Part III Analysis of PDE: Rough syllabus

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Material marked with (∗) is unexaminable, although questions may appear which make use of ideas from these sections of the course.

I. INTRODUCTION AND THE CAUCHY-KOVALEVSKAYA THEOREM

a) **Introduction** Examples of PDE; well-posedness; classification into linear/semi-linear etc.

b) **Cauchy–Kovalevskaya** Motivation through ODE theory; real analytic functions and their properties; Cauchy–Kovalevskaya theorem for first order systems (*proof*); reduction to first order systems; characteristic surfaces.

II. SPACES OF FUNCTIONS

a) **Hölder spaces** Definition; basic properties.

b) **Sobolev spaces** Weak derivatives; definition of Sobolev spaces; approximation by smooth functions; extension theorems; trace theorem; Sobolev embeddings: Gagliardo-Nirenberg-Sobolev inequality, Morrey's inequality, Poincaré inequality.

III. ELLIPTIC BOUNDARY VALUE PROBLEMS

a) **Basic solvability** Strong/uniform ellipticity; weak formulations; Lax-Milgram; energy estimates/Gårding's inequality; basic existence of solutions; *solving nonlinear problems by contraction mapping theorem*.

b) **Compactness** Weak compactness for Hilbert spaces; Rellich-Kondrachov theorem; Fredholm alternative; spectrum of L; spectral theorem for symmetric (formally self-adjoint) elliptic BVPs.

c) **Regularity** Difference quotients; elliptic regularity: interior and *boundary*.

IV. HYPERBOLIC EQUATIONS

a) **Basic solvability** Definition of hyperbolicity for second order linear operators; weak formulation of the initial-boundary value problem; uniqueness of weak solutions; *Galerkin's method for existence*.

b) **Further results and extensions** finite speed of propagation; *parabolic problems*; *solving nonlinear problems by contraction mapping theorem*.