

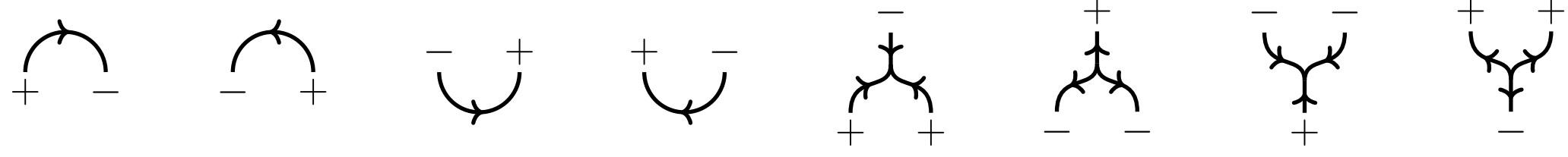
Clasps in $\mathfrak{sl}_3$ Webs at Roots of Unity

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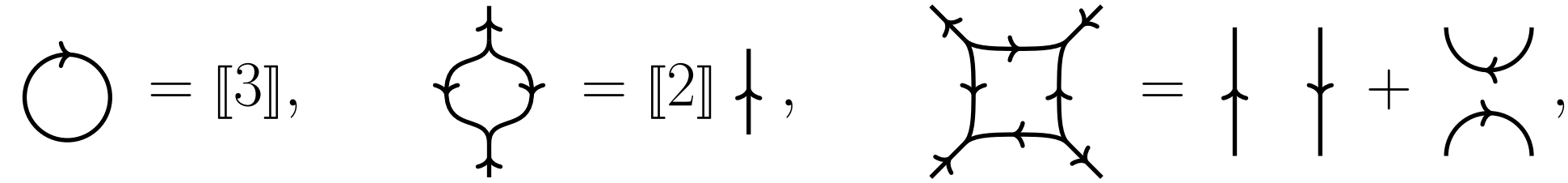
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What are $\mathfrak{sl}_3$ webs?

- Monoidal category of diagrams that gives a combinatorial model for the representation theory of the quantum group of $\mathfrak{sl}_3$, dating back to work of Kuperberg [3].
- Concretely: categories $\mathbf{Web}_k$ and $\mathbf{Fund}_k$ are equivalent.
- Objects are strings of $+$'s and $-$'s, and morphisms have generators



subject to the local relations



as well as the corresponding ones with the orientations reversed.

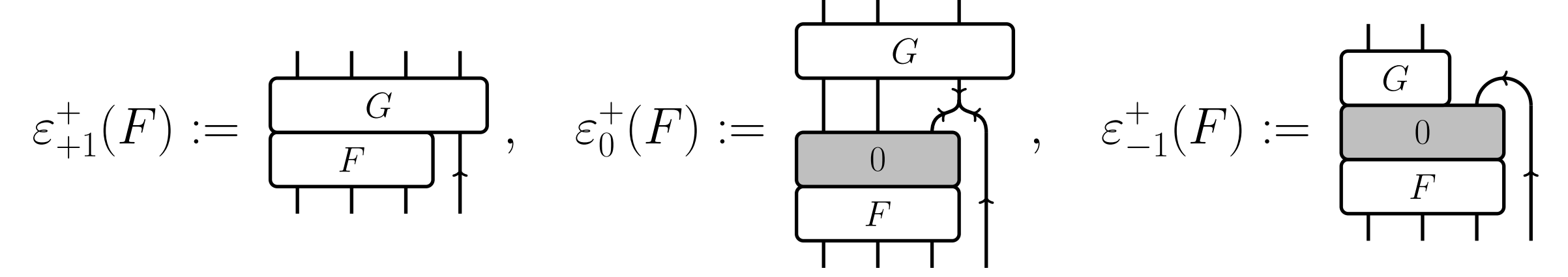
- E.g. the diagram



corresponds to a morphism of $U_q(\mathfrak{sl}_3)$ -modules $V_+ \otimes V_+ \otimes V_- \otimes V_- \otimes V_+ \rightarrow V_- \otimes V_-$.

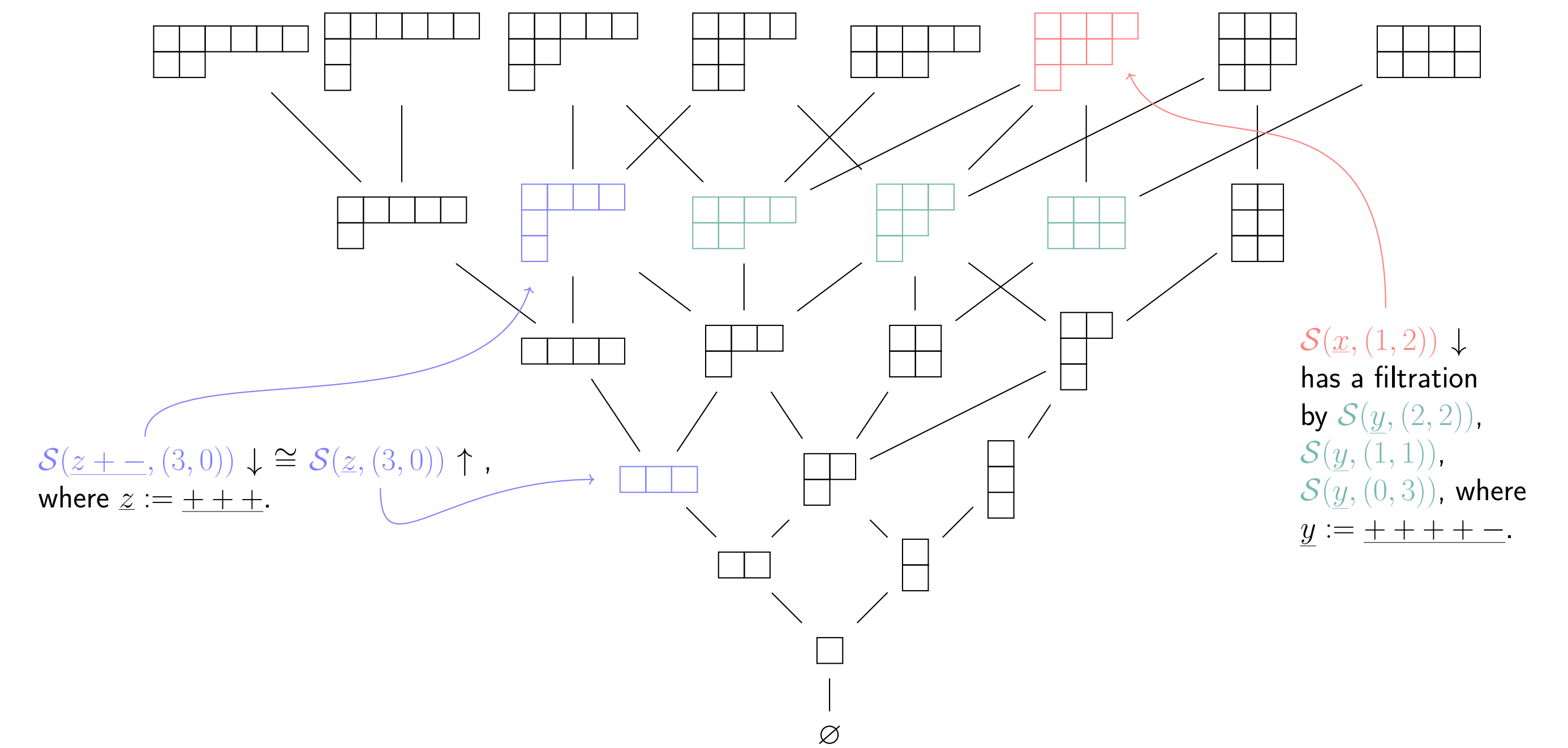
Cellularity and light ladders

- The category of webs is a *cellular category* (every endomorphism algebra is cellular in the sense of Graham–Lehrer).
- Cell modules are $\mathcal{S}(x, \lambda)$, where x is an object of $\mathbf{Web}_k$ and λ is a weight.
- Basis elements of $\mathcal{S}(x, \lambda)$ correspond to paths in the dominant Weyl chamber and can be built inductively using Elias's [2] light ladders construction:



Branching rules

Branching rules between cell modules behave as expected. E.g. for $x = ++++--$,

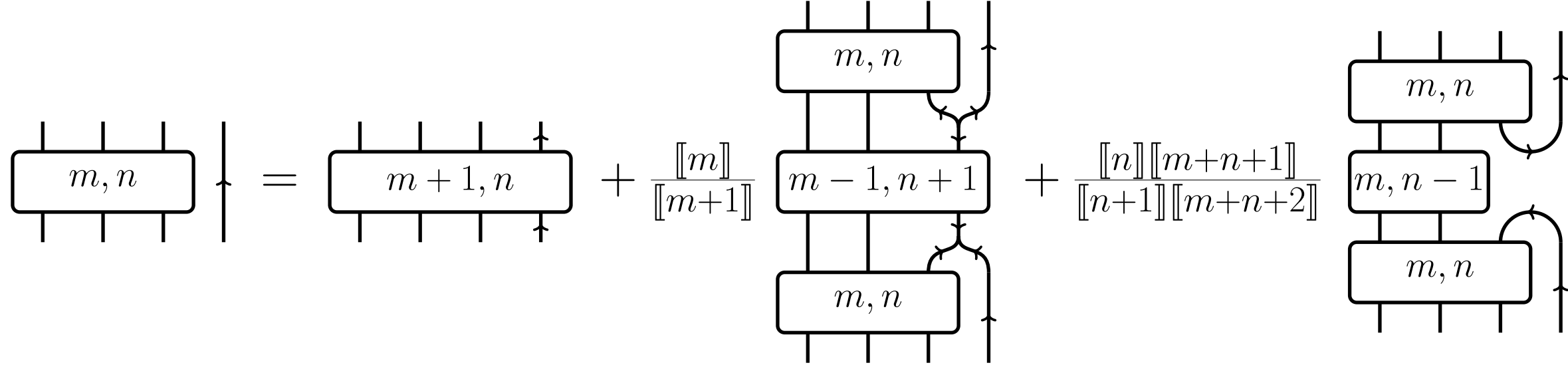


Proposition. The restriction $\text{res}_x^{\pm} \mathcal{S}(x \pm, \lambda)$ has a cellular filtration by $\{\mathcal{S}(x, \lambda')\}$ where $\lambda' \rightarrow \lambda$ in the Young poset, and we have isomorphisms

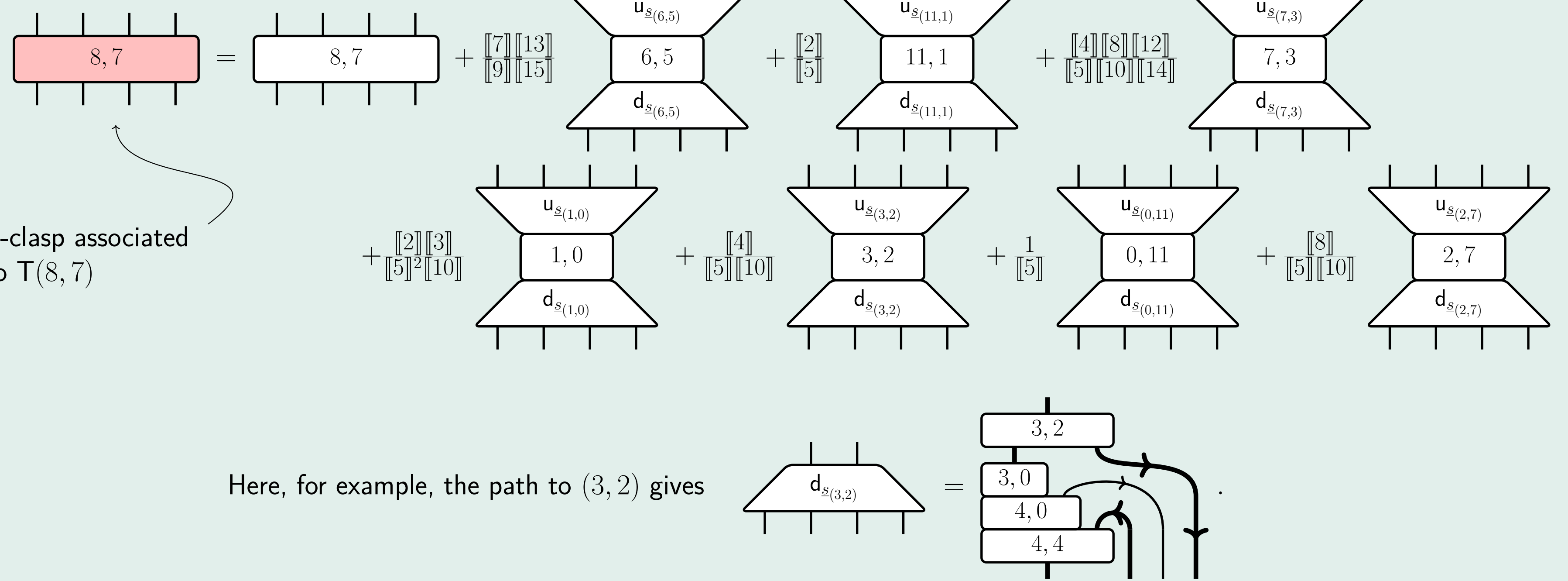
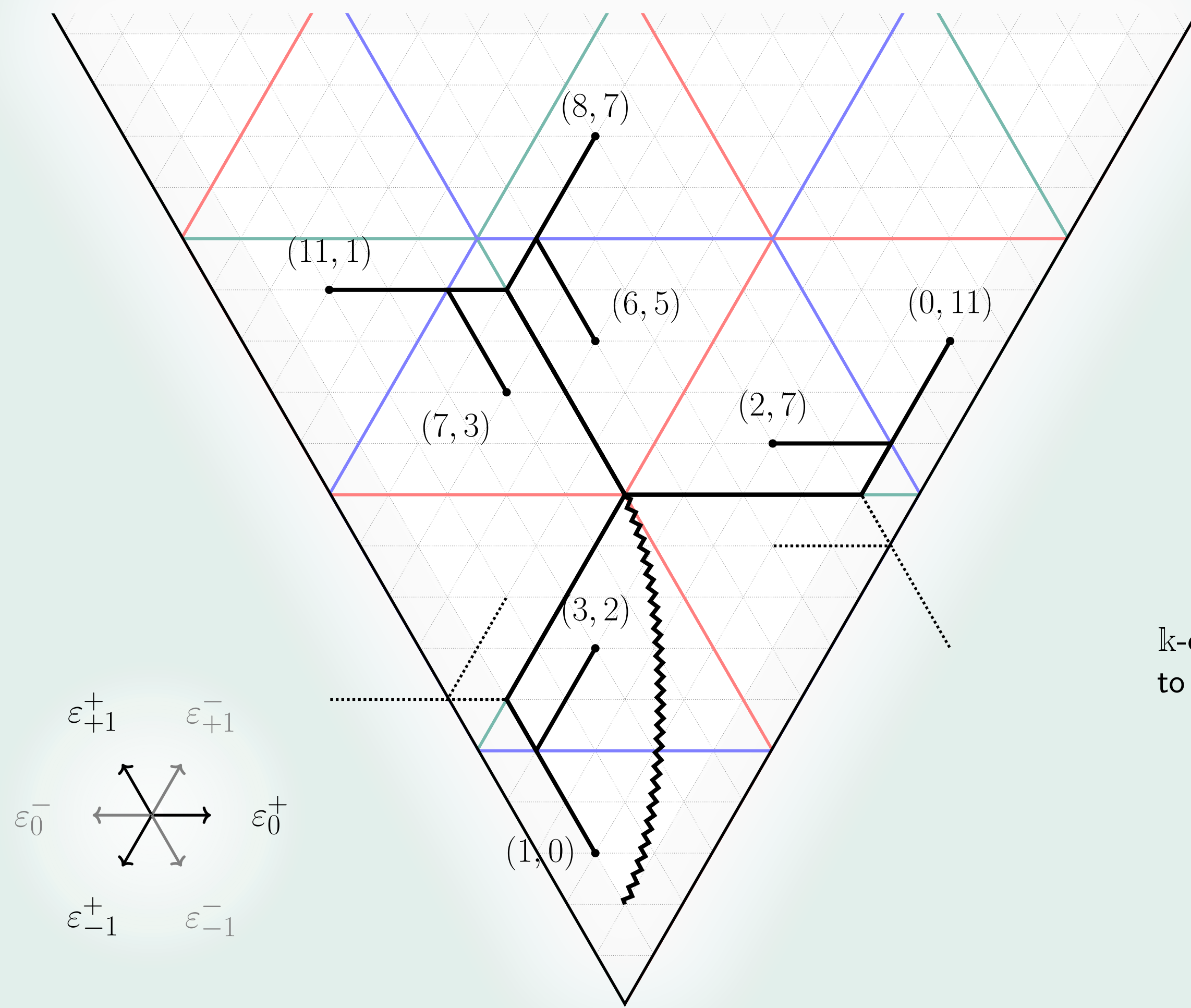
$$\text{ind}_x^{\pm} \mathcal{S}(x, \lambda) \cong \text{res}_{x \pm}^{\pm} \mathcal{S}(x \pm, \lambda) \quad \text{and} \quad \text{ind}_x^{\pm} \mathcal{S}(x, \lambda) \cong \text{res}_{x \pm}^{\pm} \mathcal{S}(x \pm, \lambda).$$

What are clasps?

- Higher rank analogues of Jones–Wenzl projectors.
- Project to the top tilting summand in V_x .
- In the semisimple case, they are given recursively by Elias's [2] triple clasp expansion:



$T(8, 7)$ when q^2 is a primitive 5th root of unity:



Main goal

To give an expression for the $\mathbb{k}$ -clasps when $\text{char}(\mathbb{k}) = 0$ and $q \in \mathbb{k}$ is a root of unity.

This relies on some numerical data telling us “which semisimple clasps need to be added together” to give a $\mathbb{k}$ -clasp. Several different approaches:

- the Weyl multiplicities in tiltings $[T(\lambda) : \Delta(\mu)]$ (in the $\mathfrak{sl}_2$ case, this is done in [8]);
- the p -canonical basis (in the $\mathfrak{sl}_2$ case, this is the approach of [1]);
- the cell filtrations of the projective cover of the trivial modules in webs (in the $\mathfrak{sl}_2$ case, this is done in [7, 5]).

Approach	Root of unity	Characteristic p
Tilting characters	AS KL polynomials [6]	billiards conjecture [4]
p -canonical basis	n/a	hard
Cell filtrations	work in progress	?

References

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