EXAMPLES OF MORPHISMS

- **Open immersions**: $A \to A_f$ a localization at $f \in A$. Then
$$\operatorname{Spec} A_f \to \operatorname{Spec} A$$
is an open immersion.

[Localizations always give open immersions]

- **Closed immersions**: $A \to A/I$ any quotient by an ideal. Then
$$\operatorname{Spec} A/I \longrightarrow \operatorname{Spec} A.$$
is a closed immersion.

[All closed immersions look like this locally]

- **Neither open immersion nor closed immersion**:

Take $U \subsetneq Z$ open (eg $\mathbb{A}^1 \subseteq \mathbb{P}^1$) and

$Z \hookrightarrow X$ closed & not open (eg $\mathbb{P}^1 \xrightarrow{\Delta} \mathbb{P}^1 \times \mathbb{P}^1$)

The composition $U \to X$ is neither.

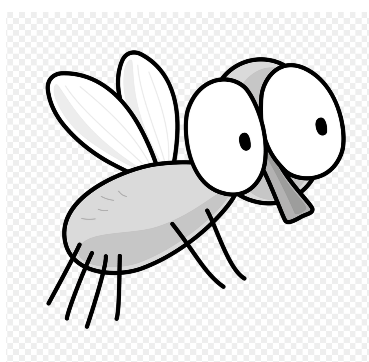
· A (nontrivial) morphism that is open & closed:

Take X any scheme and the morphism

$$X \longrightarrow X \amalg X.$$

· A morphism that is universally closed but not proper: take $\mathbb{P}^1_k$ with doubled origin, i.e. take $U_1 = \mathbb{P}^1_k \setminus pt$ & $U_2 = \mathbb{P}^1_k \setminus pt$ and $X = \mathbb{P}^1 \amalg \mathbb{P}^1 / \sim$ where $\sim$ identifies U_1 & U_2. Then $X \to \operatorname{Spec} k$ is universally closed & finite type but not separated.

[Separatedness fails b/c open subschemes of separated things are separated but this contains bug-eyed line]



- A nontrivial open subscheme that is proper: take $\mathbb{P}^1 \subseteq$ Bug-eyed-$\mathbb{P}^1$ above

[Hard for this to happen in a separated scheme]

- Morphism not of finite type: Take

$$A = k[x_1, x_2, \ldots].$$

Then $\operatorname{Spec} A \longrightarrow \operatorname{Spec} k$.

[I told you these were pathological]