Riemann surfaces - sources

There is no fixed references for the Riemann surfaces course; these are provided as additional sources to the notes you take in lecture.

1 Complex analysis prerequisites

You will need essentially the full content of a short undergraduate complex analysis course. Domains, holomorphic and meromorphic functions, power series, Cauchy's theorem, Cauchy's integral formula, the argument principle, the residue theorem. Harmonic functions. Conformality and the local behavior of analytic functions. Understanding isolated singularities and Casorati-Weierstrass.

Any reasonable complex analysis book will do all of this for you; personally, I like the first three chapters of Stein-Shakarchi [SS].

2 Topological prerequisites

You will need the very basic definitions of an undergraduate course in point-set topology. A basis for a topology, connectedness, continuity, compactness, path-connectedness, induced topologies like quotient, product, and subspace topologies. I am not conversant in this undergraduate literature, but others have recommended Mendelson's Introduction to Topology [Me]. It might be more sensible to simply use course notes from the latter half of the DPMMS Met and Top course, various versions of which can be found online.

3 Course references

The very classical parts of the course - analytic continuation and space of germs - roughly follows Ahlfors 8.1, though that presentation is very vague and your course notes will be superior. For monodromy and maps of Riemann Surfaces, see Cavalieri-Miles [CM] or Donaldson [D] or Miranda [M]. Riemann Existence is nicely covered by Cavalieri-Miles [CM]. For elliptic functions and complex tori, we follow parts of Ahlfors Chapter 7. We will not present a proof of uniformization; however, Farkas-Kra [FK] gives a clear if lengthy presentation.

4 Beyond the course

We cover a range of notions related to Riemann Surfaces in the course; here are some immediate Lent Term successors.

- If you enjoyed the classical material in the course, and I suggest Part II Topics in Analysis, Part II Analysis of Functions, and Part II Differential Geometry.
- If you enjoyed the monodromy and topological portion of the course, I suggest Part II Algebraic Topology (concurrent this year) and Part II Representation Theory.
- If you enjoyed the elliptic curves portion of the course, I suggest Part II Number Fields and Part II Algebraic Geometry.

In the longer term, for those students interested in Riemann surfaces, I would direct the analytically inclined towards complex geometry and functional analysis. For the topologically inclined, consider algebraic topology and Teichmüller theory. If you enjoyed the algebraic questions, head to algebraic geometry and algebraic number theory. To learn more about uniformization and higher genus Riemann surfaces, hyperbolic geometry and number theory.

References

[A] Ahlfors, *Complex Analysis*, McGraw-Hill.

[CM] Cavalieri and Miles, *Riemann Surfaces and Algebraic Curves*, London Mathematical Society Student Texts.

[D] Donaldson, *Riemann Surfaces*, Oxford Graduate Texts in Mathematics.

[FK] Farkas and Kra, *Riemann Surfaces*, Springer Graduate Texts in Mathematics.

[M] Miranda, *Algebraic Curves and Riemann Surfaces*, AMS Graduate Studies in Mathematics.

[Me] Mendelson, *Introduction to Topology*, Dover Books on Mathematics.

[SS] Stein and Shakarchi, *Complex Analysis*, Princeton Lectures in Analysis II.