14 October 1992, IAS Princeton

Here is a copy of my thesis. Section 4.4 is not finished—when (if) I turn it into a paper, this section will contain the calculation of the Verdier Specialization of a generic character sheaf (in the DeConcini-Procesi compactification; from G/K to the more degenerate orbits), and hence information on the monodromy of a generic character sheaf. In charactersistic zero, this goes back to Harish-Chandra.

Also, the proof of theorem 4.2 works equally well to show the rank of a generic character sheaf in $(G, H, K)_{\mathcal{L}}$ is the order of the appropriate small Weyl group—I think. There is just one small combinatorial detail I havent checked. I'll do this before publishing it.

It is shown in sections 4–5 that the double coset algebras $K^F \backslash G^F / K^F$ which have characters expressible in elementary terms (polynomials in q times roots of unity) are exactly the commutative ones; i.e. those which come from a symmetric space $X = G/K$ with precisely one K-conjugacy class of maximal torus.

These are $(G \times G, G)$, (A_n, C_n), (D_n, B_{n-1}), (E_6, F_4). The case of $(G \times G, G)$ is due to Lusztig (a slight understatement); if one places a twisted Frobenius on X one can still calculate the characters from Lusztig's work (cf. first paragraph on p9).

The case of (A_n, C_n) is due to Bannai-Kawanaka-Song [BKS]; we derive their results in section 7.4. Exactly the same technique handles (E_6, F_4) and (D_n, B_{n-1}) (and their twisted versions); this is a consequence of a result of Lawther which says that (E_6, F_4) looks like the GL_3 orbits (for conjugation) on SL_3 (respectively (D_n, B_{n-1}) looks like GL_2 orbits on SL_2.) As a result, all the character sheaves are thickly supported—no cuspidals occur.

The result is:

i) The character sheaves on (E_6, F_4), (respectively (D_n, B_{n-1})) look like the character sheaves on GL_3 (restricted to SL_3) with q replaced by q^4. (respectively, GL_2, with q replaced by q^{n-1}).

ii) The change of basis matrix from unipotent character sheaves to characters of the algebra $K^F \backslash G^F / K^F$ involves the MacDonald 2-variable symmetric functions at (q^4, q) (respectively (q^{n-1}, q)) in exactly the same way those for (GL_{2n}, Sp_{2n}) do for (q^2, q) (see [BKS]).

Actually, (ii) follows formally from (i) as in [BKS]; Srinivasan has a paper in which this is done in generality for MacDonald polynomials.

I will elaborate on this elsewhere; at the moment I am still unclear about the geometric significance of MacDonald polynomials.

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