

Percolation and related topics II

Exercises will be taken mostly from my book *Probability on Graphs*, published by Cambridge University Press in 2010, and available online at

<http://www.statslab.cam.ac.uk/~grg/teaching/perc19.html>

Solutions to Questions 3, 4, 5 to be handed in to GRG (via his pigeon-hole) by **11am on Thursday 28 November**. Class is timetabled for Tuesday 3 December at 3.30pm. Please keep an eye on the course webpage for possible changes.

You may need to read slightly ahead of the lectures to do the questions on the Ising and random-cluster models, but nothing new is needed beyond the required definitions.

1. (i) Read the proof of the BK inequality in Section 4.3. (ii) Read the sketch of the proof of the slab critical point in Section 5.2. (iii) Verify the trifurcation argument in the final part of the proof of uniqueness of the infinite open cluster, Section 5.2. (iv) Read about the critical behaviour at/near the phase transition p_c in Section 5.4.
2. *Using sharp-threshold*. Exercise 4.11 (an example of influence in action).
3. *Word percolation*. Exercise 3.11.
4. *Power-law bounds*. Exercise 5.4. (Note: There was a misprint in the *first* version of the book, the first display should be \leq not \geq .)
5. *FKG for Ising model*. Exercise 7.3.
6. *Positive association for random-cluster measures*. Exercise 8.2. (Note: Random-cluster measures are elaborations of the percolation measure. See Section 8.1.)
7. Show that the random-cluster measure $\phi_{p,q}$ satisfies the comparison inequalities of Exercise 8.7.
8. *Largest subcritical cluster in a box*. Exercise 5.2.
9. Outline a proof that $p_c = \frac{1}{2}$ for bond percolation on the square lattice.