THE SPACE Spec $\mathbb{C}[x,y]$

Recall $\text{Spec } \mathbb{C}[x,y] = \{\mathfrak{p} \subseteq \mathbb{C}[x,y] \mid \mathfrak{p} \text{ a prime}\}$

THREE TYPES OF PRIMES

- **Type I:**

$$\mathbb{C}^2 \hookrightarrow \text{Spec } \mathbb{C}[x,y]$$

$$(a,b) \longmapsto ((x-a),(y-b)) = m_{a,b}$$

The type I points are closed.

- **Type II:** $\left\{\begin{array}{l} f(x,y) \text{ irreducible} \\ \text{and nonzero} \end{array}\right\} \hookrightarrow \text{Spec } \mathbb{C}[x,y]$

The type II points are not closed

if $\eta = (f)$ then $m_{a,b} \in \overline{\{\eta\}}$ if and only

if $f(a,b) = 0$
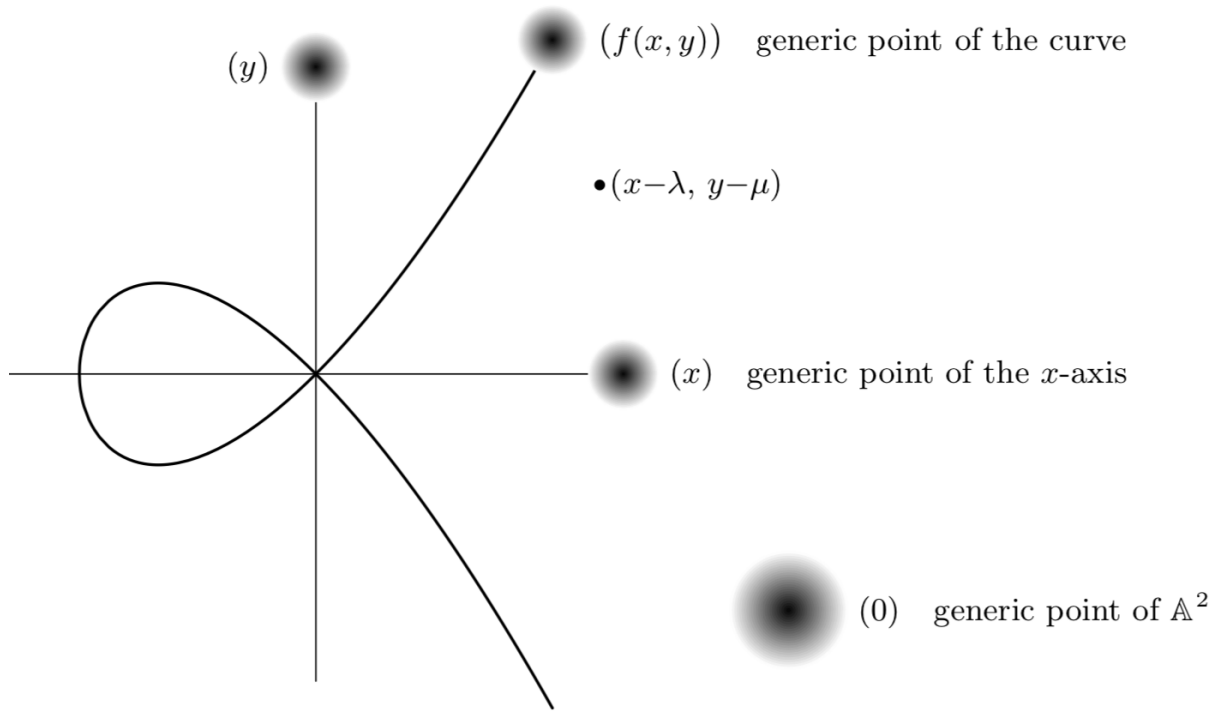
- **Type III:** The ideal $(0) \in \text{Spec } \mathbb{C}[x,y]$

The type III point — the GENERIC POINT — is

dense.

THE POINTS

(y)

$(f(x,y))$ generic point of the curve

$\bullet (x-\lambda,\ y-\mu)$

(x) generic point of the x-axis

(0) generic point of $\mathbb{A}^2$

Examples of closed sets

$V(x),\quad V(xy),\quad V(f)$

Examples of open sets

$U_x,\quad U_{xy},\quad U_f.$