MORDELL 2022 – SCHEDULE

Lectures will take place in MR2 at the CMS (Centre for Mathematical Sciences). MR2 is at basement level, and step-free access is available using the lifts in Pavilions D and G.

Coffee be served during breaks in the Central Core area of the CMS. A buffet lunch for registered conference participants will be served in the same place on Tuesday and Thursday.

The conference dinner will be at St John's College on Wednesday evening. Pre-dinner drinks will be available in the College Hall from 7:00pm with dinner to begin at 7:30pm.

Monday

2:30pm. Andrew Sutherland – *On a question of Mordell*

In 1953 Mordell asked whether one can represent 3 as a sum of three cubes in any way other than $1^3+1^3+1^3$ and $4^3+4^3-5^3$. Mordell's question inspired many computations over the following 65 years, and while none found any new solutions for 3, these computations eventually determined the positive integers $k \leq 100$ that can be represented as a sum of three cubes in all but one case: $k = 42$. In this talk I will present joint work with Andrew Booker that affirmatively answers Mordell's question and settles the case $k = 42$ along with several other previously unresolved $k \leq 1000$. I will also discuss a conjecture of Heath-Brown that predicts the existence of infinitely many more solutions and explains why they are so difficult to find.

3:30pm. Jan Vonk – *Eisenstein families and Stark-Heegner points*

In this short talk, I will discuss an analytic construction of (conjecturally) global points over class fields of real quadratic fields. The construction involves the first derivative of a p-adic family of Hilbert Eisenstein series. This is joint work with Henri Darmon and Alice Pozzi.

4:00pm. Coffee break

4:30pm. Chantal David – *On the vanishing of twisted L-functions of elliptic curves over function fields* (Joint work with A. Comeau-Lapointe, M. Lalin and W. Li.)

Let E be an elliptic curve over $\mathbb{Q}$, and let χ be a Dirichlet character of order ℓ for some prime $\ell \geq 3$. Heuristics based on the distribution of modular symbols and random matrix theory have led to conjectures predicting that the vanishing of the twisted L-functions $L(E, \chi, s)$ at $s = 1$ is a very rare event (David-Fearnley-Kisilevsky and Mazur-Rubin). In particular, it is conjectured that there are only finitely many characters of order $\ell > 5$ such that $L(E, \chi, 1) = 0$ for a fixed curve E.

We investigate in this talk the case of elliptic curves over function fields. For Dirichlet L-functions over function fields, Li and Donepudi-Li have shown how to use the geometry to produce infinitely many characters of order $\ell \geq 2$ such that the Dirichlet L-function $L(\chi, s)$ vanishes at $s = 1/2$, contradicting (the function field analogue of) Chowla's conjecture. We show that their work can be generalized to constant curves $E/\mathbb{F}_q(t)$, and we show that if there is one Dirichlet character χ of order ℓ such that $L(E, \chi, 1) = 0$, then there are infinitely many, leading to some specific examples contradicting (the function field analogue of) the number field conjectures on the vanishing of twisted L-functions. Such a dichotomy does not seem to exists for general (non-constant) elliptic curves over $\mathbb{F}_q(t)$, and we produce empirical evidence which suggests that the conjectures over number fields also hold over function fields for non-constant $E/\mathbb{F}_q(t)$.

Tuesday

9:30am. Rafael von Kanel – *Integral points on coarse Hilbert moduli schemes*
In this talk, I will first discuss various fundamental Diophantine problems, including Mordell equations and equations defining certain classical surfaces, which are related to integral points on coarse Hilbert moduli schemes. Then I will present explicit bounds for the height and the number of integral points on coarse Hilbert moduli schemes outside the branch locus. I will also explain the strategy of proof which combines the method of Faltings (Arakelov, Parsin, Szpiro) with modularity results and Masser-Wuestholz isogeny estimates. This is joint work with Arno Kret.

10:30am. Zev Klagsbrun – *Three-isogeny Selmer groups and ranks of abelian varieties in quadratic twist families*
We determine the average size of the ϕ-Selmer group in any quadratic twist family of abelian varieties having an isogeny ϕ of degree 3 over any number field. This has several applications towards the rank statistics in such families of quadratic twists. For example, it yields the first known quadratic twist families of absolutely simple abelian varieties over Q, of dimension greater than one, for which the average rank is bounded; in fact, we obtain such twist families in arbitrarily large dimension. In the case that E/F is an elliptic curve admitting a 3-isogeny, we prove that the average rank of its quadratic twists is bounded; if F is totally real, we moreover show that a positive proportion of these twists have rank 0 and a positive proportion have 3-Selmer rank 1. We also obtain consequences for Tate-Shafarevich groups of quadratic twists of a given elliptic curve. This is joint work with Manjul Bhargava, Robert Lemke Oliver, and Ari Shnidman.

11:00am. Coffee break

11:30am. Ziyang Gao – *Torsion points in families of abelian varieties*

Given an abelian scheme defined over $\overline{\mathbb{Q}}$ and an irreducible subvariety X which dominates the base, the Relative Manin-Mumford Conjecture (proposed by Zannier) predicts how torsion points in closed fibers lie on X. The conjecture says that if such torsion points are Zariski dense in X, then the dimension of X is at least the relative dimension of the abelian scheme, unless X is contained in a proper subgroup scheme. In this talk, I will present a proof of this conjecture. As a consequence this gives a new proof of the Uniform Manin-Mumford Conjecture for curves (recently proved by Kühne) without using equidistribution. This is joint work with Philipp Habegger.

12:30pm. Lunch

2:30pm. Lennart Gehrmann – *Plectic Stark–Heegner points*

I will report on a plectic generalization of Heegner and Stark-Heegner points. Inspired by Nekovar and Scholl's conjectures, these points are expected to control Mordell-Weil groups of higher rank elliptic curves. This expectation is supported by numerical evidence as well as strong theoretical evidence in case of polyquadratic CM fields.

This is joint work with Michele Fornea.

3:30pm. Holly Green – *A new rank parity computing machine*

I will present a new method to compute the parity of the rank of the Jacobian of a curve. This method involves studying the local arithmetic attached to covers of the curve. I will briefly discuss applications to the Birch and Swinnerton-Dyer conjecture, including the remaining cases of the p-parity conjecture for elliptic curves over totally real fields. This is joint work with Alexandros Constantinou, Vladimir Dokchitser, Céline Maistret and Adam Morgan.

4:00pm. Coffee break

4:30pm. Barry Mazur – *Thoughts about Mordell and uniformity of finiteness bounds*

One striking aspect of Mordell's mathematics is how he is known both for the *concrete questions* he posed in simple English sentences

as well as

- his general results; e.g.: that the group of rational points of an elliptic curve over $\mathbb{Q}$ is finitely generated—his theorem having been generalized further by André Weil to cover abelian varieties over number fields,
- and his grand finiteness conjecture (proved by Faltings), a conjecture that has enriched number theory by inspiring a host of even broader finiteness questions.

We will review some open questions concerning uniformity of upper bounds for finiteness.

Wednesday

9:30am. Alina Cojocaru – *Frobenius traces for abelian varieties*

In the 1970s, S. Lang and H. Trotter proposed a conjectural asymptotic formula for the number of primes for which the Frobenius trace of an elliptic curve defined over the rationals equals a given integer. We will discuss generalizations of this conjecture to higher dimensional abelian varieties and we will present recent results proven for abelian varieties that arise as products of non-isogenous non-CM elliptic curves and for abelian varieties with a trivial endomorphism ring. This is joint work with Tian Wang (University of Illinois at Chicago).

10:30am. Julie Desjardins – *Torsion points and concurrent exceptional curves on del Pezzo surfaces of degree one*

The blow up of the anticanonical base point on X, a del Pezzo surface of degree 1, gives rise to a rational elliptic surface E with only irreducible fibers. The sections of minimal height of E are in correspondence with the 240 exceptional curves on X. A natural question arises when studying the configuration of those curves: *If a point of X is contained in "many" exceptional curves, it is torsion on its fiber on E?*

In 2005, Kuwata proved for del Pezzo surfaces of degree 2 (where there is 56 exceptional curves) that if "many" equals 4 or more, then yes. With Rosa Winter, we prove that for del Pezzo surfaces of degree 1, if "many" equals 9 or more, then yes, but we find counterexamples where a torsion point lies at the intersection lies at the intersection of 7 exceptional curves.

11:00am. Coffee break

11:30am. Open problem session

Thursday

9:30am. Chao Li – *A higher dimensional Gross-Zagier formula*

The Gross-Zagier formula relates the central L-derivative of elliptic curves and the Neron-Tate height of Heegner points. It plays a key role in our current knowledge of the Birch and Swinnerton-Dyer conjecture. We discuss a higher dimensional generalization known as the arithmetic inner product formula. It relates the central L-derivative for unitary groups and the Beilinson-Bloch height of algebraic cycles on unitary Shimura varieties. This is joint work with Yifeng Liu.

10:30am. Joe Silverman – *Fourier Expansions and Non-negative Averages of Local Heights on Abelian Surfaces over Local Fields* Let K be a complete local field, and let E/K be an elliptic curve having split multiplicative reduction. The canonical local height on $E(K)$ is the sum of a non-negative intersection theory part and the value of a periodic Bernoulli polynomial $B(T)$. The Fourier expansion of $B(T)$ and its weighted averages have been used to study lower bounds for canonical heights in the setting of conjectures of Lang and Lehmer. In this talk I will present an analogous 2-variable periodic Bernoulli polynomial associated to an abelian surface A/K having completely split multiplicative reduction, including a formula for its Fourier expansion and some disucssion on how to obtain non-negative weighted averages. This is joint work with Nicole Looper.

11:00am. Coffee break

11:30am. Abbey Bourdon – *Minimal Torsion Curves in Geometric Isogeny Classes*

Let $E/\mathbb{Q}$ be an elliptic curve. By Mordell's 1922 theorem, the points on E with coordinates in $\mathbb{Q}$ form a finitely generated abelian group. In particular, the torsion subgroup of $E(\mathbb{Q})$ is a finite abelian group, and the groups that occur as $E(\mathbb{Q})_{\mathrm{tors}}$ are known due to work of Mazur in 1977. The past decade has seen a renewed interest in studying torsion points on rational elliptic curves: from identifying the torsion subgroups that can arise on $E/\mathbb{Q}$ under base extension to a field of higher degree to a near complete classification of the image of ℓ-adic Galois representations associated to elliptic curves over $\mathbb{Q}$. In this talk, we will discuss recent results which leverage this knowledge to begin to understand a new class of elliptic curves, namely, those geometrically isogenous to an elliptic curve defined over $\mathbb{Q}$. Motivated by the problem of producing low degree points on modular curves, we seek to characterize the elliptic curves within a fixed geometric isogeny class producing a point of prime-power order in least possible degree.

12:30pm. Lunch

2:30pm. Hanneke Wiersema – *On a BSD-type formula for L-values of Artin twists of elliptic curves*

In this talk we discuss the possible existence of a BSD-type formula for L-functions of elliptic curves twisted by Artin representations. After outlining some expected properties of these L-functions, we present arithmetic consequences for the behaviour of Tate–Shafarevich groups, Selmer groups and rational points, and we illustrate these with some explicit examples. This is joint work with Vladimir Dokchitser and Rob Evans. In the special case of Dirichlet characters our results make use of integrality properties established in joint work with Christian Wuthrich, which we also discuss.

3:30pm. Jennifer Park – *Everywhere local solubility for hypersurfaces in products of projective spaces*

Poonen and Voloch proved that the Hasse principle holds for either 100% or 0% of most families of hypersurfaces (specified by degrees and the number of variables). In this joint work with Tom Fisher and Wei Ho, we study one of the special families of hypersurfaces not accounted for by Poonen and Voloch, and we show that the explicit proportion of everywhere locally soluble $(2, 2)$-curves in $\mathbb{P}^1 \times \mathbb{P}^1$ is about 87.4%.

4:00pm. Coffee break

4:30pm. Manjul Bhargava – *Integers that are the sum of two rational cubes*

We prove that a positive proportion of integers are the sum of two rational cubes, and a positive proportion are not. This is joint work with Levent Alpöge and Ari Shnidman.

Friday

9:30am. Adam Morgan – *Parity of ranks of abelian varieties*

For an abelian variety over a number field, a consequence of the Birch and Swinnerton-Dyer conjecture is the parity conjecture: the global root number agrees with the parity of the Mordell–Weil rank. Although this remains largely open, the p-parity conjecture, which replaces the Mordell–Weil rank with the p^∞-Selmer rank for a prime p, has proven more amenable to study.

In the case when $p = 2$, however, comparatively little is known away from dimensions 1 and 2. In large part, this is due to a phenomenon observed by Poonen and Stoll that the Shafarevich–Tate group of a principally polarised abelian variety can have order twice a square. I will discuss instances where this obstruction can be overcome, presenting results for Jacobians of hyperelliptic curves and for principally polarised abelian varieties after quadratic extension of the base field.

10:30am. Tom Fisher – *Computing the Cassels-Tate pairing on 2-Selmer groups of genus 2 Jacobians*

In her thesis completed last year, my student Jiali Yan gave a practical method for computing the Cassels-Tate pairing on the 2-Selmer group of the Jacobian of a genus 2 curve all of whose Weierstrass points are rational. She also gave a second method without any assumption on the Weierstrass points, but instead assuming we can find a rational point on a certain twisted Kummer surface. The two methods can be thought of as generalising methods of Cassels and Donnelly in the elliptic curve case. I will describe some recent refinements that significantly improve the practicality of the second method.

11:00am. Coffee break

11:30am. Stephanie Chan – *Integral points in families of elliptic curves*

Given an elliptic curve over a number field with its Weierstrass model, we can study the integral points on the curve. Taking an infinite family of elliptic curves and imposing some ordering, we may ask how often a curve has an integral point and how many integral points there are on average. We expect that elliptic curves with any non-trivial integral points are generally very sparse. In certain quadratic and cubic twist families, we prove that almost all curves contain no nontrivial integral points.

12:30pm. Alexander Smith – 2^k-*Selmer groups and Goldfeld's conjecture*

Take E to be an elliptic curve over a number field whose four torsion obeys certain technical conditions. In this talk, we will outline a proof that 100% of the quadratic twists of E have rank at most one. To do this, we will find the distribution of 2^k-Selmer ranks in this family for every positive integer k.