1. Let L be a closed subset of a normal topological space K. Show that any bounded continuous function $g\colon L \to \mathbb{C}$ extends to a bounded continuous function $f\colon K \to \mathbb{C}$ with $\|f\|_\infty = \|g\|_\infty$.

2. Let L be a closed subset of a normal topological space K. Show that any (not necessarily bounded) continuous function $g\colon L \to \mathbb{R}$ extends to a continuous function $f\colon K \to \mathbb{R}$.

3. Let K be a compact Hausdorff space. Find the maximal (proper) closed subalgebras of $C^{\mathbb{R}}(K)$.

4. Let $f\colon [0,1] \to \mathbb{R}$ be a continuous function such that $\int_0^1 f(x)x^n \, dx = 0$ for every even $n \geqslant 0$. Prove that $f = 0$.

5. Let K be a compact metric space. Show that $C(K)$ is separable.

6. Let X be an inner product space, and let $T\colon X \to X$ be a linear map. Show that $\langle Tx, Ty\rangle = \langle x, y\rangle$ for all $x, y \in X$ if and only if $\|Tx\| = \|x\|$ for all $x \in X$.

7. Let X be a complex inner product space, and let $T\colon X \to X$ be a linear map. Show that if $\langle Tx, x\rangle = 0$ for all $x \in X$, then $T = 0$. Does the same conclusion hold in the real case?

8. Let X be a normed space. Show that if $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$ for all $x, y \in X$, then X is an inner product space.

9. Construct an inner product space X and a proper closed subspace Y of X such that $Y^\perp = \{0\}$.

10. Show that the unit ball of ℓ_2 contains an infinite set S such that $\|x - y\| > \sqrt{2}$ for all distinct $x, y \in S$. Can the constant $\sqrt{2}$ be improved?

11. Let S be a subset of a normal topological space X. Show that there is a continuous function $f\colon X \to \mathbb{R}$ such that $S = f^{-1}(0)$ if and only if S is a closed $\mathcal{G}_\delta$ set.

12. A topological space is *Lindelöf* if every open cover has a countable subcover, and is *regular* if for every closed subset F and point $x \notin F$, there are disjoint open sets U and V with $x \in U$ and $F \subset V$. Prove that a regular Lindelöf space is normal.

13. Let $\mathcal{O}(U)$ denote the space of holomorphic functions on the non-empty open subset U of the complex plane. Let (f_n) be a sequence in $\mathcal{O}(U)$ that is uniformly bounded on every compact subset of U. Show that some subsequence of (f_n) converges locally uniformly on U.

$^+$14. Let K be a compact Hausdorff space. Does K separable imply $C(K)$ separable?