Stable moduli spaces of high-dimensional manifolds – Universitat de Barcelona, 6th February 2012.

Stable moduli spaces of high-dimensional manifolds – MPIM Bonn, 23rd January 2012.

Stable moduli spaces of high-dimensional manifolds – Edinburgh, 26th October 2011.

Stable moduli spaces of high-dimensional manifolds – Glasgow, 24th October 2011.

Universal Picard varieties and tautological relations – ETH, Zürich, 7th October 2011.

Stable moduli spaces of high-dimensional manifolds – MIT, 26th September 2011.

Picard group of the moduli space of r -Spin Riemann surfaces – Riemann Surfaces; PCMI, Utah, 12th July 2011.

Universal Picard varieties, extended mapping class groups, and tautological relations – Stanford, 3rd May 2011.

Two lectures on spaces of manifolds – NTNU, Trondheim, 28th–29th October 2010.

Monoids of moduli spaces of manifolds, II – Topologie; Oberwolfach, 22nd September 2010.

The Picard group of the moduli space of r -Spin curves – Summer Topology Workshop; Stanford University, 5th August 2010.

Resolutions of moduli spaces – Stanford University, 13th April 2010.

Resolutions of moduli spaces – TTT meeting; University of Warwick, 23rd March 2010.

Resolutions of moduli spaces and homology stability – University of Copenhagen, 1st February 2010.

Monoids of moduli spaces of manifolds – 25th Nordic and 1st British–Nordic congress of Mathematicians; Oslo, 8th June 2009.

Monoids of moduli spaces of manifolds – Manifold Perspectives; Oberwolfach, 27th May 2009.

Monoids of moduli spaces of manifolds – University of Bonn, 12th May 2009.

Homology of the stable non-orientable mapping class group – Stanford University, 13th January 2009.

Homology of the stable non-orientable mapping class group – TTT meeting; University of Sheffield, 9th December 2008.

Geometric delooping of cobordism categories – Westfälischen Wilhelms-Universität Münster, 24th April 2008.

Spaces of manifolds and tangential structures – Moduli spaces; University of Bonn / Max Planck Institute, 10th January 2008.

Embedded cobordism categories – University of Copenhagen, 15th October 2007.

Characteristic classes of bundles of non-orientable surfaces – 22nd British Topology Meeting; Sheffield, 10th September 2007.

LOCAL
SEMINARS

Applications of cellular E_2 -algebras – Geometry seminar; University of Cambridge, 16th November 2016.

Infinite loop spaces and positive scalar curvature – Geometry seminar; University of Cambridge, 16th October 2013.

Cohomology of braid groups and mapping class groups – GDO programme; Newton Institute, University of Cambridge, 28th February 2013.

The “Mumford conjecture” package, with applications – Geometry seminar; University of Cambridge, 14th November 2012.

The Picard group of the moduli space of r -Spin curves – Algebra and Topology seminar; University of Copenhagen, 11th October 2010.

Cohomology of moduli spaces – Junior Geometry and Topology seminar; University of Oxford, 3rd June 2010.

Arc complexes – Junior Geometric Group Theory seminar; University of Oxford, 25th May 2010.

Homology stability of spaces of surfaces – Topology seminar; University of Oxford, 23rd November 2009.

Monoids of moduli spaces of manifolds – Geometry seminar; University of Oxford, 1st June 2009.

Descent in algebra, geometry and topology – Junior Geometry and Topology seminar; University of Oxford, October 2008.

The Hopf invariant one problem – Junior Geometry and Topology seminar; University of Oxford, March 2008.

Poincaré–Hopf index theorem and the transfer – Junior Geometry and Topology seminar; University of Oxford, October 2007.

Classification of smooth manifolds up to cobordism – Junior Geometry and Topology seminar; University of Oxford, February 2007.

EXTENDED
VISITS

Sabbatical at Stanford University. 4th October–5th December 2015.

“Homotopy theory, manifolds, and field theories” programme, Hausdorff Centre for Mathematics, Bonn. 27th July–21st August 2015.

Research member at the “Algebraic Topology” programme, MSRI, Berkeley. 27th January–28th February 2014.

CONFERENCES
AND
WORKSHOPS
ATTENDED

Moduli and Traces – Copenhagen, 20th–21st April 2017.

British Mathematical Colloquium – Durham, 3rd–6th April 2017.

OAC-manifolds – Oxford, 13th March 2017.

European Autumn School in Topology – Utrecht, September 19–23 2016.

OAC-manifolds – Cambridge, 9th September 2016.

Topologie – Oberwolfach, 17th–23rd July 2016.

Young Topologists’ Meeting – Copenhagen, 4th–8th July 2016.

Topology of manifolds – Lisbon, 27th June–1st July 2016.

- OAC-manifolds* – Aberdeen, 9th May 2016.
- Scottish Topology Seminar* – Edinburgh, 11th March 2016.
- OAC-manifolds* – Oxford, 22nd February 2016.
- Aspects of Homotopy Theory* – Southampton, 14th–19th December 2015.
- Manifolds Unlocking Higher Structures* – Oxford, 28th September–2nd October 2015.
- Homotopy theory, manifolds, and field theories* – Bonn, 17th–21st August 2015.
- PIMS Symposium on the Geometry and Topology of Manifolds* – Vancouver, 29th June–5th July 2015.
- Introductory School: Homotopy theory, manifolds, and field theories* – Bonn, 4th–8th May 2015.
- Teichmüller Modular Groups* – Chicago, 20th–22nd March 2015.
- ICM Satellite Conference on Algebraic Topology* – Dalian, China, 3rd–13th August 2014.
- Manifolds, K-theory, and related topics* – Dubrovnik. 23rd–27th June 2014.
- Analysis and Topology in Interaction* – Cortona. 16th–20th June 2014.
- Surgery theory and its applications in geometry* – Münster. 9th–13th June 2014.
- British Mathematical Colloquium* – Queen Mary, London. 7th–10th April 2014.
- Introductory Workshop: Algebraic Topology* – MSRI, Berkeley. 27th–31st January 2014.
- Cohomology of the moduli space of curves* – Zürich. 10th–13th October 2013.
- British Topology Meeting 2013* – Aberdeen. 9th–11th September 2013.
- Homology Stability Workshop* – Copenhagen. 26th–30th August 2012.
- BMC 2013* – Sheffield. 25th–28th March 2013.
- Aspects of Topology* – Oxford. 17th–19th December 2012.
- XIX Encuentro de Topología* – La Coruña. 19th–21st October 2012.
- Topologie* – Oberwolfach. 17th–22nd September 2012.
- 27th British Topology Meeting* – Cambridge. 6th–8th September 2012.
- Fourth Arolla Conference on Algebraic Topology* – Arolla, Switzerland. 20th–26th August 2012.
- Algebraic Topology: Applications and New Directions* – Stanford. 23rd–27th July 2012.
- 17th NRW meeting* – Münster. 4th–5th May 2012.
- Riemann surfaces* – PCMI, Park City, Utah. 3rd–17th July 2011.
- The Kervaire invariant and stable homotopy theory* – ICMS, Edinburgh. 25th–29th April 2011.
- Topologie* – Oberwolfach. 19th–25th September 2010.
- 25th British Topology Meeting* (organiser) – Oxford. 6th–9th September 2010.
- West Coast Algebraic Topology Summer School* (scientific committee) – Eugene, Oregon. 9th–15th August 2010.
- Summer Topology Workshop* – Stanford. 2nd–6th August 2010.
- 24th British Topology Meeting* – Leicester. 14th–15th September 2009.
- 25th Nordic and 1st British–Nordic congress of Mathematicians* – Oslo. 8th–11th June 2009.
- Manifold Perspectives* – Oberwolfach. 24th–30th May 2009.
- Third Arolla Conference on Algebraic Topology* – Arolla. 18th–24th August 2008.

Homotopical Group Theory and Topological Algebraic Geometry Conference and Workshop – Copenhagen / Bonn. 16th-27th June 2008.

Topology in the Swiss Alps, Young Topologists' Meeting – Le Châtelard. 26th-31st May 2008.

Moduli spaces Conference and Workshop – University of Bonn / Max Planck Institute. 1th-11th January 2008.

22nd British Topology Meeting – Sheffield. 10th-12th September 2007.

Workshop on Homological Algebra – Southampton. 8th-14th July 2007.

Postnikov Memorial Conference – Będlewo. 8th-14th July 2007.

Workshop on Stable Homotopy Theory – Strasbourg. 7th-11th May 2007.

Journal of Topology launch meeting – Oxford. 19th-21th March 2007.

SERVICE

London Mathematical Society. Editorial advisor, 2013–

Referee for *Advances*, *Annals*, *AGT*, *Compositio Math.*, *Duke*, *G&T*, *Glasgow Mathematical Journal*, *HHA*, *Inventiones*, *JAMS*, *J. Differential Geom.*, *J. Topology*, *Math. Ann.*, *Math. Proc. Cam. Phil. Soc.*, *Math. Zeit.*, *MRL*, *Proc. AMS*, *Publ. Math. IHÉS*, *QJM*, *Trans. AMS*

Organiser (with U. Tillmann, R. Hepworth) of OAC-manifolds meetings, 2016.

Organiser (with J. Giansiracusa) of 25th British Topology Meeting, Oxford, 2010.

Scientific committee for West Coast Algebraic Topology Summer School, August 2010.

Organiser of Copenhagen Topology seminar 2010-2012.

Organiser of Oxford Junior Geometry & Topology Seminar 2007-2008.

Undergraduate admissions interviewer – King's College, Cambridge, 2012–.

Undergraduate admissions interviewer – Jesus College, Oxford, 2006–2009.