17. If $\sum_{n=1}^{\infty} x_n$ is a convergent series of reals, must $\sum_{n=1}^{\infty} \frac{x_n}{n}$ be convergent?

18. Let $(x_n)_{n=1}^{\infty}$ be a real sequence with $x_n \to 0$. Prove carefully that we may choose $(\epsilon_n)_{n=1}^{\infty}$, with each $\epsilon_n = \pm 1$, such that $\sum_{n=1}^{\infty} \epsilon_n x_n$ is convergent. Does this remain true if the x_n are complex?

19. Let $x_1, x_2, \ldots$ be reals such that $\sum_{n=1}^{\infty} |x_n|$ is convergent. Show that if for every positive integer k we have $\sum_{n=1}^{\infty} x_{kn} = 0$ then $x_n = 0$ for all n. What happens if we drop the restriction that $\sum_{n=1}^{\infty} |x_n|$ is convergent?

20. Let S be a (possibly infinite) set of odd positive integers. Prove that there exists a real sequence $(x_n)_{n=1}^{\infty}$ such that, for each odd positive integer k, the series $\sum_{n=1}^{\infty} x_n^k$ converges when k belongs to S and diverges when k does not belong to S.

21. A subset of $\mathbb{R}$ is called *perfect* if it is closed and has no isolated points. Write down a (non-empty) perfect set that does not contain any (non-trivial) interval. Prove that every closed set is the union of a perfect set and a countable set.

22. Let S be a collection of subsets of $\mathbb{N}$ such that for every $A, B \in S$ we have $A \subset B$ or $B \subset A$. Can S be uncountable?

23. Let S be a collection of subsets of $\mathbb{N}$ such that for every distinct $A, B \in S$ we have that $A \cap B$ is finite. Can S be uncountable?

24. Let X be a subset of $\mathbb{R}$ such that the only order-preserving injection from X to X is the identity. Must X be finite?