A Dimensional Improvement of the Entropy Power Inequality under Marginal Assumptions

Matthieu Fradelizi (Univ Gustave Eiffel), Lampros Gavalakis (Cambridge), Martin Rapaport (Carnegie Mellon)

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Entropy and Volume

The entropy power of a random vector $X \in \mathbb{R}^n$ is

$$N(X) \ = \ \frac{1}{2\pi e} \exp\!\left(\tfrac{2}{n}\, h(X)\right)$$

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"=" iff X, Y Gaussian with equal covariances

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Brunn-Minkowski inequality

If $A, B \subset \mathbb{R}^n$ measurable, then

$$\mathrm{Vol}^{1/n}(A+B) \;\geq\; \mathrm{Vol}^{1/n}(A) + \mathrm{Vol}^{1/n}(B)$$

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$\mathrm{Vol}^{1/n}$	N
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Review in [Dembo, Cover, Thomas '91]

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Theorem (Sharp form of Young)

If $\frac{1}{r} + 1 = \frac{1}{p} + \frac{1}{q}$ *with* $1 < p, q, r < \infty$, *then*

$$\|f * g\|_r \;\leq\; C_{p,q,r} \, \|f\|_p \, \|g\|_q$$

where $C_{p,q,r}$ *corresponds to Gaussian functions (extremizers)*

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- Common proof through Rényi entropies

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$$\frac{|A+B|}{\left(|P_{\theta^\perp}A|_{n-1}^{\frac{1}{n-1}} + |P_{\theta^\perp}B|_{n-1}^{\frac{1}{n-1}}\right)^{n-1}} \geq \frac{|A|}{|P_{\theta^\perp}A|_{n-1}} + \frac{|B|}{|P_{\theta^\perp}B|_{n-1}}$$

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$\Rightarrow$ If the sets have same volume along some projection, $|P_{\theta^\perp}A|_{n-1} = |P_{\theta^\perp}B|_{n-1}$, then

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Refinements of EPI

Notation: $X_1^{n-1} = (X_1, \ldots, X_{n-1}) \in \mathbb{R}^{n-1}$

Theorem

$X, Y \in \mathbb{R}^n$ independent

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Bergström's Inequality

$A, B \in \mathbb{R}^{n\times n}$ symmetric, PSD. A_i: remove i-th raw and column

$$\frac{\det(A+B)}{\det(A_i+B_i)} \geq \frac{\det(A)}{\det(A_i)} + \frac{\det(B)}{\det(B_i)}$$

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2. Implies the EPI, $N(X+Y) \geq N(X) + N(Y)$, for $n > 1$: Using the EPI for $n-1$ and convexity of $x \to x^n$

$$N(X+Y)^n \geq \theta \frac{N(X)^n}{\theta^n} + (1-\theta)\frac{N(Y)^n}{(1-\theta)^n}$$
$$\geq (N(X)+N(Y))^n$$

where $\theta = \frac{N_{n-1}(X_1^{n-1})}{N_{n-1}(X_1^{n-1})+N_{n-1}(Y_1^{n-1})}$

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3. Remains true, via a change of axes, for any projection $A : \mathbb{R}^n \to \mathbb{R}^{n-1}$
4. If $X = (X_1, \ldots, X_n)$ and $I \subset \{1, \ldots, n\}$, write $X_I = \{X_i\}_{i \in I}$. For $|I| = k$

$$e^{\frac{2}{k} h(\sqrt{1-\lambda} X_I + \sqrt{\lambda} Y_I | \sqrt{1-\lambda} X_{I^c} + \sqrt{\lambda} Y_{I^c})}$$
$$\geq (1-\lambda) e^{\frac{2}{k} h(X_I | X_{I^c})} + \lambda e^{\frac{2}{k} h(Y_I | Y_{I^c})}$$

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Theorem (*Improved EPI under marginal assumptions*)

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- Equality case by linear algebra. What about the general inequality?
- When is the improvement significant? n large and $h(X) \ll h(Y)$

Improved Isoperimetric Inequality for Entropy

$$I(X)N(X) \geq n \quad \text{(Stam)}$$

where $I(X) = \int_{\mathbb{R}^n} \frac{(f'(x))^2}{f(x)} dx$ is the Fisher Information

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Corollary

$$I(X)N(X) \geq \left(\frac{N(X_1^{n-1})}{N(X)} \right)^{n-1} + (n-1) \frac{N(X)}{N_{n-1}(X_1^{n-1})}$$

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- By our general inequality, de Bruijn's identity and a first order Taylor expansion

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Blachman-Stam inequality

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Definition (*Conditional Fisher Information*)

$$I(X|Y) := \int f_Y(y) I\big(f_{X|Y}(\cdot|y)\big)\, dy,$$

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Theorem (*Conditional Blachman-Stam Inequality*)

$$I(X_n + Y_n|X_1^{n-1} + Y_1^{n-1})^{-1} \geq I(X_n|X_1^{n-1})^{-1} + I(Y_n|Y_1^{n-1})^{-1}$$

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- Obtained by applying the Fisher information inequality to a particular operator

Thank you!