

Study group on quantitative uniform Mordell, after Yu–Yuan–Zhou

The first half of the seminar will discuss ideas related to Vojta’s proof of the Mordell conjecture. The second half will give an overview of Part I of [JY26].

Some guidelines for the seminar:

- Each talk should be ≤ 1 hour long.
- As a corollary, focus on giving precise definitions and statements over detailed proofs.
- There is no need to strive for maximum generality in your talk; a well chosen example can be more illustrative.

Some general references that might be useful:

- Bombieri–Gubler [BG06]
- Hindry–Silverman [HS00]
- Niven Achenjang’s notes for a similar study group: https://people.math.harvard.edu/~achenjang/assets/pdf/STAGE_Vojta_Notes.pdf.
- Ziyang Gao’s survey: <https://ziyangjeremygao.github.io/articles/SurveyUnifML.pdf>

1 May: Overview

Discuss the aims of the study group, give an overview of the proof, place the result in a broader context and distribute the talks.

Speaker: Jef

7 May: Weil and canonical heights

Discuss the Weil height machine and define canonical heights on abelian varieties.

References: Sections B.2, B.3 and B.5 of [HS00].

Speaker: TBC

14 May: Mumford Gap principle

Discuss the classical Mumford gap principle, its proof and applications to rational points.

References: Original paper [Mum65], Niven’s notes, and [HS00, Theorem B.6.5 and Proposition B.6.6]

Speaker: TBC

21 May: Vojta’s inequality

Give an overview of the proof of Vojta’s inequality, which Vojta used to give a second proof of Mordell’s conjecture. This will necessarily be a cartoon version of the proof. You should make clear how Siegel’s lemma is used in the proof.

References: [BG06], [HS00] and Niven’s notes

Speaker: TBC

28 May: Admissible adelic line bundles on curves and their intersections

Survey Sections 2.3, 2.4 and 2.5 of [JY26]. Omit details surrounding Berkovich spaces. Focus on the definition of the intersection product $\bar{L} \cdot \bar{M}$ for two adelic line bundles \bar{L}, \bar{M} on a curve X/K , the definition of the height function $h_{\bar{L}}$, and the connection with the canonical height, namely Theorems 2.5, 2.6 and 2.7.

Speaker: TBC

4 June: Quantitative versions of the Mumford and Vojta inequality

Armed with the background of adelic line bundles, state the quantitative versions of the Mumford and Vojta inequalities of Yu–Yuan–Zhou. Briefly sketch the archimedean ingredients that go into the proof.

References: Section 3 of [JY26].

Speaker: TBC

11 June: Quantitative Bogomolov conjecture

Sketch the proof of the quantitative Bogomolov conjecture in Section 4 of [JY26].

Speaker: TBC

18 June: Putting everything together

Combine the results from the previous two lectures to get the bound for $\#C(\mathbb{Q})$ stated in the overview lecture. Emphasize the results from sphere packings that go into the proof.

References: Section 5 of [JY26]

Speaker: TBC

References

- [BG06] Enrico Bombieri and Walter Gubler. *Heights in Diophantine geometry*, volume 4 of *New Mathematical Monographs*. Cambridge University Press, Cambridge, 2006.
- [HS00] Marc Hindry and Joseph H. Silverman. *Diophantine geometry*, volume 201 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2000. An introduction.
- [JY26] Shengxuan Zhou Jiawei Yu, Xinyi Yuan. Quantitativity on the number of rational points in the mordell conjecture. Arxiv Preprint, available at <https://arxiv.org/abs/2602.01820>, 2026.
- [Mum65] David Mumford. A remark on Mordell’s conjecture. *Amer. J. Math.*, 87:1007–1016, 1965.