Corrections to
A Course in Mathematical Analysis

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I wish to thank Bentley Eidem and Michael Mueger, who pointed out many of the errors.

Correction to Pages 46-47

The As and Bs have become interchanged. The formula in Line 3* should be

$$n_1 = \inf(B) \leq \inf(A) = m_1.$$

Line I* should be

$$n_{k+1} = \inf\{b \in B : b > n_k\} \leq \inf\{a \in A : a > m_k\} = m_{k+1}.$$

The formula in Page 47, Line 1 should be

$$n_{k+1} = \inf\{b \in B : b > n_k\} \leq m_k < m_{k+1}.$$

Of course, Proposition 2.4.1 is just a simpke technicality.

Correction to Page 65, line 13

Two brackets are missing. The first term should be $-(\psi(m))\psi(l))$, and vthe last but one should be $\psi((-m)l)$.

Correction to Page 67, line 3*

'upper bound for U' should be 'upper bound for L'.

Addition to Page 70,line 7

For clarity, add 'and so $D(x)$ is the Dedekind cut equal to x'. Of course, one forgets that a real number is defined as a Dedekind cut.

Correction to Page 74, lines 5*-4*

'By the lemma' justifies the formula in line 5*, not its consequence in line 4*.

Correction to Page 84 lines 10-11
Replace $l > 0$ by $\epsilon > 0$ and the formula by $0 < 1/n \leq 1/n_0 < \epsilon$.
Correction to Page 108, line13*
It should read 'derived from $(z_j)_{j=0}^{\infty}$'.
Misprint in Page 122, line 2*
It should be 'transcendental'.
Correcxtion to Page 123, line 2
Replace the second bracket by

$$\left(\frac{1}{q+1} + \frac{1}{(q+1)(q+2)} + \cdots\right).$$

Correction to Page 157
6. Let f be the *saw-tooth function*

$$f(x) = \begin{cases} \{x\} & \text{for } 2k \leq x < 2k+1, \\ 1 - \{x\} & \text{for } 2k+1 \leq x < 2k+2, \end{cases}$$

for $k \in \mathbf{Z}$. Let $g(x) = f(1/x)$ for $x \neq 0$, and let $g(0) = 0$. Then g has a discontinuity at 0: $g((x)$ oscillates in value between 0 and 1 as $x \to 0$.
Correction to Pages 205-6
Thus

$$f_\alpha(x) = 1 + \sum_{j=1}^{n-1} \binom{\alpha}{j} x^j + r_n(x).$$

We need to show that the remainder $r_n(x)$ tends to 0 as $n \to \infty$. The Lagrange form of the remainder is

$$r_n(x) = \binom{\alpha}{n}(1+\theta_n x)^{\alpha-n}x^n = (1+\theta_n x)^\alpha \binom{\alpha}{n}\left(\frac{x}{1+\theta_n x}\right)^n,$$

where $0 < \theta_n < 1$. If $0 \leq x < 1$ then $\sup_n |x/(1+\theta_n x)| < 1$, and so $r_n(x) \to 0$ as $n \to \infty$ (see Exercise 3.2.5). If $-1 < x \leq -1/2$, this argument does not work.

Instead, we use Cauchy's form of the remainder. Choose $k > |\alpha|$. We find that

$$r_n(x) = \frac{\alpha}{k}(1-\theta_n)^{n-k}\binom{\alpha-1}{n-1}(1+\theta_n x)^{\alpha-n}x^n.$$

Since $1 - \theta_n < 1 + \theta_n x$, it follows that if $n \geq \alpha$ then

$$|r_n(x)| \leq \left|\frac{1}{(1-|x|)^{k-\alpha}}\binom{\alpha-1}{n-1}x^n\right|,$$

and so $r_n(x) \to 0$ as $n \to \infty$.

Correction to Page 243, line 4

This should read '$a_j(f_o) = 0, \quad b_j(f_o) = b_j(f)$'.

Correction to Page 250.

Thus

$$\sum_{n=1}^{N} a_n(I_\delta)\cos nt = \frac{2}{\delta}\sum_{n=1}^{N}\frac{\sin n\delta \cos nt}{n} = \frac{1}{\delta}\sum_{n=1}^{N}\frac{\sin(n(t+\delta)) - \sin(n(t-\delta))}{n}.$$

Do these sums converge, as $N \to \infty$? If $0 < \alpha < \pi$ then

$$|\sum_{n=1}^{N}\sin n\alpha| \leq |\sum_{n=1}^{N} e^{in\alpha}| = \left|\frac{e^{i(N+1)\alpha} - e^{i\alpha}}{e^{i\alpha} - 1}\right| = \frac{2}{|e^{i\alpha/2} - e^{-i\alpha/2}|} = \frac{1}{\sin \alpha/2}.$$

Thus if $|t| \leq \pi$, and if $|t-\delta| > \eta$ and $|t+\delta| > \eta$, then

$$|\sum_{n=1}^{N}(\sin(n(t+\delta)) - \sin(n(t-\delta)))| \leq 2\sin \eta/2.$$

Correction to Page 263.

We can clearly suppose that f is increasing in I. By a change of variables, we can suppose that $t = 0$, so that we need to show that $\sum_{j=-n}^{n}\hat{f}_n = \frac{1}{2}(f(0+) + f(0-))$. We can also suppose that $f(0) = \frac{1}{2}(f(0+) + f(0-))$. Let $j(0) = j(\pi) = 0$, let $j(s) = 1$ for $0 < s \leq \pi$ and $j(s) = -1$ for $-\pi < s < 0$, and extend j by periodicity. Then j is an odd function, and so $\sum_{j=-n}^{n}\hat{j}_n = 0$ for all $n \in \mathbf{N}$. Now let

$$g(s) = f(s) - \tfrac{1}{2}(f(0+) - f(0-))j(s) - f(0).$$

Then

$$\sum_{k=-n}^{n}\hat{g}_k = \sum_{k=-n}^{n}\hat{f}_k - f(0),$$

and so we need to show that $\sum_{j=-n}^{n}\hat{g}_n = 0$.

f̲Correction to Pasge 308, line 7

It should read $j(+\infty) = \pi/2$.

Addition to Pages 337and 354

We also need to define the limit point of a sequence $(x_n)_{n=0}^{\infty}$ in a metric space or topological space: x is a limit point of the sequence $(x_n)_{n=0}^{\infty}$ if, whenever N is a neighbourhood of x and $n \in \mathbf{N}$, there exists $m \geq n$ such that $x_m \in N$.

Correction to Page 385

In the table, it should read that a subspace of a send countable space is second countable.

Addition to Page 389

In line 4 it should be '$X = \prod_{i=1}(X_i, d_i)$'.

Correction to Pages 401-2

In Theorem 14.3.4, V_1 and V_2 should be replaced by normed spaces $(E_1, \|.\|_1)$ and $(E_2, \|.\|_2)$