PROJECTION FROM A POINT

$$
\begin{tikzcd}
& C = \mathbb{V}(X_0X_2 - X_1^2) \subseteq \mathbb{P}^2 \arrow[dl, dashed, "\pi_1"'] \arrow[dr, dashed, "\pi_2"] & \\
\mathbb{P}^1 & & \mathbb{P}^1
\end{tikzcd}
$$

π_1 : Projection from $[0:1:0]$ ▸ Does not lie on C

π_2 : Projection from $[0:0:1]$. ▸ Lies on C.

$$
\begin{tikzcd}
\pi_1 : & \mathbb{P}^2 \arrow[r, dashed] & \mathbb{P}^1 \\
& C \arrow[u, phantom, "\cup" description] \arrow[ur, "f"'] &
\end{tikzcd}
\qquad [X_0 : X_1 : X_2] \mapsto [X_0 : X_2].
$$

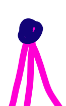
$f : C \longrightarrow \mathbb{P}^1$

Inspect preimages: Fix $[a_0 : a_2]$ on $\mathbb{P}^1$,

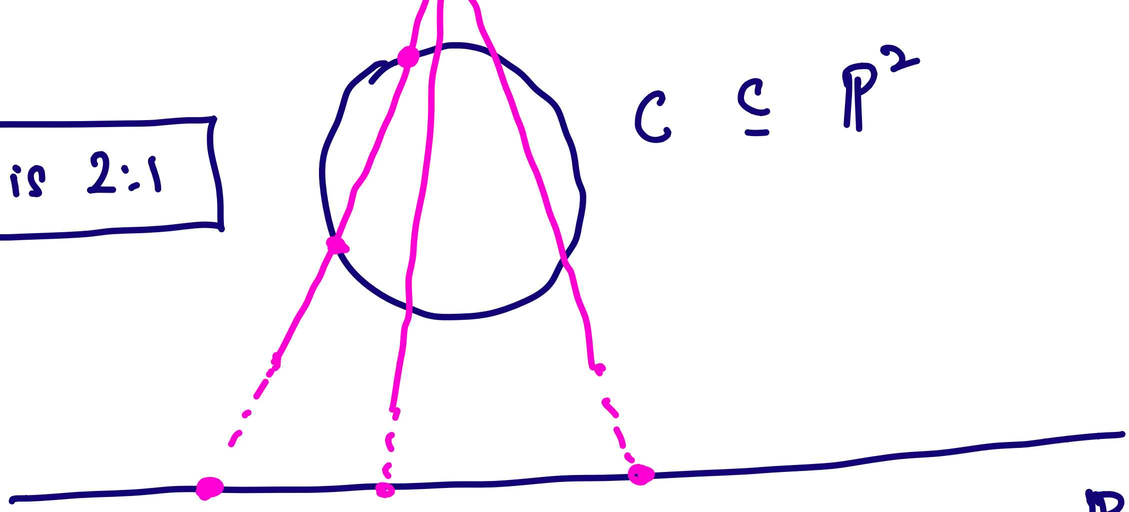
then $f^{-1}([a_0 : a_2]) = \{ [a_0 : s : a_2] \mid s^2 = a_0 a_2 \}$

Typically two choices for s.

$[0:1:0]$



$\boxed{\text{Map is } 2:1}$



$C \subseteq \mathbb{P}^2$

$\mathbb{P}^1$

$\pi_2 : \mathbb{P}^2 \dashrightarrow \mathbb{P}^1 \qquad [x_0 : x_1 : x_2] \longmapsto [x_0 : x_1].$

At the moment π_2 restricts to a morphism only on $C \setminus \{[0:0:1]\}$.

$$\begin{cases} C \overset{g}{\dashrightarrow} \mathbb{P}^1 \\ [x_0 : x_1 : x_2] \longmapsto [x_0 : x_1]. \end{cases}$$

<u>Preimages of g</u>: Fix $[a_0 : a_1]$ then

$$g^{-1}([a_0 : a_1]) = [a_0 : a_1 : \frac{a_1^2}{a_0}].$$

Map looks 1:1 where defined.

Is g regular at $[0:0:1]$?

We need to look for other pairs $[F_0:F_1]: C \dashrightarrow \mathbb{P}^1$, such that $[F_0:F_1]$ and $[X_0:X_1]$ determine the same rat'l map.

Need $F_0X_1 - F_1X_0 = 0$ on C

i.e. $\in I(C)^h$
$\quad = \langle X_0X_2 - X_1^2 \rangle$

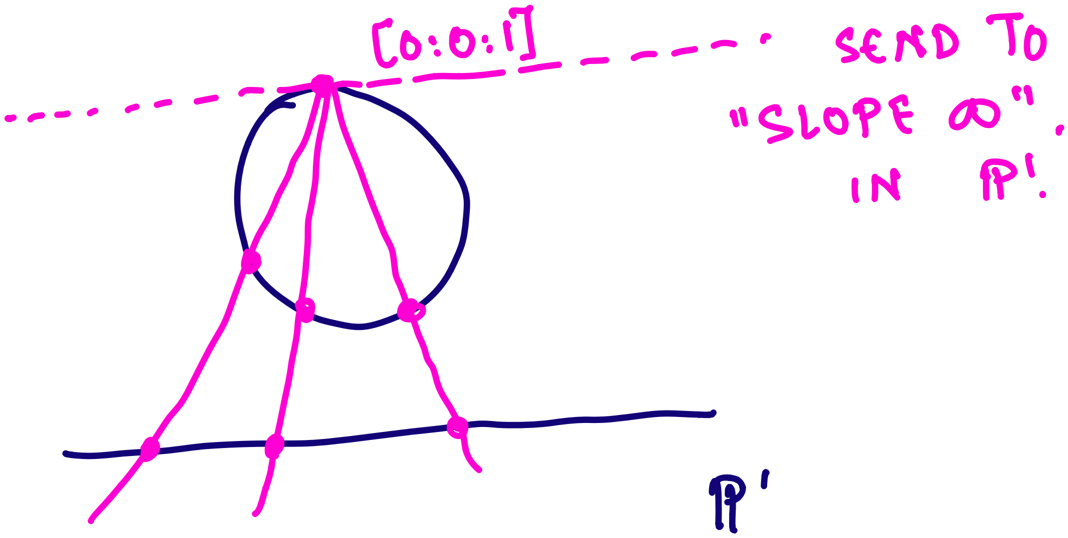
Take $[F_0:F_1] = [X_1:X_2]$

this determines $C \overset{g'}{\dashrightarrow} \mathbb{P}^1$ that is defined (i.e. regular) at $[0:0:1]$

g & g' agree when both defined and together give a morphism

$$\boxed{C \longrightarrow \mathbb{P}^1.}$$

b/c one of them is always defined



[0:0:1]
SEND TO "SLOPE ∞". IN P!
P'