

Further remarks on Bourbaki

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A preliminary version of my essay “The Ignorance of Bourbaki” was read to an undergraduate mathematical society, the Quintics, in Cambridge on October 29th, 1986, and that version was printed in the Cambridge undergraduate mathematical periodical *Eureka*, shortly afterwards.

A revised version appeared in *Physis* in 1991 and in the *Mathematical Intelligencer* in July 1992; the text of that version may be found at

<http://www.dpmms.cam.ac.uk/~ardm>

in the files *bourbaki.dvi* or *bourbaki.ps*.

That revised version has been reviewed by Professor Sanford L Segal of Rochester for the *Zentralblatt*. Light has been shed on numerous points in this debate by recently published remarks by two members of Bourbaki: the article by Armand Borel entitled “Twenty-Five years with Nicolas Bourbaki, 1949–1973”, in the *Notices of the American Mathematical Society*, March 1998, pp 373 – 380, and *The Continuing Silence of Bourbaki — an interview with Pierre Cartier, June 18, 1997*. in a recent issue of the *Mathematical Intelligencer*.

Alas, these items have come to my notice too late for me to incorporate appropriate changes in the Hungarian translation of the 1992 version of *The Ignorance* that is due to appear this month. So I aim to show here that my first thesis is confirmed by the statements of Cartier, and that my second thesis is not refuted by the criticisms my article has received.

I am grateful for factual help given by Professor Segal during the preparation of this reply to his criticism.

PLAN OF MY ANSWER:

- I. The purpose and the two theses of my essay.
- II. Answers to fine points raised by Professor Segal.
- III. My first thesis supported by recent revelations.
- IV. Bourbaki and the possibility of intellectual terrorism.
- V. Ethical questions raised by Professor Segal.
- VI. My second thesis: Bourbaki not an adequately functioning individual.

1: The purpose and the two theses of my essay.

1-0 The first question I ask on reading Professor Segal’s review is this: where in my essay is the diatribe into which he says my essay degenerates ? Like Lord Clive before me, I stand astounded at my own moderation. I am at a loss to understand the meaning that Professor Segal attaches to the word “unfortunately”, with which he condemns some of my phrases. Does he mean that a sober historian would not use such phrases ? I was not aiming to be a sober historian; I was aiming to describe the present-day consequences of the past behaviour of an influential group.

1-1 Perhaps I should state explicitly that my piece was not intended to be a historical study. It was intended to challenge the near-divine status of Bourbaki, which in my view has meant that Bourbaki besides the good they have done, have had a harmful effect. The phrase about the tall oak in the shade of which nothing will grow comes to mind.

1-2 My message is a simple one. No one denies that the Bourbachiste œuvre is a collection of very wonderful books written by very wonderful people; nevertheless I contend that they have given generations of mathematicians a stunted conception of logic. So I am saying to the adventurous youth of today: if you want to know what has been going on in logic in this century do not go to Bourbaki.

1-3 In my essay I asked two questions, which I reformulated in various ways, and offered tentative answers. One version of the first question was why the foundational understanding of Bourbaki did not advance as foundational studies advanced; one version of the second was why the Bourbachistes did not notice the inadequacy of their chosen set theory as a foundation for mathematics. Before commenting further on these, let me turn to some points of detail raised by Professor Segal.

2: Answers to fine points raised by Professor Segal

2.0 I think the division I had in mind was that all the German scholars in the first list were dead by the end of the First World War, whereas all those in the second list survived it. Let me add the dates:

Riemann	1826–1866	1849–1925	Klein
Frobenius	1849–1917	1862–1943	Hilbert
Dedekind	1831–1916	1885–1955	Weyl
Kummer	1810–1893	1898–1962	Artin
Kronecker	1823–1891	1882–1935	Noether
Minkowski	1864–1909	1877–1938	Landau
Cantor	1845–1918	1868–1942	Hausdorff

I agree that Klein is anomalous: he outlived Poincaré but was an established figure before the latter’s meteoric rise by which Klein felt threatened, and hence I think of him as in some sense of an older generation. Of course there really are no generations, the division is arbitrary, as is the choice of date — date of birth, date of death, date of greatest activity, date of greatest influence.

2.1 About the French analysts, for not mentioning whom Professor Segal reproves me: Montel (1876–1975), Fréchet (1878–1973), Emilio Borel (1871–1957), Baire (1874–1932), and Lebesgue (1875–1941) were all in their forties at the end of the First World War, when Dieudonné was but 12, so that it is surely fair to suggest that he and the others arriving on the mathematical scene after that holocaust would see its survivors as of an older generation, despite the remarkable longevity with which many were blest.

2.2 The purpose of these opening paragraphs with their lists of names was to set the scene: were they to be deleted on the grounds that I am playing fast and loose with history, it would in no wise affect my two main points about Bourbaki.

2.3 Professor Segal says that some of my speculations contradict the statements in Weil’s reminiscences (which were not available to me at the time.) Well, I have read interviews with Chevalley, Weil, Armand Borel, etc. etc, and they contradict each other even about the names of the founding members.

Armand Borel names the founders as Henri Cartan, Chevalley, Delsarte, Dieudonné, Weil.

Chevalley gives Borel’s list plus Mandelbrojt and de Possel.

Cavaillès gives Chevalley’s list plus Ehresmann.

Cartier agrees with Armand Borel, and goes on to list subsequent generations as follows:

second: Schwartz, Serre, Samuel, Koszul, Dixmier, Godement, Eilenberg.

third: Borel, Grothendieck, Bruhat, Cartier, Lang, Tate.

fourth: a group of students of Grothendieck, who by then had left in anger.

2.4 I was unaware that Herbrand was a close friend of the founders; but alas he had died by the time they started their crusade; had he not been killed on the mountain he might have brought them to a more balanced conception of the rôle of logic. I do not see, incidentally, why his being a friend means that they would necessarily have been intellectually in agreement with him. I have many close friends who haven’t a clue about mathematical logic, much as I admire their achievements in other domains.

3: My first thesis supported by recent revelations.

3-0 Professor Segal allows that I have a point concerning the Bourbachiste neglect of Gödel. Hurrah. Someone has admitted it. Supposing that Gödel is indeed mentioned in the 1950's version, that means that his theorem had been in print for more than twenty years before the Bourbachistes noticed it. If one counts in biological generations, that is one; and in mathematical generations it may be more.

3-1 This relates to the running argument in FOM: do discoveries in logic affect mathematics ? I think they do; many mathematicians go into extraordinary contortions in order to maintain the belief that they do not.

3-2 Cartier in his *Intelligencer* interview makes numerous thoughtful points; for the purpose of the present article the most significant are these:

“Bourbaki never seriously considered logic. Dieudonné himself was very vocal against logic.”

“Dieudonné was the scribe of Bourbaki.”

Borel in his *Notices* essay confirms the dominant rôle of Dieudonné: he mentions shouting matches, generally led by Dieudonné with his stentorian voice, and says

“There were two reasons for the productivity of the group: the unflinching commitment of the members, and the superhuman efficiency of Dieudonné.”

3-3 [A peripheral question: Borel says that there were no majority votes, and that all decisions had to be unanimous. I should love to know: were people ever expelled ? or did they just leave if they felt out of place ?]

3-4 After those statements I think that the finger points at Dieudonné. In his last book, *The Music of Mathematics* he makes the same mistake that he made in his position papers of fifty years previously: he went to his grave believing that truth and provability are identical.

That is an intuitionist position: so I confess to a feeling of glee when I found the following passage in the interview with Cartier:

“The Bourbaki were Puritans, and Puritans are strongly opposed to pictorial representations of their faith. The number of Protestants and Jews in the Bourbaki group was overwhelming. And you know that the French Protestants especially are very close to Jews in spirit. I have some Jewish background and I was raised as a Huguenot. We are people of the Bible, of the Old Testament, and many Huguenots in France are more enamoured of the Old Testament than of the New Testament. We worship Jaweh more than Jesus sometimes.”

My reason for glee is this: I made during a lecture at Oxford in 1976 some remarks on a possible connection between religious and mathematical positions; they are summarised in the text of that lecture in the Oxford volume edited by Gandy and Hyland. Put crudely, my equations were Platonism = Catholicism; Intuitionism = Protestantism; Formalism = Atheism; Category Theory = Dialectical Materialism.

For saying that, I was exposed to derision from certain quarters, though more recently people have been kind enough to say they found the remarks interesting. I think the above paragraph from Cartier vindicates me.

3-5 The above disclosures concerning Dieudonné confirms the quotation from Quine's autobiography that I circulated previously. Here it is again.

“A Logic Colloquium was afoot in the Ecole Normale Supérieure. [...] Dieudonné was there, a harsh reminder of the smug and uninformed disdain of mathematical logic that once prevailed in the rank and vile, one is tempted to say, of the mathematical fraternity. His ever hostile interventions were directed at no detail of the discussion, which he scorned, but against the enterprise as such. At length one of the Frenchmen asked why he had come. He replied ‘*J'étais invité.*’ ”

[I should say that I have received this week an eye-witness account of the seminar concerned, which suggests that Quine may have been over-reacting to Dieudonné's characteristic behaviour.]

3-6 To me it now seems certain that there was a bias against logic; it may be that the bias was due solely to one extremely energetic man, but there is a hint in Quine's remark that Dieudonné was not the only opponent.

3-7 In my essay I wondered whether these attitudes might stem from the influence of Hilbert or from some nationalist or chauvinist feeling, and Professor Segal suggests that I am thereby contradicting myself.

3-8 Perhaps I should first state that I see a distinction between nationalism and chauvinism. Consider, for example, Janiszewski, who at the end of the First World War called for a small poor country to make its mark in foundational studies: I see him as a Polish nationalist but not a chauvinist. It is one thing to say "Good things are going on elsewhere in the world: let us try and do as well or better." It is another to say "Everything that is worth knowing is known by us; let us ignore the activities of others".

3-9 Now Cartier's interview makes it clear that Hilbert and German philosophy were held up as models by Weil and others:

"The general philosophy as developed by Kant. Bourbaki is the brainchild of German philosophy. Bourbaki was founded to develop and propagate German philosophical views in science. All these people ... were proponents of German philosophy."

3-10 So I really do not see that there is a contradiction between wishing to strengthen French mathematics and saying that the Germans do it better. One might say that the Bourbachistes were nationalist but not chauvinist. Further evidence comes from "Claude Chevalley described by his daughter", where she says that the Bourbaki movement was started essentially because rigour was lacking among French mathematicians by comparison with the Germans, that is, the Hilbertians.

3-11 I am delighted that Professor Segal should note my footnote commenting on the dearth of logic in England; the response I have had from leading English academics to that has resembled the wriggling of tobacco companies confronted with evidence of the dangers of smoking. Perhaps one day someone will do something. It makes a sad contrast with the positive response given in Poland to Janiszewski's manifesto.

4: Bourbaki and the question of intellectual terrorism.

Perhaps the most moving of the comments I have received from readers of *The Ignorance of Bourbaki* is one that came from the holder of a (C4) chair at a leading German University, who told me that as a young man he had been reduced to a state of intellectual paralysis by reading Bourbaki and that he had had to retire from mathematics for six months before making a fresh start. It may not be an exaggeration to say that he was thrilled to find support in my essay for the notion that it is not necessary to worship at the Bourbachiste shrine in order to do serious mathematics.

That it might ever have been thought so necessary can be divined from fleeting remarks by Miles Reid in his book *Undergraduate Algebraic Geometry*, London Mathematical Society Student Texts, 12, first published by the Cambridge University Press in 1988. I quote from the historical remarks on pages 114–117 of the 1994 reprint, which provide independent evidence of unwholesome tensions within the mathematical community.

"Rigorous foundations for algebraic geometry were laid in the 1920s and 1930s by van der Waerden, Zariski and Weil. (van der Waerden's contribution is often suppressed, apparently because a number of mathematicians of the immediate post-war period, including some of the leading algebraic geometers, considered him a Nazi collaborator.)"

"By around 1950, Weil's system of foundations was accepted as the norm, to the extent that traditional geometers (such as Hodge and Pedoe) felt compelled to base their books on it, much to the detriment, I believe, of their readability."

"From around 1955 to 1970, algebraic geometry was dominated by Paris mathematicians, first Serre then more especially Grothendieck."

"On the other hand, the Grothendieck personality cult had serious side effects: many people who had devoted a large part of their lives to mastering Weil foundations suffered rejection and humiliation. ... The study of category theory for its own sake (surely one of the most sterile of all intellectual pursuits) also dates from this time."

“I understand that some of the mathematicians now involved in administering French research money are individuals who suffered during this period of intellectual terrorism, and that applications for CNRS research projects are in consequence regularly dressed up to minimise their connection with algebraic geometry.”

Let us set against Reid’s remarks a comment of Armand Borel:

“Of course there were some grumblings against Bourbaki’s influence. We had witnessed progress in, and a unification of, a big chunk of mathematics, chiefly through rather sophisticated (at the time) essentially algebraic methods. The most successful lecturers in Paris were Cartan and Serre, who had a considerable following. The mathematical climate was not favourable to mathematicians with a different temperament, a different approach. This was indeed unfortunate, but could hardly be held against Bourbaki members, who did not force anyone to carry on research in their way.”

I wonder if there is an element of complacency in Borel’s statement that no-one was forced to carry on research in the Bourbaki way. This opens a theme that is difficult to discuss, but I believe is necessary to do so.

Suppose it were the case that over a certain period in numerous universities the Bourbachistes seized power and pursued a policy of denying jobs to non-Bourbachistes. How would one obtain evidence of that ? The poor non-Bourbachistes, being excluded from employment which would permit them to research would be likely to move away from universities and find jobs in industry or elsewhere, and indeed to lose touch with research mathematics. So they would be excluded from any figures that might be produced. People would be saying that the Bourbachiste view is the standard one; what would not be said is the subtext, that that state of affairs has come about because the opposition has been suppressed. In such a case there would be a political component to what Graham White calls mathematical practice.

So I should very much like to hear from anyone who believes that unfair pressure of the kind Armand Borel says does not exist has been brought to bear upon them; in whatever degree of confidence they would like.

4-0 Professor Segal says that I am unhappy with the neglect of logic by mathematicians. I wonder whether I dare to be more specific or will I again be accused of using “unfortunate” terminology ? It is not the neglect — surely all are free to be as ignorant as they choose — to which I object but the interference by the high-placed ignorant with the teaching of logic to those who wish to learn it, and the denial, through the mechanism farcically known as “peer review”, of research funds for work in this area.

5: Ethical questions raised by Professor Segal.

5-0 Now let me comment on what Professor Segal calls “the reprehensible practice of anonymous citation.” This is indeed serious: I was brought up to tell the truth and shame the Devil. It has cost me dear.

So what should I do when someone offers comments that I find interesting, but asks not to be named ? Many people do not dare openly to challenge the Bourbachiste hegemony for fear of losing their livelihood; so I do not think I should betray the identity of my correspondents. I was warned not to publish my essay when I first drafted it, as I was told I would be “murdered” by the Bourbachistes. Professor Segal’s review is the first opposition to my thesis that I have seen in print; but to my certain knowledge Bourbachistes have intrigued against me covertly, to the detriment of my career.

5-1 Let me give another example. A mathematical logician has confided in me that he obtained tenure at his University by pretending that despite retaining an eccentric interest in logic, in reality he subscribed to his Department’s view that ”real men don’t do logic”. He believes, and I with him, that had he revealed the depth of his commitment to logic he would not have been given tenure. I could wish that now that he has landed safely in the Realm of the Blessed, he would speak up for logic, but it appears that the habit of caution is too deeply ingrained. Still, it is not for me to “out” him.

5-2 Personally I think I can do more good by respecting the confidentiality of anonymous communications. I think that if I betrayed such confidences I should soon cease to receive any. For example, here is one comment that was sent to me, about which I have yet to do anything.

“I was talking to someone at high table the other day about metamathematics (or, at least, the fact that I was interested in it). He remarked that at an unspecified meeting it had been said by Sir Michael Atiyah that there was no interest in metamathematics in Cambridge and that the subject wasn’t worth supporting.

This was the first time I had heard (second-hand) views actively AGAINST meta-mathematics/logic. Certainly I was surprised (maybe naively) that it came from Atiyah, who is oherwise a bright guy.”

6: My second thesis: Bourbaki not an adequately functioning individual.

Now we come to the part of my essay dealing with my second point against Bourbaki, that his chosen foundations are restrictive.

6-0 Left brain, right brain: Professor Segal states that in an adequately functioning individual the two halves of the brain communicate and are integrated via the corpus callosum. That is exactly my point: I contend that Bourbaki is not an adequately functioning individual; there is a gross imbalance in his mathematical personality.

6-1 This is related to my debate with Mac Lane, and though I have in more recent essays been able to argue my case rather better than I did in the essay on Bourbaki, I cannot claim to have succeeded in conveying to devotees of category theory the limitations they are putting on their conceptual universe by slavishly adopting that mode of thought. How does one prove to someone that he is colour-blind ? The victim has to be willing to notice that others have perceptions denied to him.

6-2 But there are signs of these different perceptions. Cartier writes

“ (Following the collapse of the Soviet Union) the Russians have brought a different style to the West, a different way of looking at the problems, a new blood”.

The group I failed to mention in my essay, centred around Baire, E. Borel, and Lebesgue created a new view of analysis growing out of the insights of Cantor. Both Lusin and Janiszewski came from the East to sit at their feet, and returned home with a positive message. I wonder to what extent the Russian style that Cartier has noticed descends through Lusin from Baire, just as there is in France a similar descending chain: Baire, Denjoy, Choquet, Louveau.

6-3 Cartier again:

“ Most people agree now that you do need general foundations for mathematics, at least if you believe in the unity of mathematics. I believe now that this unity should be organic, while Bourbaki advocated a structural point of view.”

“In accordance with Hilbert’s views, set theory was thought by Bourbaki to provide that badly needed general framework. If you need some logical foundations, categories are a more flexible tool than set theory. The point is that categories offer both a general philosophical foundation — that is, the encyclopaedic or taxonomic part — and a very efficient mathematical tool to be used in mathematical situations. That set theory and structures are, by contrast, more rigid can be seen by reading the final chapter in Bourbaki set theory, with a monstrous endeavour to formulate categories without categories.”

These two quotations will really have to be the starting point of a new essay. In the second one it is plain that what Cartier means by set theory is the feeble bunch of trivialities in the Bourbaki volume of that name; a far cry from what set theorists mean by set theory. On the other hand, Cartier may be saying in the first one what I said in *What is Mac Lane missing ?*, that unity is desirable but not uniformity.

I ended my earlier essay with a quotation from Dieudonné saying that we have not begun to understand the relationship between combinatorics and geometry. As in a classical tragedy, the Bourbachistes are looking for something, but do not realise that what they seek is already to hand: it is ironical, but pleasing, that despite Bourbaki’s dead hand, Paris has now acquired one of the largest concentrations of logicians on the planet.

Meanwhile, according to Cartier, Bourbaki is struggling with dead projects. What about Bourbaki making an attempt to understand the logicians ?