Part III Analysis of PDEs: Examinable syllabus

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Chapter 1: Introduction

1. The Cauchy problem for ODEs (cf. Ch. 1, Section 2 of notes)
2. Statement of the Cauchy-Lipschitz / Picard-Lindelöf (Theorem 2.4 of Ch. 1 of notes)

Chapter 2: Cauchy–Kovalevskaya Theorem

1. Basic properties of real analytic functions (radius of convergence, etc.) (cf. Ch. 2, Section 1 of notes)
2. Statement and proof of Cauchy–Kovalevksaya (non-characteristic condition, reduction to flat Cauchy hypersurfaces, reduction to first order systems, method of majorants of Cauchy)
3. Application of Cauchy–Kovalevksaya to simple examples
4. Hadamard examples of ill-posedness

Chapter 3: Second order elliptic equations

1. The Sobolev spaces $H^k(\mathcal{U})$ and $H^1_0(\mathcal{U})$ for $\mathcal{U} \subset \mathbb{R}^d$. Sobolev inequalities for these spaces.
2. The Poincare inequality on a bounded domain $\mathcal{U}$ (cf. Ch. 3, Theorem 1.14 of notes)
3. Ellipticity for second order linear operators
4. Definition of weak solution for the Dirichlet problem for second order elliptic operators (cf. Ch. 3, Definition 4.2 and Definition of notes)
5. Proof of existence and uniqueness of weak solutions by Riesz representation or Lax-Milgram
6. Difference quotients (cf. Ch. 3, Definition 7.2 of notes),
7. Interior elliptic regularity, H^{s+2} estimates (cf. Ch. 3, Section 7.2 of notes)
8. Boundary regularity (cf. Ch. 3, Section 7.3 of notes)

Chapter 4: Scalar transport and wave equations

1. Linear transport with constant coefficient: classical and weak L^∞ solutions with or without source term (definition, existence, uniqueness with proofs) (cf. Ch. 4, Section 2 of notes)

2. Linear transport equation with variable coefficients: definition of characteristic curves, existence and uniqueness with proof of classical and weak L^∞ solutions (cf. Ch. 4, Section 3 of notes)

3. Nonlinear scalar 1D transport equation: local-in-time classical solutions (statement and proofs), development of shocks and non-existence of global classical solutions in general, non-uniqueness of weak L^∞ solutions, definition of entropic solutions(cf. Ch. 4, Section 4 of notes)

4. Wave equation: global and localised energy estimate in $\mathbb{R}^n$ (cf. Ch. 4, Sections 5.2 and 5.3 of notes)