LOGIC AND TERROR [1]

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I T IS THE MOST SEVERE PLEASURE for a mathematical logician to read a paper to a gathering of historians and philosophers. Mr Golding has asked me to spare you mathematical arguments: his very request is indicative of the gulf between mathematical reasoning and ordinary reasoning that I might take as a starting point.

Many mathematicians have been proud of their separateness: G.H.Hardy wrote with considerable satisfaction in his *Mathematician's Apology* "I have never done anything 'useful'."

A more provocative illustration of the difference between life and mathematics, more particularly the difference between logic and mathematical logic, is this:

When I was first a Fellow of this august College, Mr Cowling said to me in the course of a discussion "Why do logicians always give such extraordinary examples ?" I replied "Because reasoning, if correct, should be valid in all instances." He said "That's totally untrue." I have been pondering his words ever since.

Try adding the word 'mathematical' to my thesis: "mathematical reasoning, if correct, should be valid in all mathematical instances".

Now that is much more true: indeed it is practically a required belief for anyone taking the Mathematics Tripos, for how else may a supervisor criticise fallacious reasoning that reaches a true conclusion ?

I should mention, though, that as one passes beyond the meagre demands of the Mathematical Tripos, one finds that even my amended thesis is not entirely true, for there is more than one mathematical logic. There is, for example, intuitionistic logic, which though equiconsistent with classical logic is in some disagreement with it. Problems therefore arise of deciding which logic is appropriate for the concept under consideration: some people get into comic contortions trying to impose an alien logic on unco-operative material. "Why not", I have heard it said, "use the logic you've got ?"

Mathematical logic is logic restricted in language, subject matter and method. Subject matter and method would both come under Mr Golding's ban, but the restriction of language is easy to illustrate: take the word 'and'.

In a natural language, such as English, the word 'and' has causal and temporal overtones. "John went to London and caught a plane" is not the same as "John caught a plane and went to London". The order of events is different, and indeed the mode of transport used by John to go to London appears to be train, car or bus in the first case, plane in the second.

In a mathematical language, (for I should discourage my listeners from thinking that there is a single mathematica language to which mathematics might be restricted) the word 'and' is a simple connective, used when one wants to assert the joint truth of two statements A and B. There is no suggestion of any relationship of the content of the two statements. Thus $A\&B$ is equivalent to $B\&A$.

Anyone who, asked if he would like tea or coffee, answers "Yes", or if asked "Is it a boy or a girl ?" replies "I hope so" has grasped the (classical) mathematical use of the word 'or'.

This restriction of language, adumbrated in Leibniz, may be credited to Frege, and it has enabled mathematical logic to develop enormously in the present century. The study of the logic of natural languages has progressed much more slowly because the problems raised by natural languages are so very difficult in comparison with those of mathematical languages. Try, for example, analysing the sentence "Unpunctuality is a fault." This is rich in sociological allusions wholly beyond the more modest ideas suggested by such mathematical truths as "$2 + 2 = 4$". Natural languages contain countless concepts, each with its own logic.

The mathematical use of the word 'and' that I have described is determined truth-functionally: that is, the sentence $A\&B$ is true in this usage if and only if both A and B are true. The truth value of $A\&B$ is a function just of the truth values of A and B, and not of their content.

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[1] read to the Perne Club of Peterhouse on February 12th 1987 and subsequently revised.

Truth functions in this sense were known in antiquity to the logicians of Megara and the Stoa, though not to Aristotle. A truth-functional treatment of implication, as given by Philo of Megara, leads to the paradox of material implication: if grass grows at the north pole then the moon is made of green cheese. Such sentences are felt to be paradoxical; and such was the row about them in antiquity that Callimachus, the poet and cataloguer of the Library at Alexandria in the second century BC, was heard to remark that the very ravens on the roof tops were cawing about the meaning of conditional sentences.

People are fond of saying that there has been, even that there can be no development of logic since Aristotle. The French are prone to say so, it being an important ingredient of rationalism to believe that there is only one logic. Among others guilty of this *canard* are an anonymous mathematician who made that statement at Princeton to an audience that included the most important mathematical logician of the century, Kurt Gödel; Immanuel Kant, who is recorded as saying that since Aristotle logic has been unable to advance a step, and thus to all appearances has reached its completion; Carl Prantl, who, as it is said, devoted a lifetime to writing a comprehensive history of logic in order to prove that logic has no history at all; and Professor Kilmister, who in *Language, Logic and Mathematics* writes "Logic developed hardly at all after it left the hands of Aristotle until the 19th century."

But it appears that when human thought reaches an impasse, and a thinker wishes to propose a system that can rise above contemporary troubles, he begins by writing a work on logic: thus Hegel wrote one, and so did Mill. Indeed one might say that it is only when you realise that your current vocabulary is inadequate for your problems that you can make progress; and that once you do realise that, you naturally wish to elaborate the logic of your expanded range of ideas.

There is a distinction to be made between *enlargement of vocabulary* which entails no enlargement of logic, and *enlargement of logic*. The bureaucrat, who thinks that there is no problem sees no need to enlarge his vocabulary; the conservative politician who thinks that there is a problem but existing procedures are adequate for it wishes to enlarge his vocabulary but not his logic; the revolutionary who holds that existing instutions are inadequate wishes to enlarge his logic.

System builders, whatever their merits, have in time an effect that is damaging in proportion to their authority. So long as their mental construction is adequate for the world, they are a good thing; but as the world grows beyond their language, they become restricting, then stultifying and finally tyrannous. Thus Aristotle's authority led to a harmful belief in the fixity of logic and had a bad effect on the Thomists; Euclid had a bad effect on Kant and Kant on the further development of geometry; Hegel in explaining one Terror paved the way for the next.

So crises precipitate new systems, which become old and engender new crises.

Having said that logic has a history, let me now summarise it. [2]

The initial impetus to Greek logic appears to have come from the paradoxes of motion of the Eleatic school — Zeno (circa 460 BC) and his chums. So embarrassing were these paradoxes that the geometer Euclid excludes from his list of axioms for plane geometry the notions of motion that he needs to prove that if two triangles have equal sides they have equal angles. He appeals to the reader, surreptitiously, to imagine sliding one triangle over the other.

Schools then developed from two pupils of Socrates (*ob.* 399 BC), namely, Plato (428-348 BC) and Euclid of Megara. Of Plato's great pupil, Aristotle, (384-322 BC) I need say nothing; of the school of Megara, I may mention Eubulides, who is credited with the discovery of the Liar paradox, Diodorus Cronus, who died in 307 BC of despondency, and Philo, whose truth-functional definition of implication has already been mentioned.

The first founder of the Stoa, Zeno of Citium, (336-246 BC), was taught by scholars of both schools, Aristotle's pupil Theophrastus (*ob.* 287 BC) on the one hand and Diodorus and Philo on the other. He in turn taught Chrysippus of Soli (281-208 BC).

Three hundred and fifty years later, Herminus (AD 130-190), who reportedly disagreed with Aristotle on many important points, had two pupils, Alexander of Aphrodisias (AD 160-220), later Professor of Philosophy at Athens, and Galen of Pergamon (AD 129-199), the physician. The two had a good row, naturally on

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[2] I have greatly benefited from reading Father Bocheński's *History of Formal Logic* and the books of Dr Marenbon of Trinity College, Cambridge.

the question of motion, Galen attacking and Alexander defending Aristotle's view that everything which is moved is moved by something. In the best modern academic style this row included not only doctrinal controversy but also personal hostility; and it lasted, in a sense, for centuries: Alexander's line was followed by Averroes and the school of Baghdad, among whom Maimonides wrote that Galen was driven by his superiority in medical matters to fancy himself comparably expert in logic, physics and metaphysics, and so foolishly to gainsay Aristotle. Galen was followed by Avicenna and the eastern school.

That Galen should write on logic is part of a trend, found later among the Syriac speaking sects, whose scholars took up the study and propagation of Greek learning - chiefly Aristotle - from the fourth century onwards, for logic to become a part of medical studies, medicine being seen as a bridge between science and theology.

A contemporary of Galen's, Porphyry, wrote a *précis* of Aristotle; and much more substantial treatises and translations were compiled by Boethius, whose execution in AD 524 marks a halt to the development of logic in the West.

Five years later, in AD 529, the school of philosophy at Athens was closed by the Emperor Justinian on suspicion of fomenting paganism.

Thereafter Alexandria was the place for Aristotelian studies until the burning of the Library under Omar. Heretical Christian (Nestorian and Monophysite) scholars carried Greek logic to Edessa in the fifth century and to Baghdad in the ninth century.

Our knowledge of Greek logic reached us first via Latin summaries, secondly via Arabic translations: Adelard of Bath went in the twelfth century on a seven year trip to Salerno, Syracuse, Tarsus and Jerusalem, and imbibed deeply of Arab scholarship. Similarly Hermann of Carinthia travelled to Toledo in about 1256 to work on the Arabic manuscripts now in the Escorial; and only later through Byzantium.

The advanced state of Arab civilisation may be gauged from the following fine precept found in a thirteenth century Arab treatise on Aristotelian logic: "a syllogism composed of preconceptions is called a sophistry, and its purpose is to silence the opponent."

Among mediaeval logicians we should mention Abelard (1079-1142) who wrote "my logic has made me odious to the world"; and then with the return in around 1150 from the Arabs of the works of Aristotle in Boethius' translations there was a surge in the study of logic, leading to the *Summulae Logicales* of Peter of Spain, (*ob.* 1277) and the *Logica Magna* of Paul of Venice (*ob.* 1429), an enormous work of 180 folios in the pages of which are to be found no fewer than fifteen solutions to the Liar paradox.

It should be emphasised that, contrary to vulgar belief, the Schoolmen added considerably to the achievement of the Greeks. Starting from the term logic of Aristotle they rediscover the propositional logic of the Stoics. They are interested in semantic paradoxes, and the language-metalanguage distinction is beginning to appear. They define and study the linguistic processes of supposition, ampliation, copulation and appellation.

Thereafter, there does seem to be a period of relative inactivity, otherwise known as a decline: the Middle Ages now being over, people were too stirred up to think about logic. Such activity as there was seems to be moving in the direction of the emergence of mathematical logic in the nineteenth century. Leibniz foreshadowed Frege and Bolzano Cantor.

So that is one line of development, in which the concept of motion has been a recurrent problem. There have been other logics: for example, the logic of the Bantu, who have no word for "accident"; let me look at another, independent, school of logic: that of India.[3]

This has two main periods of development, the gap between which is something like the gap between Aristotle and Frege: in the first period, roughly from the fourth to the tenth centuries, there was some rivalry between Jain logic, Buddhist logic and the logic of the Nyāya Sutra (or Logic treatise) which, as I read, directed its arguments to a great extent against Buddhism.

The logic of the second period shows Buddhist influence, though Buddhism itself had by then almost disappeared from India. Of scholars of the Navya-Nyāya, or new Nyāya, I might mention (1) Gangesopādhyāya, of the thirteenth century, (2) Raghunātha Siromani (*c.*1425 − *c.*1500), a pupil of Vāsudeva Sārvabhauma, who would be of great importance were he better known: he apparently forestalled Brouwer in his attitude to negation and Frege in his definition of number; and (3) Mathurānātha Tarkavāgisa (*c.*1600 − *c.*1675).

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[3] I am grateful to Dr Tillotson of Peterhouse for helpful discussions of Indian civilisation.

In *Materials for the study of Navya-Nyāya logic*, by Daniel Henry Holmes Ingalls, Harvard U.P. 1951, we read:

> "The newness of Gangesa is one of style and method rather than of theory. The great revolution in the doctrines of the school comes with Raghunātha: and indeed some Indians use the term Navya-nyāya only of him and his followers."

The third scholar, Mathurānātha, seems to have been a pedantic codifier.

Indian logic, unlike Greek logic, had no variables and developed no formalism. From our point of view, an important characteristic is its obsession, believed to derive from the Buddhist doctrine of exclusion, with negation: Dinnāga, in the fifth or sixth century AD, writes that the meaning of a word consists of a repudiation of the discrepant meaning. The flavour of the second period may be illustrated by the following: in a discussion of the derivation of the proposition that "the mountain possesses fire" from the proposition that "the mountain possesses smoke", the proposition, familiar to all in the West, that there is no smoke without fire is expressed first as "All bodies of smoke occur in a locus of fire" and then analysed as "In smoke there is generic absence of occurrent-ness described by locus of absence of fire, which absence describes a counter-positiveness limited by fireness and contact."

It would be interesting to know to what extent the thought of Ramanujan, whose notion of proof so baffled Hardy as to make him declare that he had none, derived from this tradition.

We must now consider the question of negation.

Given any proposition A, and its negation *not A*, we see four possibilities: neither is true; exactly one; both. The classical position is that exactly one is true, and achieves that through two laws. The Law of Contradiction states that at most one is true: not both A and *not A*. The precept known as *tertium non datur* or the Law of the Excluded Middle says that at least one is true: either A or *not A*. Thus given both these laws, *not not A* is equivalent to A.

The law of the excluded middle is a principle found in many cultures: for example, among the ancients, Chrysippus fought hard for the view that every proposition was either true or false, and instanced the behaviour of dogs in support of his view. A hound in pursuit of its quarry, coming to a branching of the road into three, would be observed to sniff two branches and finding no scent set off down the third *without prior sniffing*. Aristotle, indeed, had had reservations about contingent statements about the future, holding that while the statement "either there will be a sea battle tomorrow or there will not be a sea battle tomorrow" is true and necessary today, neither the statement "there will be a sea battle tomorrow" nor the statement "there will not be a sea battle tomorrow" is true today.

Among the Schoolmen, monks practising detachment, prayer and the presence of God, and therefore likely to have a very different relationship to their language and to external events from that of modern materialists, we nevertheless find Robert Kilwardby (*ob.* September 10th 1279 in Viterbo) writing that "It is to be said that a negation can be negated, and so there is a negation of negation, but this second negation is really an affirmation, though accidentally and vocally a negation. For a negation which supervenes on a negation destroys it, and in destroying it posits an affirmation."[4]

In Indian logic, we find an interesting conflict: whereas believers in the law of the excluded middle can find statements with which they can agree, for in the old Nyāya Kamalasīla (who, however, distinguished three sorts of negation) in the 8th century AD says "simple negation means for example that a cow is not a non-cow", and in the new Nyāya Mathurānātha in the seventeenth century declares that "absence of constant absence of pot is essentially identical with pot.", in between those two scholars, Raghunatha Siromani in the fourteenth century took a most interesting view, namely that *not not A* was not identical with A. He continued, though, to think that *not not not A* is identical with *not A*, which suggests to me (who have not read his works) that he kept the Law of Contradiction while jettisoning *tertium non datur*.

The ideas of Raghunatha were revived in the twentieth century by Brouwer, a Dutch logician. He too denies that *not not A* is identical with A while maintaining that *not not not A* is identical with *not A*. This logical stance is the basis of the mathematical school of *intuitionism*; today, intuitionistic logic is much in vogue in the unexpected world of computer science: the slogan of a leading contributor, Robert Constable, being **the proof is the program**.

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[4] Bocheński, *op.cit.*

We may rephrase the intuitionist position as denying *Tertium non Datur*, while retaining the Law of Contradiction. It need not be the case that *A* or *not A*.

Brouwer's standard example, if you will permit me a little mild mathematics, is this: it is as yet unknown whether there are in the decimal expansion of π seven consecutive 7's. Brouwer contends that to say "Either there are or there are not" is meaningless until we are in a position either to exhibit a computation that establishes the existence of such a string, or to prove that no such string exists. Thus in a way Brouwer goes even further than Aristotle with his sea battle. The classical hound set off down the third path without sniffing; an intuitionist hound would sniff first.

Intuitionism has flourished in Holland, together with gin and tulips. It seems slightly risky to talk of an intuitionist culture, since Brouwer held views about the immorality or indeed impossibility of communication that make the word 'culture' seem hardly appropriate, and which I believe reflect his Calvinist background. "After the age of sixteen", he wrote, "I read nothing." The Universities of Holland at the end of the nineteenth century were apparently sunk in academic torpor — Stieltjes, for example, had to make his career in France, being wholly unable to obtain a post at Dutch Universities — and one can see how an intensely intelligent adolescent such as Brouwer might, surrounded by the sleepy dons of Leyden, come to believe in the impossibility of communication and sink into the morass of solipsism.

Brouwer acknowledged his debt to the Bhagavad-gita, a text of the 4th century BC on morality. But his attempt to see mathematics as an integral part of the life of the mathematician must have startled the honest English pragmatists:

> "Mathematics is an essentially languageless activity of the mind having its origin in the perception of the falling apart of a life moment into two distinct things, one of which gives way to the other, but is retained by memory." [5]

Hardy, who disliked Poincaré's lauding of the subconscious over the conscious, would, one feels, have liked this even less.

At the other extreme from Brouwer's denial of the Law of the Excluded Middle is the dialectician's denial of the Law of Contradiction. This was of extreme importance in the Soviet Union between the Revolution and the rehabilitation of bourgeois logic in 1946. In that country in those years it was held that '*A* and *not A*' is the correct dialectical opinion, for by the time you have determined that *A* is true, things have probably changed and *A* has become false.

"To think dialectically" writes Somerville in his book on Soviet Philosophy, "is to think in terms of movement, change, development, transformation, history, and hence, also, in terms of interconnectedness."

We have again to go back to the Greeks, to Zeno of Elea, regarded by Aristotle as the father of dialectic, and to Heraclitus, who was quite clear that we live in a changing world: "all things are a-flowing", he said, "harmony consists of opposing tensions, war is universal, jurisdiction is strife, everything comes about by strife and necessity".

But there is something very problematic about a wholesale abandonment of constancy: in antiquity, Epicharmus, a Sicilian writer of plays and mimes of the fifth century BC, made merry with the idea that a host might reasonably refuse his guest admission to a party on the grounds that the guest was no longer the person to whom the invitation had been issued.

The word "dialectic" itself is problematic: seven different meanings of it have been identified in the works of Engels. One very attractive meaning is that given to it by Plato, namely the way in which, in the most rewarding discussions, all parties change the meaning of their terms as they talk, ending with a consensus: that is to say, interpretations of words that have absorbed and reconciled both points of view.

This process is capable of great degeneration, seen even in Plato's own progress from debates with substantial opponents through dialogues with yes-men to monologues; in pragmatic England, where ideas are hated, arguments are seen as games with a winning and a losing side.

This sort of dialectic, with its recognition of the creativity of another's mind, is attractive and valuable: Brouwer with his concept of the isolated mind represents the degradation of logic into pure deduction.

Still it is difficult to be a solipsist with a bayonet at your throat, and in the Communist East dialectic, as developed by Hegel Marx and Engels has been taken seriously. We do not expect Governments to occupy

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[5] *South African Journal of Science* **49** (1952–3), pp 139–146.

themselves with such apparently academic concerns as the teaching of logic, nor do we expect mathematics to have political content yet here in the sober columns of the *Journal of Symbolic Logic*, **24** (1959), p 174, is the following report, by J.G.Kemeny, of a lecture given in Hungary in 1952 by a scholar whose name will be known to logicians and computer scientists since it is borne by the class of elementary functions that he defined.

> "In Professor L.Kalmár's lecture it is said that for a long time Church's result on the unsolvability of the decision problem was thought to be inconsistent with dialectical materialism; but in fact this result is not qualitatively different from those of Gödel and hence it is perfectly orthodox dialectical materialism. Kolmogoroff's great contribution to probability theory is said to be the establishment of a dialectical materialist foundation (replacing previous idealist doctrines). The author also exhorts Hungarian physicists not to use capitalist infinitesimals in their work but to employ the socialistic ϵ–δ method in their proofs.
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> In the discussion, Rényi argues that Kalmár exaggerates the importance of mathematical logic; a mathematician is helped much more by a knowledge of dialectical materialism than by a knowledge of mathematical logic. Alexits points out that Church's theorem shows a shortcoming of idealist mathematics, but it in no way restricts a mathematics that is based on dialectical materialism. He also argues that Kalmár is tinged with idealism and has misapplied Marx's and Lenin's deep insights into the nature of mathematics.
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> In reply, Kalmár argues that while dialectical materialism is the one indispensable tool of the mathematician, this does not preclude the possibility of his finding use for mathematical logic."

Actually by 1952 things were calming down behind the iron curtain, and to be accused of idealism was no longer as dangerous as it had been in say 1937. But to understand how a scholarly discussion of mathematics could follow such lines, let us go back a century or so: I have not been able to get my hands on Marx's deep mathematical insights, but here are some from Engels, in his attack on Herr Dühring.

> "In its operation with variable magnitudes, mathematics itself enters the field of dialectics, and it is significant that it was a dialectical philosopher, Descartes, who first introduced this advance in mathematics. The relation between the mathematics of variable and the mathematics of constant magnitudes is in general the same as the relation of dialectical to metaphysical thought. But this does not prevent the great mass of mathematicians from recognising dialectics only in the sphere of mathematics, and a good many of them from continuing to work in the old, limited metaphysical way with methods that have been obtained dialectically. [p 137]
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> The mathematics of variable magnitude, whose most important part is the infinitesimal calculus, is in essence nothing other than the application of dialectics to mathematical relations [p151]
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> I negate the negation, i.e. I integrate the differential formula [page 154] Once again therefore it is no one but Herr Dühring who is mystifying us when he asserts that the negation of the negation is a stupid analogy invented by Hegel, borrowed from the sphere of religion and based on the story of the fall of man and redemption. Men thought dialectically long before they knew what dialectics was, just as they spoke prose long before the term *prose* existed.
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> The law of the negation of the negation which is unconsciously operative in Nature and History, and until it has been recognised, also in our heads, was only clearly formulated for the first time by Hegel. And if Herr Dühring wants to use it himself on the quiet and it is only the name which he cannot stand, let him find a better name, But if his aim is to expel the process itself from thought, we must ask him to be so good as first to banish it from Nature and history and to invent a mathematical system in which $-a \times -a$ is not a^2 and in which the differential and integral calculus are prohibited under severe penalties. [p159]"

Lenin in his *Conspectus of Hegel's lectures on the History of Philosophy* writes:

> "It is that characterisation of motion which correctly expresses the continuity of time and space, whereas the concept of motion as the presence of a body one place at one time, in another place at another time describes only the result of motion and does not contain an explanation of motion itself."[6]

Naturally dialectic, being destructive of all loyalty, was a concept of great use to those who overthrew the Tsar, and it continued after the Revolution to be the basis of much of Soviet thought. In a country run on the principle that all morality and all truth are comprehended in the Party, dialectic could be used to justify, regardless of consistency, pacts with Germany, wars on Germany, free enterprise, collectivisation, and so on.

In the twenties a debate developed between two groups of Marxists, the mechanists, who thought there had already been enough dialectic and the dialecticians, who wanted more.

In a debate in Kharkov in 1930, the mechanist Professor S.Yu.Semkovsky was accused of underestimating Hegel. He replied that he fully recognised all three of the Hegelian laws of dialectic. This statement was greeted with indignant cries by the dialecticians: "What Soviet scholar", some shouted, "would dare in the thirteenth year of the Bolshevist revolution to deny the three laws of Hegelian dialectic ?" [7]

These three laws are, it will be remembered : the Law of the Contradiction of Things, sometimes called the Conflict and Unity of Opposites – *in my beginning is my end*; the Law of the Negation of the Negation, which we have seen Engels illustrating with his grain of barley; and the Law of the Transition from Quantity to Quality, – the *heap of sand* argument.

Professor Asmus, whom we shall meet again, wrote in 1929 against the Law of the Excluded Middle, agreeing with Hegel that it was banality and nonsense.

In the thirties, with the very rigidity that dialectic is meant to combat, formal logic was condemned in the Soviet Union in these terms:

> "Formal logic is always a most trustworthy weapon in the hands of the predominant exploiting classes. a bastion of religion and obscurantism" (from a 1934 work on *Dialectical and Historical Materialism*)

> "the laws of formal logic are opposed to the law of dialectical logic. formal logic is empty, poor, abstract, for the laws and categories which it sets up do not correspond to objective reality." (*Concise philosophical Dictionary*, 1940)"

and from the *Large Soviet Encyclopaedia* (1936):

> "formal logic is a metaphyical form of thinking ... the lowest stage in the development of human knowledge, replaced by dialectic as the highest form of thinking.

> "Formal logic, as we have seen, is not included in dialectic, but is displaced refuted and overcome by it.

> "Just as materialist dialectic which is the highest product of the whole productive and revolutionary scientific practice of the proletaria serves as a powerful weapon in the struggle for the general line of the Party, so formal logic is also the methodological basis for the anti-Leninist deviations in the All-Union Communist Party (VKP(b)) Formal logic thinking is a characteristic trait of Menshevism frequently noted by Lenin who levelled devastating dialectical criticisms at the Menshevik formal-logical deductions of syllogisms and sophisms."

The celebrated philologist Nikolai Marr levelled these charges at formal logic:(*Selected Works* 1934)

> "Formal logic, a product of class thinking, together with the class that created it, is swept away by the dialectical materialist thinking of the proletariat in which thought gains ascendancy over language."

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[6] *Collected Works* Volume 38, p 259 or thereabouts.

[7] For this and for much of the information below about academic conditions in the U.S.S.R. I am greatly indebted to *Logic and Dialectic in the Soviet Union* by Alexander Philipov, Research Program on the U.S.S.R., Varangian Press, New York, 1952.

So by the early thirties, formal logic was well under attack, and Soviet logicians such as Markov had to give their papers titles suggesting that the content was not logic but group theory or other harmless concern.

But, as one might expect from the principle of the unity of opposites, the laws of dialectic embodied the seeds of their own overthrow. If all is change, should not change also overcome that most perfect Bolshevik *régime* established in October 1917 ? This conundrum, whether the process of constant change and dying away referred also to the Soviet system, was put to A.S.Kocharovskaya, Professor of Dialectical Materialism at the Kharkov Technical Institute by her pupils. She smiled graciously and said she would have to think about it. For this reply she was dismissed from her position.

Nevertheless the difficulty remained: clearly the Party would have to declare that dialectic, having done its work, was no longer needed, and in 1938 the *History of the Communist Party of the Soviet Union (Bolshevik) – Short Course* was published by Stalin with the approval of the Party Central Committee. Stalin remarks in this work that

> "if everything is found in continuous movement, in the dying away of the old and the growth of the new, then there are no more 'immutable social systems, eternal principles of private property.' "

Chapter IVB, Section 2 of this work contained remarks on dialectical and historical materialism that were intended to answer all the questions that might arise in connection with the applicability of dialectic to the Soviet system. Philipov writes:

> "It is astonishing, for example, that the entire section contains no criticism of the laws of formal logic, a criticism in which dialecticians had previously seen a proof of dialectic. Also absent is any examination of the concept of motion, which had served dialectical materialism both as the most telling refutation of the laws of formal logic and as a bridge to the laws of dialectic. In addition there is no mention of the Law of the Negation of the Negation."

Thus not even the laws of dialectic are permanent. Formal logic, with its capacity for keeping kings on their thrones and commissars in their dachas, began to return. In 1946, to meet the immediate need, the tenth, 1918 edition of a *Textbook of Logic* by a Tsarist scholar, Professor G.I.Chelpanov, was reprinted by the Moscow State Publishing House for Political Literature. Its reappearance was officially explained as due to the "personal instructions of Comrade Stalin and the Central Committee of the Communist Party on the teaching of logic", which were to put an end to the "abnormal situation" in which "for a long time many of our philsophers had had an erroneous nihilistic attitude toward logic." In the same year, Professor Strogovich's *Logic* was published and used in the Academy of Military Law. New textbooks followed, one by V.F.Asmus in 1947 and in 1949 a revised edition of Strogovich's text.

These two "modern" books differ in tone: Strogovich states that modern bourgeois logic, like all bourgeois philosophy, is decadent, has a class character, pursues the aim of preserving the bourgeois social system. This hostility is reflected in his choice of material: he quotes copiously from the works of Marx, Engels, Lenin Stalin, Molotov and the U.N. speeches of Vyshinsky. Asmus draws on more diverse and less politically charged materials, for which he was in due course officially rebuked.

The problem of reconciling logic and dialectic was starting to be felt, and the Georgian Bakradze wrote:

> "Logicians place the burden of deciding this question on the shoulders of authors of textbooks on dialectical materialism. Authors of textbooks on the dialectical method apparently feel that this problem is in the province of logicians and on this basis also remain silent about it."

"Soviet thinkers" (i.e. commentators on Party-prescribed ideas) immediately began to attack the dialecticians. Professor Kursanov accused Hegelian logic — so admired before the Central Committee's decree — of "indefiniteness, diffuseness and vagueness". M. Rosenthal suddenly attacked Zeno – till lately held in such esteem – for absolutizing "continuity and discontuinuity, setting them against each other, proceeding now from absolute continuity and now from absolute discontinuity." Professor Tavanets, daringly, supported J.S.Mill's theory of judgements: "anyone who admits that logical judgments have an extra-conceptual reference is a materialist" — a statement to complement the Soviet view that "any logician who is concerned solely with the consistency of ideas is an idealist."

The rehabilitation of formal logic proceeded:

courses in formal logic were introduced into secondary schools and universities; departments of logic were set up in leading state universities; a section of logic was added to the Institute of Philosophy of the Academy of Sciences; logic teachers were recruited and textbooks were commissioned. ...

A number of meetings were held to discuss the relationship of dialectic (or dialectical logic) to formal logic. The principal questions discussed were (1) Are formal logic and dialectic distinct disciplines, or is one included in or dependent on the other ? (2) Do dialectic and formal logic differ with respect to subject matter, or method, or both ?[8]

Naturally, the dialecticians fought back. In a report[9] of a 1948 congress we read a summary by the reviewer, Max Black, of a paper by G. Alexits and E. Fenyö entitled *Les fondements des mathématiques et la philosophie du matérialisme dialectique*:

The formalism of Hilbert and his school is the purest form of idealism, and Gödel has shown that Hilbertian mathematics and Brouwerian mathematics are intertranslatable; the results of Gödel and Church concerning unsolvable problems should have shown the idealists the folly of their ways. Mathematical knowledge is a reflection of the movement of matter and real numbers are abstractions from material facts. The foundations of mathematics require the methods of dialectical materialism.

and in a review [10] by G.L.Kline of two monographs on Marx's mathematics by L.P.Gokieli, we read:

Axiomatic logic is criticized from purely Marxist premises. ... the force of the author's strictures is wholly negative. His positive recommendations are based on an intuitionist view of logic derived mainly from Aristotle, Engels and Lenin: logic is an ontological science which "reflects" external reality. "For dialectical materialism logic is bound up with the dialectical character of reality itself, which can be rationally known."

In the second monograph: "the concept of infinitesimal cannot be understood mechanically; it must be interpreted dialectically [11] ... by a twofold negation of zero we obtain not the same zero but a concept richer in content."

In 1948 a decree of the USSR Ministry of Higher Education condemned the work of the Section of Logic at Moscow University for its "perverse, formalist tendency to estrange logical forms from the content of thought, from the living thought of the Soviet people" and stated that

"the ruling classes throughout all history have used science, logic included, to strengthen their class rule, in consequence of which formal logic in antiquity defended the ideology of the slaveholders, in the Middle Ages was the handmaiden of theology. and in capitalist society was adapted by the bourgeoisie to keep the workers in ignorance, in order to hold the oppressed masses captives of bourgeois ideology.".

In consequence of these attacks, Asmus in 1949 had to confess

that his book characterised logic as a science cut off from socialist practice, that he did not show logic to be one of the means of the Communist education of Soviet students, did not point out that the content of the thinking of Soviet people must be socialist reality, ... that he did not characterize logic as a partisan science, that he did not develop the sharp, militant, irreconcilable criticism, indispensable for a Soviet author, of the reactionary, idealistic and formalist doctrines that are spread by present day logicians in capitalist countries, especially the reactionary foreign "logic of relations" ... that his book did not show the true role of Marxism-Leninism in developing the science of logic.

Strogovich, noting with satisfaction Professor Asmus' recognition of his errors, demanded that Tavanets also ackowledge his error in basing the materialistic theory of judgment on the "bourgeois" theories of Mill, Karinsky, and Povarnin.

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[8] *Journal of Symbolic Logic* **17** (1952), p 125.
[9] *Journal of Symbolic Logic* **14** (1949), p 183.
[10] *Journal of Symbolic Logic* **14** (1949), pp 243–4.
[11] This casts doubt on the correctness of Kalmár's attitude to infinitesimals, cited above.

The Party cannot have been pleased with this counter revolutionary trend: clearly the hash of dialectic would have to be settled authoritatively, and in 1950, in the early months of the Korean war, Stalin made his first public pronouncement for five years. It was *On Marxism in Linguistics*, mentioned in a scene in Solzhenitsyn's novel *The First Circle*. Like parts of *Ulysses* it is cast in the form of question and answer.

> *Is it true that language is a superstructure on the base ?*

> "No, it is not true."

Stalin then proceeds, helpfully, to give a definition of these Marxist terms:

> "The base is the economic structure of society at a given stage of its development. The superstructure consists of the political, legal , religious artistic and philosophical views of society and the political legal and other institutions corresponding to them.

> "if the base changes or is eliminated, then following this its superstructure changes or is eliminated.

> "the base creates the superstructure precisely in order that it [the superstructure] may serve it [the base].

> "Language is not a product of one or another base .. Language has been created to serve society as a whole.

> "Language is connected with man's productive activity directly .. Language or rather its vocabulary is in an almost constant state of change."

He goes on to point out that

> "the Russian language did not change after the revolution",

and concludes that:

> "(a) a Marxist cannot regard language as a superstructure on the base;
> (b) to confuse language and superstructure is a serious error."

In response to a question of Comrade Krasheninnikova,

> *Your article convincingly shows that language is neither the base nor the superstructure. Would it be right to consider that language is a phenomenon peculiar to both the base and the superstructure, or would it be more correct to regard language as an intermediate phenomenon ?*

Stalin answers,

> "Briefly, language cannot be ranked either among bases or among superstructures. Neither can it be ranked among "intermediate" phenomena, as such "intermediate" phenomena do not exist."[12]

> *Did Pravda act correctly in inaugurating an open discussion on questions of linguistics ?*

> "It did.

> The discussion has brought out that in linguistic bodies both in the capital and in the republics a regime has prevailed which is alien to science and men of science. The slightest criticism of the state of affairs in Soviet linguistics, even the most timid attempts to criticize the so-called "new doctrine" in linguistics, were persecuted and suppressed by the leading linguistics circles. Valuable workers and researchers in linguistics were dismissed from their posts or demoted for being critical of N.Y.Marr's legacy or expressing the slightest disapproval of his teachings. Linguists were appointed to responsible posts, not on their merits, but because of their unqualified acceptance of N.Y.Marr's theories."

We see that Marr has fallen. Stalin continues:

> "It is generally recognised that no science can develop and flourish without a battle of opinions, without freedom of criticism. But those generally recognised rule was ignored and flouted in the most outrageous fashion. A tight group of infallible leaders, having insured themselves against all possible criticism began to act arbitrarily and highhandedly.

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[12] That is of course a modern instance of the law of the excluded middle.

> "To give one example: the so called Baku course, by N.Y.Marr, unreservedly included in the lists of manuals recommended to students. This means that the students were deceived by having a rejected "course" presented to them as a first class text-book."

And then a sentence at the sight of which many faces must have turned white:

> **"If I were not convinced of the integrity of comrade Meshchaninov and the other linguistic leaders, I would say that such conduct is tantamount to sabotage."**

He continues:

> "N.Y.Marr introduced into linguistics the incorrect non-Marxist formula that language is a superstructure, and got himself into a muddle and put linguistics into a muddle. Soviet linguistics cannot be advanced on the basis of an incorrect formula.

> "I think that the sooner our linguistics is rid of N.Y.Marr's errors the sooner will it be possible to extricate it from its present crisis.

> "An overestimation of semantics and abuse of the latter led N.Y.Marr to idealism."

> *In your article you quite correctly qualify Marr as a vulgarizer of Marxism. Does this mean that linguists should discard the whole of the linguistic legacy of Marr ?*

> "Of course the works of N.Y.Marr do not consist only of errors. N.Y.Marr conscientiously and capably studies individual languages.

> "N.Y.Marr and his disciples accuse of formalism all linguists who do not accept N.Y. Marr's new doctrine. This is of course frivolous and foolish.

> "I think that formalism was invented by the authors of the new doctrine to make it easier for them to struggle against their opponents in linguistics."

The difficulties of academic life under such a *régime* are apparent: Stalin quotes a letter from Comrade Kholopov, who, he says, is driven to despair.

> "From your article",

the Comrade writes in his letter to Stalin, and one must here admire the delicacy with which he addresses the god that has contradicted himself,

> "I understand that the crossing of languages can never result in some new language, whereas before the article I was firmly convinced that, according to your speech at the sixteenth congress of the communist Party of the Soviet Union, languages will fuse into one common language under communism".

Comrade Stalin's "historic" declaration on linguistics was immediately proclaimed by *Pravda* to be the contribution of a genius. Though the Deity had spoken only of linguistics, logicians were not slow to convert his utterances into a licence to develop logic. Osmyakov, Head of the Philosophy Division of the Administration for the Teaching of the Social Sciences of the USSR wrote:

> "Comrade Stalin's work on linguistics opens up the broadest perspectives not only for the development of linguistics but for other sciences, particularly logic. Now many questions of logic become so clear, that the existing stagnation in this science can be fully overcome, and the elaboration of problems of logic must move ahead at full speed.

> "The logic of thinking, like language, cannot be considered a class partisan logic: it is one for all people, independent of their national or social affiliation. The logic of thinking serves identically all sciences, all classes, all parties, all societies, the whole of mankind."

An editorial in *Problems of Philosophy* said:

> "The remarkable works of J.V.Stalin on questions of linguistics provide a geniuslike resolution of problems having a direct relation to the science of thinking, to logic. The new Stalinist works point out to our philosophers the path to the creative elaboration of questions of logic."

The Head of the Section of Logic of Moscow University declared that

> "J.V.Stalin's works on linguistics provide the key to the solution of all the questions of logic
> which have been the subject of lengthy debates."

"It seems to me", Professor Strogovich said, "that from Comrade Stalin's work *On Marxism in linguistics* a conclusion with regard to the laws of thought can also be drawn, that they, like language, are not a superstructure and do not have a class characteristic."

Thus, with Marxism remaining the official Soviet world view, the logicians were left with the difficult task of reconciling Marxism with traditional "bourgeois" logic.

Poor Bolshevik logicians !

They cannot repudiate logic, writes Philipov, because the unchallenged supremacy of dialectic would threaten the stability of their social and political order. They cannot repudiate dialectic, because a full recognition of logic would show up the inconsistency of their constantly changing policies. Thus they are condemned to continuing vacillation.

There is a striking passage in Philipov where he cites details of the career of Asmus to illustrate the unbearable and humiliating character of the conditions under which Soviet logic was expected to develop.

Asmus graduated from Kiev before the Revolution and was retained by Professor A.N.Gilyarov, author of a monograph on the Greek sophists, for training as a professor in the department of Philosophy. After the Soviet *régime* came into power, professor Gilyarov thoroughly understood the impossibility of continuing his work in philosophy under the Bolsheviks and, although he retained his membership in the Ukrainian Academy of sciences, occupied himself not with philosophy but with the preparation of perfumes.

Asmus, however, entered into Soviet philosophy, and, since dialectic was completely in the ascendancy at the time, he began to attack and revile formal logic. Since he had been retained by the University in pre-Soviet days, he certainly understood the absurdity and injustice of his charges.

When logic was finally introduced in the Soviet union, he completely forgot his earlier condemnation of logic and, just as if nothing had happened, wrote a textbook on formal logic. All went very smoothly at the beginning: the State Publishing House for Political Literature published his book in an edition of 100,000 copies. But then it turned out that logic was to be "partisan", "political" etc., and since these features were nowhere to be found in his *Logic*, Asmus forthwith "admitted his errors" and "promised to rectify them," recognizing that logic was a "partisan" and "political" science. Then with Stalin's revelations on language, it again appeared that logic was "non-partisan" and "non-political", and Asmus was compelled to retract his earlier retraction.

But under a totalitarian *régime*, if it is dangerous to speak, it is also dangerous to be silent. So

> "in November 1950, the editors of *Voprosy filosofi* invited Soviet philosophers and logicians
> to air their views in print, reminding them of Stalin's statement[13] that "no science can
> develop or mature without a clash of opinions and freedom of criticism."
>   The articles under review represent the first response to this appeal. The authors
> devote a good deal of effort and ingenuity to exegesis of key passages in Engels and
> Lenin."[14]

From the reviewer's summary of the various articles it will be seen how Stalin's pronouncement imposed Marxist categories on the debate, and it will be sensed how dangerous in such times is it to articulate a new idea. Mitin, for example, finds it prudent to end his contribution to the debate with the following doxology:

> "Glory to the great leader and teacher of the workers, the geniuslike scholar and warrior,
> Joseph Vissarionovich Stalin !"

Tavanets is concerned to refute the "idealistic" errors committed by both Soviet and western logicians.

Strogovic, following Engels, had asserted in his textbook that formal logic deals with the "simple everyday relations of things" relations which remain stable and self-identical "for all practical purposes" over a given period of time, so that the law of identity holds. But, according to Strogovic, in the case of more complex relations, formal logic fails and must be supplanted by dialectic.In fact dialectic is theory of knowledge. Stalin's statement that language is not part of the ideological superstructure may be applied to logic;

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[13] cited above

[14] *Journal of Symbolic Logic* **17** (1952), p 125.

the forms of logical thought are not class forms (as was asserted in the logic curriculum published by the Ministry of Higher education in 1949); they are universal forms.

Bakradse, after Stalin's declaration, also denied the class character of logical laws.

> "The forms of logical, i.e. valid, consistent thought, certainly are non-class forms. The thinking of the ancient Greeks who first discovered and established these rules was subject to these laws and rules; we the Soviet people make use of these forms; ... the forms of logical thinking are universally human.

> "There can be no doubt that all knowledge of the simplest or of the most complex relations always proceeds in conformity with the laws and rules of logic."

> Professor Cherkessov:

> "The universal character of the rules and laws of formal logic is perfectly obvious .. Only ignorance in the field of Marxism and a complete misunderstanding of the nature of the forms and laws of thinking can explain the attempt of some comrades to call formal logic a partisan class science (either bourgeois or Soviet proletarian and Marxist). For all this I am indebted to Comrade Stalin ... It is surprising that any comrades want to create a new Soviet Marxist logic when one has already been created by Marx and Engels .. the highest truly scientific logic is Marxist dialectic."

Osmakov distinguishes between the "logic of thought" and the "science of logic", asserting that "the former has remained unchanged since the time of Aristotle, and indeed throughout human history, whereas the latter is a part of class science and changes with socio-economic development. ... Formal logic is a special science of thought; dialectic is a general science, the methodology of all the sciences, including logic."

Popov appears to feel the difficulties involved in the transfer of the Hegelian concept of "contradiction" as an ontological category from an idealistic to a materialistic ontology.

Spasov writes that the laws of formal logic are not sufficient to ensure correct thinking for they fail to capture the flux and development of phenomena.

Anovskaa considers that formal logic is to dialectic as arithmetic is to algebra and that Hilbert and Ackermann's error lies in interpreting mathematical logic as a true logic rather than an apparatus for formalising mathematical calculi and proofs. Hilbert's attempt to finitize his "metalogic", she insists, has proved a complete failure.

Ostroub calls "A is A" a relative truth; the absolute truth is "A is A and *not A*". Osmakov, he says, is wrong: language and logic are quite different. Marxist logicians insist on retaining the category of substance. Bourgeois logicians regard deductive inference as the only scientific method. ... even mathematics has an empirical basis.

A.D.Aleksandrov, who seems to be the most intelligent contributor to the discussion, writes that thinking is correct only if it provides a correct reflection of reality. Dialectic and inductive inference as well as Aristotelian logic should all be included under logic. .. It is ... difficult ... to identify the stable elements in the complicated flux of interconnected phenomena. Formal logic is not restricted to any particular subject matter, but its use must be justified in any given case — i.e. it must be shown that the objects in question remain stable and distinct over a specified time interval. This justification cannot be provided within the system of formal logic. ... Concepts develop ... in the process of deductive inference.

Aleksandrov admits the contradiction involved in the assertion that a concept changes between premises and conclusion, but he calls this a "vital" contradiction as distinguished from an "absurd" contradiction. Thinking, as it develops, bursts through the framework of formal logic; e.g. in the paradox of Epimenides the Cretan.

In 1951, Aleksandrov published two further articles entitled *Leninist dialectic and mathematics* and *On idealism in mathematics*, in which, to quote from the review by G.L.Kline,[15]

> the author undertakes to apply Lenin's views on dialectic to the historical development of mathematics, representing this development as a dialectical "conflict and unity of opposites", *viz* of the abstractly discrete and the abstractly continuous. Like Lenin he em-

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[15] *Journal of Symbolic Logic* **18** (1953), p 271

phasizes that the unity of these polar opposites is temporary whereas their conflict is absolute.

In the second paper, there is a popular critique of "idealism" in "bourgeois" philosophies of mathematics. The author distinguishes three species of "mathematical idealism": (1) set-theoretic idealism, which springs from Cantor and is essentially Platonic; (2) Hilbert's formalism, which is of Kantian inspiration, and (3) Brouwer's intuitionism, which the author regards as a variety of subjective idealism.

All of these, it is maintained, develop and flourish in capitalist society, serving the class interests of the bourgeoisie; they result in attempts to reconcile religion and science, and end in mysticism and obscurantism. The author does not mention J.S. Mill, who is regarded by Soviet writers as a "bourgeois" thinker but whose views of the relationship of mathematics to the "real world" is much closer to Lenin's view than to that of the idealists.

Luzin is criticised for "idealism", Kolmogorov for "formalist tendencies". [16]

He asserts that the "crisis in mathematics" precipitated by the paradoxes of set theory cannot be overcome in capitalist conditions. He further asserts that crises of science are impossible in socialist society, since Marxism is a scientific ideology which harmonizes with the objective content of science.[17]

The hostile passage of the 1940 edition of the *Concise Philosophical Dictionary* quoted on page 7 above is replaced in the 1951 edition by this:

> "The laws and rules of formal logic are the laws and rules of the natural processes of thought. Formal logic teaches us to think correctly, by avoiding ambiguity and inconsistency. ... If thinking is not internally consistent, scientific knowledge and reasoned discourse become impossible."

This changed attitude spread to the satellites: at a philosophical congress in Jena in November 1951, Klauss and Hoffmann read papers on the relationship between formal and dialectical logic, both generally taking the line that formal logic deals with relatively stable phenomena, and dialectic with changing ones. Criticism was mild, Klauss being criticised for neglecting Hilbert's positive achievements (while, correctly, attacking his "idealist" ones) and Hoffmann being criticised for not making use of Stalin's paper on linguistics. "Since Stalin does not consider language part of the superstructure, the same should apply to logic. The consequences of this position should be explored."

Thus by the 1950's the height of the storm, so far as logic was concerned, is past. The puzzling thing about the timetable is that while formal logic was being restored to favour, genetics and other sciences continued to be under attack.

Finally here is Aleksandrov writing in 1966, after the fall of Khruschev:

> "My professional activity involves mainly the proof of new theorems. And for me Marxist-Leninist philosophy is an unquestioned guide in comprehending general questions of my science. Dialectical materialism, needless to say, does not offer methods for solving specific problems in mathematical science, but it indicates true reference points for searches for scientific truth and arms one with methods for elucidating the true import of theories and the content of scientific concepts.

> "I could cite examples showing how philosophy helps one master the mathematical theory of infinite numbers, Einstein's theory of relativity or quantum mechanics, but this would require the introduction of complicated, specialised concepts. I shall say only that as a student studying in a physics department I was able to undestand quantum mechanics to a significant degree thanks to the fact that at the same time I was studying philosophy which helped me to comprehend this difficult theory in the spirit of dialectical materialism.

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[16] a reference to his axiomatic reduction of probability to measure theory

[17] a prudent echo of Stalin's view that "explosions" are impossible in the Bolshevik state.

> "Unfortunately in our scientific milieu one still encounters a pragmatic or, to put it more simply, a narrow-minded view of science in which people turn away from its general and philosophical questions, reducing it to the solution of individual particularised problems. This view is a decided hindrance to the posing of fundamental theoretical problems and to searches for new lines of research."

We are left with many questions. We have seen a political attempt to integrate mathematics with life. Can such an attempt ever succeed ? Is there a mathematical rather than a political definition of logic ? Can there be a mathematical dialectic ?

The school of Lawvere attempts to achieve a mathematical dialectic with its doctrines of topos theory. Comically, the logic underlying much of topos theory is often intuitionistic: a wholly unexpected link between the self-absorption that produced intuitionism and the tyranny that promoted dialectic.

I hope that this study of the difficulty of pursuing Truth in the face of hostile Authority has been of interest to the votaries of Andrew Perne.