Theorem 5.47 of *The Random-Cluster Model* **corrected**

I am grateful to Remco van der Hofstad for pointing out a problem with Theorem 5.47, which has lost two important conditions. Here is a corrected but weaker version.

A probability measure μ on $(\Omega, \mathcal{F})$ is said to satisfy the 'uniform insertion-tolerance condition' if, for some $\alpha, \beta \in (0, 1)$,

$$(1') \qquad \alpha \le \mu(J_e \mid \mathcal{T}_e) \le \beta, \qquad \mu\text{-almost-surely, for } e \in \mathbb{E}^d,$$

where J_e is the event that e is open. Let E be a finite set of edges, and let $K_1, K_2, \dots, K_I$ be the components of the graph $(\mathbb{Z}^d, \mathbb{E}^d \setminus E)$. We say that μ has the 'empty-boundary Markov property' if: for all such sets E, given that every edge in E is closed, the configurations on the K_i, $i = 1, 2, \dots, I$, are independent. [We shall need a slightly weaker form of this property in the proof of Theorem 5.47′, following.]

Theorem 5.47′. *Let μ be a translation-invariant, positively associated probability measure on $(\Omega, \mathcal{F})$ satisfying $(1')$ for $\alpha, \beta \in (0, 1)$, and with the empty-boundary Markov property. The limit*

$$(5.48) \qquad \zeta(\mu) = \lim_{n\to\infty} \left\{ -\frac{1}{n} \log \mu(|C| = n) \right\}$$

exists and satisfies

$$(5.49) \qquad \mu(|C| = n) \le \frac{(1-\alpha)^2}{\alpha} n e^{-n\zeta(\mu)}, \qquad n \ge 1.$$

Furthermore, $0 \le \zeta(\mu) \le -\log[\alpha(1-\beta)^{2(d-1)}]$.

The quoted proof remains valid. One needs one further step in addition to those in the proof of Lemma 6.102 of reference [154]. In proving (6.105) of [154] in the current context, one needs to condition on the event that all edges in the external boundary of $\sigma * \tau$ are closed, to break this probability into a product of two terms, using positive association, and then to use the empty-boundary Markov property.

A small note concerning the second display of Theorem 5.51, which should read

$$\phi^0_{p,q}(|C| = n) \le \frac{q^2(1-p)^2}{p(p + q(1-p))} n e^{-n\zeta}.$$

Proof. In the terminology of (6.105) of [154], it suffices to prove that

$$(2') \qquad \mu(C = \sigma * \tau) \ge \frac{\alpha}{(1-\alpha)^2} \mu(C = \sigma)\mu(C = \tau).$$

Let E_σ be the set of edges of $\mathbb{L}^d$ that do not belong to σ but have one or more endpoints in σ, and let F_σ be the event that every edge in E_σ is closed. Let $e = \langle x, y\rangle$ denote the edge with endpoints $x = \mathrm{tr}(\sigma)$ and $y = \mathrm{bl}(\tau)'$. Then

$$\mu(C = \sigma) = \mu(F_\sigma)\mu(C = \sigma \mid F_\sigma),$$

and, by the empty-boundary Markov property and positive association,

$$\begin{aligned}\mu(C = \sigma,\ C_y = \tau + y) &= \mu(F_{\sigma*\tau} \cap F_e)\mu(C = \sigma,\ C_y = \tau + y \mid F_{\sigma*\tau} \cap F_e)\\ &= \mu(F_{\sigma*\tau} \cap F_e)\mu(C = \sigma \mid F_\sigma)\mu(C_y = \tau + y \mid F_{\tau+y})\\ &\geq \frac{\mu(E_{\tau+y} \setminus \{e\} \text{ closed})}{\mu(F_{\tau+y})}\mu(C = \sigma)\mu(C_y = \tau + y)\end{aligned}$$

where $F_e = \{e \text{ is closed}\}$. By the uniform insertion-tolerance property (1′),

$$\frac{\mu(E_{\tau+y} \setminus \{e\} \text{ closed})}{\mu(F_{\tau+y})} \geq \frac{1}{1-\alpha},$$

and therefore, by the translation-invariance of μ,

$$\mu(C = \sigma,\ C_y = \tau + y) \geq \frac{1}{1-\alpha}\,\mu(C = \sigma)\mu(C = \tau).$$

Inequality (2′) follows by a further application of (1′). □