A brief Note on Doubles Tournaments

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The purpose of this note is to describe a short solution of the Doubles Tournament problem and the Spouse-Avoiding Doubles Tournament problem for $4k$ and $2k$ players respectively, when k is a power of 2. Similar tournaments are known for almost all values of k — see [1] and [2].

The Doubles Tournament Problem: Given $4k$ players, to arrange a tournament of $(4k-1)$ rounds, each of k double matches, so that any two players play with each other exactly once and against each other exactly twice.

Rule 1: Let F be a finite field of order $4k$. We index the players by the members of F and the rounds by the set F' of non-zero elements of F. Pick $\xi \in F$, with $\xi \neq 0$ or 1. In round ψ, player θ will play with player $\theta + \psi\xi$ against players $\theta + \psi$ and $\theta + \psi + \psi\xi$.

Proof: To see that Rule 1 is coherent, note that the sets $\{\theta, \theta + \psi\xi\}$ are the cosets of the additive subgroup $\{0, \psi\xi\}$, itself a subgroup of $\{0, \psi, \psi\xi, \psi + \psi\xi\}$; of which larger subgroup the matches $\{\theta, \theta + \psi, \theta + \psi\xi, \theta + \psi + \psi\xi\}$ are the cosets.

The Spouse-Avoiding Doubles Tournament Problem: Given $2k$ married couples, to arrange a tournament of $(2k-1)$ rounds, each of k mixed doubles, so that no couple ever play in the same game, but such that each person plays *against* each other person, spouse excepted, exactly once and plays *with* each person of the other sex, spouse excepted, exactly once.

Rule 2: Let G be a finite field of $2k$ elements; index the players by the members of G and the rounds by the set G' of non-zero elements of G. Pick $\xi \in G\backslash\{0,1\}$ as before, and call the members of couple θ Man θ and Woman θ. In round ψ, Man θ and Woman $\theta + \psi\xi$ play Man $\theta + \psi$ and Woman $\theta + \psi + \psi\xi$.

The coherency and adequacy of this rule may be verified as above.

[1] Construction of Balanced Double Schedules, P. Healey
Journal of Combinatorial Theory Series A 29, 280-286, 1980

[2] Spouse-avoiding Mixed Double Tournaments, W. D. Wallis
Annals of the New York Academy of Sciences, 549-554, 1979