9. Let f be a function from $\mathbb{N}^2$ to $\mathbb{N}$. Does there always exist an infinite set $S \subset \mathbb{N}$ such that $f(S^2)$ is *not* the whole of $\mathbb{N}$?

10. In a *tournament* on n players, each pair play a game, with one or other player winning (there are no draws). Construct a tournament in which, for any two players, there is a player who beats both of them. Is it true that for any k there is a tournament in which, for any k players, there is a player who beats all of them?

11. Let f be a real function of two real variables. Suppose that for each fixed y the function $f(x, y)$ is a polynomial in x and for each fixed x the function $f(x, y)$ is a polynomial in y. Must $f(x, y)$ be a polynomial in x and y?

12. Let f and g be real polynomials such that the set of values taken by f on the rationals is the same as the set of values taken by g on the rationals. Prove that there exist rationals a and b such that $g(x) = f(ax + b)$.

13. Construct a function f from the rationals to the rationals that takes every value on every interval – in other words, for every $a < b$ and every c there is an x with $a < x < b$ such that $f(x) = c$. Is there such a function from the reals to the reals?

14. Show that it is impossible to write the open interval $(0, 1)$ as the disjoint union of a family of non-trivial closed intervals.

15. Can the open square $(0, 1)^2$ be written as the disjoint union of a family of non-trivial closed straight-line segments?

16. Let $[a_1, b_1], [a_2, b_2], \ldots$ be a sequence of closed intervals whose union contains $[0, 1]$. Does it follow that $\sum_{n=1}^{\infty}(b_n - a_n) \geq 1$?