Uniform Game Semantics for Computation and Reasoning: EPSRC grant GR/L27848 Summary of Final Report

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This project was funded by the EPSRC jointly with a project at Oxford: the principal investigators were Hyland (DPMMS, University of Cambridge) and Ong (Oxford University Computing Laboratory). The two projects shared the case for support and the workplan provided a rough division of topics between the two sites. The Cambridge side of the project began in January 1997 and finished in December 1999. The main areas in the project were as follows.

- **Games for Computation:** Mainly the concern of the Oxford side.
- **Games for Proofs:** Of concern to both sides of the project.
- **Structures and Foundations:** Mainly the concern of the Cambridge side.

At Cambridge research was carried out mainly by Schalk (Post Doctoral RA) in collaboration with Hyland. Other researchers at Cambridge include Bierman (Research Fellow) and Ayer, Tan, Urban (Research students). There were regular meetings with the main Oxford team comprising Ong, Nickau (Post Doctoral RA) and in the early stages Wallen (Co-Investigator). The work of the project has been presented at MFPS 1998, Daghstuhl (Seminar on Linear Logic and Applications) 1999, CTCS 1999, Darmstadt Workshop on Domains 1999, Troelstra Meeting 1999.

Games for Proofs

The project's main focus was on classical proof. Substantial work concerning games and the $\lambda\mu$-calculus was done in particular at Oxford. We also have given categorical characterisations of sequent calculi with deterministic reduction strategies which fall outside the $\lambda\mu$ paradigm. However it seems that there are computational aspects of classical proof which were not adequately handled by these approaches. Results of case studies indicate that current algorithmic readings of proofs may be dominated by the choice of reduction strategy. To avoid this distortion, we have developed term calculi for classical proofs and a strong normalization result for a non-deterministic reduction procedure with the crucial property that cuts can be permuted with cuts. At an abstract level this connects with work of Power and Robinson.

Structures and Foundations

This area forms the main thrust of the Cambridge research, the work being driven by the concept of abstract games. This covers models for the Geometry of Interaction, and provides a general framework for concurrent games. We have made successful analyses of sequentiality, and have worked out detailed connections with more standard notions of game. One striking result of the investigation is the discovery of a variety of new categories of games, where strategies are determined by positions. We have given detailed accounts of the central techniques of glueing and orthogonality. A further achievement is to give general constructions of exponentials covering for example self dualization and general Chu and Dialectica categories.

Another aim was to understand proofs of full completeness results in general terms. We have succeeded in giving abstract accounts of full completeness results for the multiplicative fragment of classical linear logic (without units) and of intuitionistic linear logic (with units). We have also established a basic result for compact closed categories with biproducts and shown how many full completeness results for multiplicative linear logic can be derived from it. Generally we have made progress in the categorical treatment of proof theory.