Martin Hyland : Linear Logic & GOI

Aim : Model dynamics of computat^n.
static (denotat^{n}al) vs dynamic (rewriting)

Cut Rule : main protagonist.

^{n}al view vs. Process view of Cut.

Cut-Elimination as rewriting ~~as~~ ^{or} categorical ; op^n on terms.

Model: $*$-aut. cat. + every free coalg. naturally carries the struc. of a comm. ~~monoid~~ monoid s.t. every coalg.

What is the dynamics of cut?

Cut = Contract^n (in Physics)

$$A \xrightarrow{f} B . \quad B \xrightarrow{g} C$$

$$\Downarrow$$

$$A \otimes B \xrightarrow{f \otimes g} B \otimes C \;\sim\; A \longrightarrow B^* \otimes B \otimes C$$

$$A \longrightarrow B^* \otimes B \otimes C \xrightarrow{ev \otimes 1} I \otimes C \cong C$$

is $g \circ f$. "Contract^n" = Comm. + hiding

$\therefore$ Comp^n = tensor + "Comm of hiding"

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simplificat^n: $\otimes = \wp$

$\& = \oplus$

$! = ?$

Then deal with a "compact closed category"
– can do tensor calc. & sums = prods &
self dual comonad.

Fermionic Model: Fin dim. vectr spaces.

$\otimes$ for $\otimes$ and $\wp$

$\oplus$ for $\&$ and $\oplus$

(ext. power) Λ for $!$ and $?$

$\Lambda(V)$ has an almost comm. monoid structure
– only graded. ∴ commutative if field has
char 2

For each $\Gamma \vdash_{[\Pi]} \Delta$ in LL where Π reps
cuts, we obtain a map of Vs's

$$\Gamma \otimes \Pi \longrightarrow \Delta \otimes \Pi$$

(all depending on choices for atomic formulae).

s.t. $\Gamma \longrightarrow \Delta \otimes \Pi \otimes \Pi^* \longrightarrow \Delta$

get "the" cut-free pf.

But: Where's the dynamics? What does
it mean?

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- Can give a nicer model using Fermionic Fock spaces....
But claims dynamics there will not be good.

Girard's GOI model : a precursor.

Objects $A = (A_*, A^*)$
inputs outputs

Maps $A \longrightarrow B$ =

$$\begin{tikzcd} A_* \arrow[r, bend right] \arrow[d] & A^* \\ B_* & B^* \arrow[l, bend right] \arrow[u] \end{tikzcd}$$

= partial f^ns $A_* + B^* \longrightarrow B_* + A^*$

Composit^n : mediated by feed back between B_* & B^*

$$\begin{tikzcd} A_* \arrow[r] \arrow[d] & A^* \\ B_* \arrow[d] & B^* \arrow[l] \arrow[u] \\ C_* & C^* \arrow[l] \arrow[u] \end{tikzcd}$$

This cat is compact closed, but little else....
Dynamics is externally monitored.

The Real model :
- "Based" on his pf of SN
- Info. "flow dynamics".

- Equip A_* A^* w/ sets of internal dynamics.
Haske

$\alpha : A_* \longrightarrow A^*$, $\beta : A^* \longrightarrow A_*$ s.t. $\forall \alpha, \beta$

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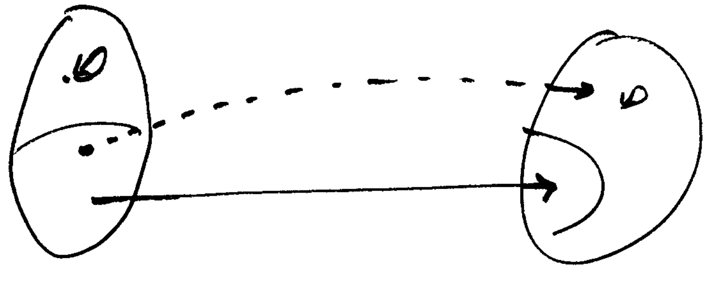
- $\beta\alpha : A_* \longrightarrow A_*$, $\alpha\beta : A^* \longrightarrow A^*$ passes by iterat^n to a stable state (Nilpotent operators)

E.g. of partial f^ns



- sets of states of dynamical system - defined on "non-idling" part ; - - - - - - > idles & nothing happens. ∴ partial f^ns = info flow in this sense.

Now take maps = (singularity-free) dynamics as before which stabilizes w/ help of internal dynamics.

- This model: still compact closed but not much else (no +, !'s are fudged) — but there is an internal control of dynamics.

Abramsky – Jagadeesan model.

Objects: $A = (A_*, A^*)$

maps $A \to B$:
$$\begin{tikzcd} A_* \arrow[dr] & & A^* \\ & {} \arrow[ur] \arrow[dl] & \\ B_* & & B^* \arrow[ul] \end{tikzcd}$$

- f^ns $A_* \times B^* \longrightarrow B_* \times A^*$

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Compⁿ: $A \to B \to C$.
mediated by
fixed pts.

$$\begin{tikzcd} A_* & & A^* \\ B_* & & B^* \\ C_* & & C^* \end{tikzcd}$$

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e.g.

$f : A \times S \longrightarrow B \times S$ (fⁿ w/states).

IF fixed pts $\exists$, can pass to a fⁿ:

~~[illegible]~~ e.g. $\dfrac{A \times S \longrightarrow S}{A \longrightarrow S^S \xrightarrow{Y} S}$

$A \xrightarrow{\sigma} S$

Define:

$\bar{f} : A \longrightarrow B$

$a \longmapsto f(a, \sigma(a))$.

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above: $A_* \times C^* \times \underbrace{(B_* \times B^*)}_{=S} \to C_* \times A^* \times (B_* \times B^*)$

$\therefore$ get a fⁿ as above

Abstract derivatⁿ of A - J.

· Take cat. $\mathbb{E}$: from self dual cat.

$\mathbb{D} = \mathbb{E} \times \mathbb{E}^{op}$. ~~[illegible]~~.

· IF $\mathbb{E}$ ccc., take comonad on $\mathbb{D}$.

$(U, X) \xmapsto{G} (U, X^U)$ & dual

monad $(U, X) \xmapsto{Q} (U^*, X)$

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playing second in $\Gamma^* \Delta$ –
Get compact closed cat – not much else.

Query: can we fudge?
We do control dynamics internally: g ~~games~~ are wf & ∴ ~~[illegible]~~ det. by ordinal of game.

General framework:

$A \xrightarrow{f} B$, $B \xrightarrow{g} C$ . . . .

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other models:

· Blass's Theory of Games
· Lamarche's seq. algms

· Realiz. models.
· Girard's SN