

Study group 2021: ‘Modularity lifting theorems’

Time: 14.30 UK time every Thursday

Notes: available at <https://users.math.yale.edu/~rz289/>

Possible additional references:

- Patrick Allen’s video lectures: [here](#)
- Toby Gee’s Arizona Winter School lectures: [here](#)

Lent term.

1 Overview (28/01)

Speaker: Jack

2 Weil-Deligne representations and l-adic monodromy theorem (04/02)

Speaker: Sam

Lecture 1: basics of Galois representations (Brauer-Nesbitt)

Lecture 2: basics of Galois representations (semisimplification, Chebotarev)

Lecture 3: ℓ -adic representations over p -adic fields, monodromy and Weil-Deligne representations

3 Deformations of Galois representations: basics (11/02)

Speaker: Jef

Lecture 4: Galois deformation theory basics, basic representability.

4 Deformations of Galois representations: further properties (18/02)

Speaker: Lukas

Lecture 6: Galois deformation theory: representability for Schur deformations, state Brauer-Nesbitt for coefficients.

5 Even further properties (25/02)

Speaker: Dmitri

Lecture 7: Galois deformation theory: complete the proof, base change, tangent spaces.

6 Deformation problems and global deformations (04/03)

Speaker: Vaughan

§8.2 and Lecture 9: Galois deformation theory: deformation problems, global deformations.

7 Adjoint selmer groups (11/03)

Speaker: Jun

Lecture 10: Galois deformation theory: global deformation problems via adjoint Selmer groups.

8 Geometric properties of deformation rings (18/03)

Speaker: Guillem

§8.1 (generic fibres of deformation rings) and Lecture 11, plus brief summary of Lecture 12 (importantly Theorem 12.5).

Easter term.

9 Representation theory of p -adic groups (08/04)

Lectures 13 and 14: Representation theory of p -adic groups: definitions, Hecke algebras, the case $GL_n(K)$, parabolic induction, hierarchy of smooth representations.

Speaker: Art

10 The local Langlands and local Jacquet-Langlands correspondences (15/04)

Lecture 15 and §16.1: Representation theory of p -adic groups: stating local Langlands, unramified local Langlands, local Jacquet-Langlands correspondences.

Speaker: Lukas

11 Automorphic forms for quaternion algebras (22/04)

§16.2 and Lecture 17: Automorphic forms for quaternion algebras: definitions, global Jacquet-Langlands. Local Langlands correspondence for GL_n .

Speaker: Guillem

12 Integral theory of automorphic forms (29/04)

Lecture 18: Automorphic forms for quaternion algebras: integral theory.

Speaker: Vaughan

13 Automorphy lifting (06/05)

Lecture 19: Automorphy lifting: minimal ALT (start of proof).

Speaker: Dmitri

14 Automorphy lifting: patching (13/05)

Lecture 20: Automorphy lifting: proof, patching.

Speaker: Jun

15 Automorphy lifting: the Taylor-Wiles method (20/05)

Lecture 21: Automorphy lifting: proof continued, Taylor-Wiles method for patching.

Speaker: Jef

16 Fermat's last theorem (27/05)

Lectures 22, 23 and 24: Fermat's last theorem.

Speaker: ??

Contents of the notes

Lecture 1: basics of Galois representations (Brauer-Nesbitt)

Lecture 2: basics of Galois representations (semisimplification, Chebotarev)

Lecture 3: ℓ -adic representations over p -adic fields, monodromy and Weil-Deligne representations

Lecture 4: Galois deformation theory basics, basic representability

Lecture 6: Galois deformation theory: representability for Schur deformations, state Brauer-Nesbitt for coefficients.

Lecture 7: Galois deformation theory: complete the proof, base change, tangent spaces

Lecture 8: Galois deformation theory: generic fibers, deformation problems

Lecture 9: Galois deformation theory: deformation problems, global deformations

Lecture 10: Galois deformation theory: global deformation problems via adjoint Selmer groups

Lecture 11: Galois deformation theory: geometric properties of deformation rings away from p , Galois-theoretic conditions, Taylor-Wiles

Lecture 12: Reminder of p -adic Hodge theory

Lecture 13: Representation theory of p -adic groups: definitions, Hecke algebras and the case $GL_n(K)$.

Lecture 14: Parabolic induction, hierarchy of smooth representations

Lecture 15: Representation theory of p -adic groups: stating local langlands, unramified local langlands and Satake parameters

Lecture 16: Representation theory of p -adic groups: local Jacquet-Langlands. Automorphic forms for quaternion algebras: definitions

Lecture 17: Automorphic forms for quaternion algebras: global Jacquet-Langlands. Local Langlands correspondence for GL_n .

Lecture 18: Automorphic forms for quaternion algebras: integral theory

Lecture 19: Automorphy lifting: minimal ALT (start of proof)

Lecture 20: Automorphy lifting: proof, patching

Lecture 21: Automorphy lifting: proof continued, Taylor-Wiles method for patching

Lecture 22: Eichler-Shimura theory, Galois representations associated to modular forms

Lecture 23: Fermat's last theorem

Lecture 24: Fermat's last theorem: putting everything together