$0^{\#}$ and the p-point problem

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A p-point is an ultrafilter F on ω which has the following properties:

(.1) F contains the Fréchet filter of all cofinite subsets of ω

(.2) if X_i $(i < \omega)$ are elements of F, there is an $X \in F$ such that for each i $X \setminus X_i$ is finite.

p-points have been constructed on ω using the continuum hypothesis (Rudin [12]) or Martin's axiom (Booth [1]). It is however unknown whether their existence is provable in ZFC.

A filter F which enjoys the properties (.1) and (.2) need not be an ultrafilter; for example, F might be the Fréchet filter. However the Fréchet filter is clearly much smaller than an ultrafilter. In [10], the author introduced the concept of a <u>feeble filter</u> : a filter F is <u>feeble</u> if there is some weakly monotonic map f of ω onto ω such that $\{X \mid f^{-1}\text{"}X \in F\}$ is the Fréchet filter. Evidently no ultrafilter is feeble. It is shown in [10] that every $\underset{\sim}{\Sigma}^1_1$ filter is feeble, and that provided that $\omega \to (\omega)^{\omega}$, every filter is feeble. Jalali-Naini [5] and Talagrand [14] have independently discovered the following characterization:

a filter F is feeble if and only if considered as a subset of the Cantor space 2^{ω} it is of the first category.

Thus the following property is a reasonable notion of largeness of a filter F:

(.3) F is not feeble.

In [6], Kanamori introduced the concept of a <u>coherent</u> filter and remarking that the proof of Theorem 1.9 of [6] shows that a filter is coherent if and only if it has properties (.1), (.2) and (.3), has asked as an approach to the p-point problem whether the existence of a coherent filter is provable in ZFC. We shall use the term coherent filter in this paper to mean one having properties (.1), (.2) and (.3).

The principal result of this paper is the following partial answer to Kanamori's question:

THEOREM 1. If $0^{\#}$ does not exist, there is a coherent filter.

Here $0^{\#}$ is the real number defined in Solovay [13]. Its existence is unprovable in ZFC, and it lies in no Boolean extension of L. We denote the assertion of its non-existence by $\neg 0^{\#}$. The paper [3] of Dodd and Jensen shows that the hypothesis $\neg 0^{\#}$ can be considerably weakened, e.g. to the non-existence of an inner model with a measurable cardinal. Our method does not permit us to prove the existence of a coherent filter in ZFC alone, but we have the following partial result:

THEOREM 2. If $2^{\aleph_0} \leq \aleph_{\omega+1}$, there is a coherent filter.

Kunen shows in [8] that the existence of rare p-points[1] is unprovable in ZFC, where a filter F is rare if it satisfies (.1) and every weakly monotonic map f of ω onto ω is, restricted to some $X \varepsilon F$, one-to-one. Our method gives the following amusing result:

THEOREM 3. If $2^{\aleph_0} = \aleph_2$, either there is a p-point or there is a rare filter.

A.Miller [11] has shown that in Laver's model [9] of Borel's conjecture there is no rare filter, or indeed 2-rare, where we call F 2-rare if it satisfies (.1) and every weakly monotonic map f of ω onto ω is, restricted to some $X \varepsilon F$, at-most-two-to-one; or equivalently, if given a partition of ω into disjoint finite intervals s_i $\exists X \varepsilon F$ with each $X \cap s_i$ of power at most 2. No feeble filter is 2-rare.

If U is a subset of $P(\omega)$, we write [U] for $\{x \mid \exists y \varepsilon U\ y \subseteq x\}$. If $\pi: \omega \to \omega$ we write $\pi_* U$ for $\{x \subseteq \omega \mid \pi^{-1}{}''x \varepsilon U\}$.

We say that f <u>dominates</u> g if f is strictly monotonic, and for all m, $g(m) \le f(m)$. We call a subset $\mathcal{F}$ of ${}^{\omega}\omega = \{f \mid f: \omega \to \omega\}$ <u>dominant</u> if every $g \in {}^{\omega}\omega$ is dominated by some member of $\mathcal{F}$. Note that the minimum cardinality of a dominant family is the minimum cardinality of a family G such that $\forall g \in {}^{\omega}\omega\, \exists f \in G\, \exists m\, \forall n > m\; f(n) > g(n)$; as given such a family G, the set of all functions h of the form $h(n) = r + \sum_{i \le n} (f(n)+1)$ for $r \in \omega$ and $f \in G$ is a dominant family of the same cardinality as G.

Following Ketonen [7] we write (H) for the assertion that no dominant family is of cardinality less than $2^{\aleph_0}$. We shall use the following result of [7]:

(K1) If (H) holds then there is a p-point.

<u>Definition 4</u> For $f: \omega \to \omega$ define $\bar{f}: \omega \to \omega$ by recursion thus: $\bar{f}(0) = 0$, $\bar{f}(n+1) = f(\bar{f}(n) + 1)$.

<u>Lemma 5</u> If f dominates π and $\forall i\; i < \pi(i)$, then for no i and n can the following hold:

$$\pi(i) \le \bar{f}(n) < \bar{f}(n+1) \le \pi(i+1) \tag{6}$$

Thus for all n with $\bar{f}(n) \ge \pi(0)$, there is a value of π strictly between $\bar{f}(n)$ and $\bar{f}(n+1)$.

<u>Proof</u> $\bar{f}$ is strictly monotonic, as f is. From (6), $\bar{f}(n+1) = f(\bar{f}(n)+1)$. $\le \pi(i+1) \le f(i+1)$; as f is monotonic, $\bar{f}(n) + 1 \le i + 1$; so $\bar{f}(n) \le i \le \pi(i) \le \bar{f}(n)$; so $i = \pi(i)$ - a contradiction. ⊣

<u>Lemma 7</u> Suppose that M is an inner model of ZFC such that ${}^{\omega}\omega \cap M$ is dominant, and let $F \in M$ be a filter in M.

(i) If F is not feeble in M, neither is [F] in V, the universe.

(ii) If F is rare in M, [F] is 2-rare; consequently any ultrafilter extending [F] is rare, and [F] is rare if F is, in M, an ultrafilter.

<u>Proof</u> Let π be strictly monotonic, and let $f \in M$ dominate it. Then

$\bar{f} \in M$.

(i) Define $g(n) = \bar{f}(2n)$. If F is not feeble in M, there is an $X \in M$ such that for infinitely many n, $[g(n),g(n+1)) \cap X$ is empty. By Lemma 5, for all such n with $\pi(0) \leq \bar{f}(2n)$, there are k_n, ℓ_n such that $g(n) = \bar{f}(2n) < \pi(k_n) < \bar{f}(2n+1) < \pi(\ell_n) < \bar{f}(2n+2) = g(n+1)$; consequently $[\pi(k_n), \pi(k_n+1)) \cap X \subseteq [\pi(k_n), \pi(\ell_n)) \cap X \subseteq [g(n), g(n+1)) \cap X = \emptyset$. As the set of such k_n's is infinite, and $X \in [F]$, it is clear that $\pi_*[F]$ is not the Fréchet filter.

(ii) If F is rare in M, tnere is an $X \in F$ such that $\forall i\ [\bar{f}(i), \bar{f}(i+1)) \cap X$ has cardinality 1. Let h enumerate X monotonically. Then for all i $\exists$ at least one j with $h(i) < \bar{f}(j) \leq h(i+1)$. (Recall that $\bar{f}(0) = 0$). It follows that for all i, $[\pi(i), \pi(i+1)) \cap X$ has at most 2 members; for if $\exists k\ \pi(i) \leq h(k) < h(k+1) < h(k+2) < \pi(i+1)$, we would have j, j' with $\bar{f}(j) \in (h(k), h(k+1)]$ and $\bar{f}(j') \in (h(k+1), h(k+2)]$, and by Lemma 5 $\pi(i') \in (\bar{f}(j), \bar{f}(j+1))$ for some i', contradicting the monotonicity of π. Thus $[F]$ is 2-rare.

If F is an ultrafilter in M, either $\{h(2n) \mid n \in \omega\}$ or $\{h(2n+1) \mid n \in \omega\} \in F$, and both meet each $[\pi(i), \pi(i+1))$ in at most one element. Thus $[F]$ is rare. An ultrafilter extending $[F]$ is rare for the same reason. $\dashv$

<u>Lemma 8</u> Let M be an inner model of ZFC such that every countable subset $\mathcal{X}$ of $P(\omega) \cap M$ is a subset of some $\mathcal{Y} \in M$ with $\mathcal{Y}$ countable in M. If $F \in M$ has property (.2) in M, $[F]$ has (.2) in V.

<u>Proof</u> Let $X_i \in [F]$. Pick $Y_i \in F$ with $Y_i \subseteq X_i$, and set $\mathcal{X} = \{Y_i \mid i < \omega\}$. Ex hypothesis, $\mathcal{X} \subseteq \mathcal{Y} \in M$ where $\mathcal{Y}$ is countable in M. As F has (.2) in M, $\exists Z \in F$ such that $\forall W \in \mathcal{Y} \cap P(\omega) \cap F\ Z \setminus W$ is finite. A fortiori $Z \setminus X_i$ is finite for each i. $\dashv$

For the rest of this paper, let G be a dominant family of least possible cardinality, and let κ be its cardinal.

<u>Proposition 9</u> $\mathrm{cf}(\kappa) > \aleph_0$

Proof Suppose the contrary and let $\kappa = \sup\{\lambda_i | i < \omega\}$ where $\lambda_0 < \lambda_1 < \lambda_2 \ldots$ and the λ_i are regular cardinals. Write G as the union of sets G_i with $\overline{\overline{G}}_i = \lambda_i$, and let $H_i = \{\lambda n(r+f(n)) | f \in G_i \ \& \ r \in \omega\}$.[2] Then $\overline{\overline{H}}_i = \lambda_i$. By the minimality of κ, there is a function f_i not dominated by any member of H_i. Define $f(n) = \sup\{f_i(n) | i \le n\}$. Suppose $\exists g \in G_{i_0}$ $\forall n \ f(n) \le g(n)$. $\forall n \ge i_0 \ f_{i_0}(n) \le f(n)$. Let $h = \lambda n.(g(n) + \sum_{m \le i_0} f(m))$. Then $h \in H_i$ and $\forall n \ h(n) \ge f_{i_0}(n)$, contrary to the choice of f_{i_0}. $\dashv$

Proposition 10 If $\kappa = \aleph_1$ then there is a filter which is both rare and coherent.[3]

Proof For each countable ordinal ζ let $h_\zeta \in {}^\omega\omega$ code ζ; let A be a subset of $\aleph_1$ coding each h_ζ and all elements of G. Set $M = L[A]$. Then $\aleph_1^M = \aleph_1$; $G \in M$, and in M, $2^{\aleph_0} = \aleph_1$. There is consequently a Ramsey ultrafilter U in M. $G \subseteq ({}^\omega\omega) \cap M$ which is therefore dominant. By Lemma 7(ii), [U] is rare. Lemma 8 will tell us that [U] is coherent provided we know that each countable subset X of $P(\omega) \cap M$ is a subset of a set countable in M. But let $\langle x_\nu | \nu < \omega_1 \rangle$ be an enumeration in M of $P(\omega) \cap M$ in order type ω_1^M. Set $\zeta = \sup\{\nu | x_\nu \in X\}$. $\zeta < \omega_1 = \omega_1^M$ so $X \subseteq \{x_\nu | \nu \le \zeta\}$ which is countable in M as required. $\dashv$

PROOF OF THEOREM 3 If $2^{\aleph_0} = \aleph_2$, $\kappa = \aleph_1$ or $\aleph_2$. If $\kappa = \aleph_1$, by Proposition 10 there is a rare filter; if $\kappa = \aleph_2$ by Ketonen's result (K1) there is a p-point.

Proposition 11 If $\aleph_1 < \kappa < \aleph_\omega$ there is a coherent filter.

Proof Let $\kappa = \aleph_n$. For each infinite $\nu < \aleph_n$, let f_ν be a 1-1 map of $\overline{\overline{\nu}}$ onto ν, and let A be a subset of $\aleph_n$ coding each f_ν and each $g \in G$. Set $M = L[A]$. Then for $1 \le i \le n$, $\aleph_i^M = \aleph_i$; in M, $2^{\aleph_0} = \aleph_n$; as $G \in M$, $({}^\omega\omega) \cap M$ is dominant, and so by the minimality of κ and the transitivity of dominance, there is in M no dominant family of power less than $\aleph_n$. Consequently the hypothesis (H) holds in M, and therefore there is in M a p-point U. As U is an ultrafilter in M it is not feeble in M, and so by Lemma 7(i) [U] is not feeble. Trivially [U] has property (.1).

We now show that any countable subset X of $P(\omega) \cap M$ is a subset of a set countable in M; Lemma 8 will immediately imply that [U] satisfies (.2) and is thus coherent.

Let $\langle x_\nu^n | \nu < \omega_n \rangle$ be an enumeration in M of $P(\omega) \cap M$ in order type ω_n, and set $A_n = \{\nu | x_\nu^n \in X\}$. Set $\zeta_n = \sup A_n$; by the regularity of $\aleph_n$, $\zeta_n < \aleph_n$, and $\overline{\overline{\zeta_n + 1}}^M = \overline{\overline{\zeta_{n+1}}} = \aleph_{n-1} = \aleph_{n-1}^M$. Now let $\langle x_\nu^{n-1} | \nu < \omega_{n-1} \rangle$ be an enumeration in M of $\{x_\nu^n | \nu \leq \zeta_n\}$ of order type ω_{n-1}, and set $A_{n-1} = \{\nu | x_\nu^{n-1} \in X\}$. As before, $\zeta_{n-1} = \sup A_{n-1}$ is less than $\aleph_{n-1}$. We are assuming that $n > 1$; if $n-1 = 1$, $\{x_\nu^{n-1} | \nu \leq \zeta_{n-1}\}$ is the desired set countable in M of which X is a subset; if $n-1 > 1$, we enumerate $\{x_\nu^{n-1} | \nu \leq \zeta_{n-1}\}$ in order type ω_{n-2} as $\langle x_\nu^{n-2} | \nu < \omega_{n-2} \rangle$ and repeat the argument. After a total of n-1 steps we obtain an enumeration in M of a subset of $P(\omega) \cap M$, in order type ω_1, of which X is a bounded subset, and thus of which a suitable countable initial segment is the desired superset of X countable in M. ⊣

PROOF OF THEOREM 2: The only case not covered by Propositions 9, 10 and 11 is that when $\kappa = 2^{\aleph_0} = \aleph_{\omega+1}$, when Ketonen's theorem (K1) immediately yields a p-point.

In proving Theorem 1 we shall use the following result of Jensen, which is Theorem 1 of Devlin and Jensen [2]:

<u>THE COVERING LEMMA</u> Suppose that $0^\#$ does not exist. If X is an uncountable set of ordinals, there is a set $Y \in L$ of ordinals such that $X \subseteq Y$ and $\overline{\overline{X}} = \overline{\overline{Y}}$.

The covering Lemma has the following consequence:

<u>Proposition 12</u> $[\neg 0^\#]$ Let M be an inner model of ZFC such that $\aleph_1^M = \aleph_1$ and $\aleph_2^M = \aleph_2$. Any countable subset of $P(\omega) \cap M$ is a

subset of a set countable in M.

Proof: Let $(2^{\aleph_0})^M = \lambda$, let $\langle x_\nu | \nu < \lambda \rangle$ be an enumeration in M of $P(\omega) \cap M$ in order type λ, and let X be a countable subset of $P(\omega) \cap M$. Let $A = \{\nu | x_\nu \in X\} \cup \omega_1$. A is an uncountable set of ordinals, so by the Covering Lemma, there is a B in L, which is therefore in M, such that $A \subseteq B$ and $\bar{\bar{A}} = \bar{\bar{B}}$. As $\bar{\bar{A}} = \aleph_1$ and $\aleph_2^M = \aleph_2$, the cardinal of B in M cannot be $\aleph_2^M$ or more, and so must be $\aleph_1^M$. Enumerate the elements of B in M in order type ω_1 as $\langle \eta_\xi | \xi < \omega_1 \rangle$. There is a $\zeta < \omega_1$ such that $\{\nu | x_\nu \in X\} \subseteq \{\eta_\xi | \xi < \zeta\} \in M$; consequently $X \subseteq \{x_{\eta_\xi} | \xi < \zeta\}$ which is countable in M. ⊣

PROOF OF THEOREM 1: In view of Proposition 10 we may assume that $\kappa \geq \aleph_2$. We first establish Theorem 1 on the further assumption that κ is regular, and then show how to do the singular case.

Suppose then that κ is regular. Let $A \subseteq \kappa$ be such that $G \in L[A]$, $\aleph_1^{L[A]} = \aleph_1$ and $\aleph_2^{L[A]} = \aleph_2$. Put $M = L[A]$. As κ is regular we may apply the theorem of Hajnal [4] that $P(\omega) \cap L[A] = \bigcup\{P(\omega) \cap L_\zeta[A \cap \theta] | \zeta, \theta < \kappa\}$ to conclude that in M $2^{\aleph_0} = \kappa$. $({}^\omega\omega) \cap M$ is dominant and so Ketonen's hypothesis that there is no dominant family of power $< 2^{\aleph_0}$ holds in M: then there is in M a p-point, U. By Proposition 12 and Lemmas 7 and 8, [U] is coherent.

The case of singular κ would be easy were an affirmative answer known to the following, which appears to be an open question:

<u>Problem 13</u> Let $A \subseteq \theta$ where θ is a cardinal of uncountable cofinality. Is it true that in $L[A]$ $2^{\aleph_0} \leq \theta$?

Fortunately we can by-pass that problem in proving Theorem 1. Suppose that κ is singular; by Proposition 9 we know that $cf(\kappa) > \aleph_0$. Let λ_ν $(\nu < cf(\kappa))$ be an increasing sequence of successor cardinals with supremum κ and with $\lambda_0 = \aleph_2$. Enumerate G without repetitions as $\langle g_\xi | \xi < \kappa \rangle$ and let $G_\nu = \{g_\xi | \xi < \lambda_\nu\}$.

We shall define a sequence of filter bases F_ν such that $\bar{\bar{F}}_\nu = \lambda_\nu$

and $\nu < \nu' < cf(\kappa) \Rightarrow F_\nu \subseteq F_{\nu'}$.

We shall need a variant of Ketonen's result. If $\mathcal{B}$ is a subalgebra of $P(\omega)$ and $\mathcal{H}$ a subset of ${}^\omega\omega$ such that for all $h \in \mathcal{H}$ and all $i \in \omega$ $h^{-1}``\{i\} \in \mathcal{B}$, we shall say that a subset F of $P(\omega)$ is a $(\mathcal{B},\mathcal{H})$-<u>point</u> if it is an ultrafilter in $\mathcal{B}$ containing all cofinite sets and for all $h \in \mathcal{H}$ either $h^{-1}``\{i\} \in F$ for some $i < \omega$ or there is an $X \in F$ with $X \cap h^{-1}``\{i\}$ finite for all i, so that in the latter case $\omega \setminus h^{-1}``\{i\} \in F$ for each i. The following is implicit in Theorem 1.2 of [7] and is readily provable in ZFC by transfinite induction using Proposition 1.3 of [7]:

(K2) Let $\mathcal{B}$ be a subalgebra of $P(\omega)$ containing all finite sets and of cardinality $\le \kappa$; let $\mathcal{H}$ be a subset of ${}^\omega\omega$ of cardinality $\le\kappa$. Then every filter in $\mathcal{B}$ generated by fewer than κ sets can be extended to a $(\mathcal{B},\mathcal{H})$ point.

If M is an inner-model of ZFC we shall mean by an M-<u>point</u> an $(M \cap P(\omega),\ M \cap {}^\omega\omega)$ - point. An M-point need not be a member of M. We shall say that M is λ-<u>correct</u> if $\aleph_1^M = \aleph_1$, $\aleph_2^M = \aleph_2$ and $(2^{\aleph_0})^M = \lambda$.

We proceed to the constuction of the sequence F_ν.

<u>$\nu = 0$</u> Let M_0 be a λ_0-correct inner model with $G_0 \in M_0$, and let F_0 be an M_0-point: F_0 exists by (K2). $\bar{\bar{F}}_0 = \lambda_0$.

<u>$\nu = \xi+1$</u> Let $M_{\xi+1}$ be a $\lambda_{\xi+1}$-correct inner model with $G_{\xi+1} \in M_{\xi+1}$ and $F_\xi \in M_{\xi+1}$. Let $F_{\xi+1}$ be an $M_{\xi+1}$-point extending F_ξ. $F_\xi \subseteq F_{\xi+1}$ and $\overline{\overline{F_{\xi+1}}} = \lambda_{\xi+1}$.

<u>$\lim(\nu)$</u> Let $F'_\nu = \bigcup\{F_\xi \mid \xi < \nu\}$. Let M_ν be a λ_ν-correct model containing F'_ν and G_ν, and let F_ν be an M_ν-point extending F'_ν. Note that $\bar{\bar{F}}'_\nu = \sup\{\lambda_\xi \mid \xi < \nu\} < \lambda_\nu$; so such an F_ν may be found by (K2). $\bar{\bar{F}}_\nu = \lambda_\nu$.

Let $F = \bigcup\{F_\nu \mid \nu < cf(\kappa)\}$. I assert that [F] is coherent. By construction of F_0,[F] satisfies (.1). Let X be a countable subset of F. As $cf(\kappa) > \aleph_0$, $X \subseteq F_\nu$ for some ν, so that X is a countable subset of $P(\omega) \cap M_{\nu+1}$. By Proposition 12, there is a Z , countable in $M_{\nu+1}$, with $X \subseteq Z$. $F_\nu \in M_{\nu+1}$, so $\{X \in Z \mid X \in F_\nu\} \in M_{\nu+1}$ and is countable in $M_{\nu+1}$. Enumerate it in $M_{\nu+1}$ as $\langle X_i \mid i < \omega \rangle$. Still working in $M_{\nu+1}$ set $Y_j = \bigcap_{i \le j} X_i$ and $Z_j = Y_j \setminus Y_{j+1}$. $\bigcap_{i<\omega} X_i \in M_{\nu+1}$; if $\bigcap_{i<\omega} X_i \in F_{\nu+1}$ there is nothing left to prove; otherwise $\omega \setminus \bigcap_{i<\omega} X_i \in F_{\nu+1}$, and we may define $f \in M_{\nu+1}$ by $f(n) = 0$ if $n \ (\bigcap_{i<\omega} X_i \cup (\omega \setminus Y_0))$, and $f(n) = i$ if $n \in Z_{i-1}$. In that case, $\omega \setminus f^{-1}"\{i\} \in F_{\nu+1}$ for each i, and so $\exists Z \in F_{\nu+1}\ \forall i (Z \cap f^{-1}"\{i\}$ is finite); such a Z has the desired property that $\forall X \in X (Z \setminus X$ is finite). As X was arbitrary we have shown that F and therefore $\lfloor F \rfloor$ have property (.2).

To see that $\lfloor F \rfloor$ is not feeble, let $f\colon \omega \to \omega$ be strictly monotonic. There is a $g \in$ some $M_{\nu+1}$ that dominates f. Define $\bar{g}$ as before; $\bar{g} \in M_{\nu+1}$. Recall that $\bar{g}(0) = 0$. Let X be whichever of $\bigcup \{[\bar{g}(4n), \bar{g}(4n+2)) \mid n \in \omega\}$ and $\bigcup \{[\bar{g}(4n+2), \bar{g}(4n+4)) \mid n \in \omega\}$ belongs to $F_{\nu+1}$. As strictly between any two values of $\bar{g}$ greater than $f(0)$ there is a value of f, the set $\{i \mid [f(\nu), f(\nu+1)) \cap X = \emptyset\}$ is infinite, which shows that $\{Y \mid f^{-1}"Y \in \lfloor F \rfloor\}$ is not the Fréchet filter; and thus by the arbitrary nature of f, $\lfloor F \rfloor$ is not feeble. ⊣

Notes

1 Rare p-points are called 'Ramsey ultrafilters' and 'selective ultrafilters' in the literature. See [1], [8] and the author's paper 'Happy Families' in Volume 12, part 1, of the Annals of Mathematical Logic.

2 Here and three lines lower the λ notation for functions is being used.

3 Taylor has also noticed that if $\kappa = \aleph_1$ there is a rare filter. Arguments of this type were first adumbrated by Blass in conversations at Oberwolfach in 1975.

References

[1] D.Booth, Ultrafilters over a countable set, Annals of Mathematical Logic 2 (1970), 1-24.

[2] K.J.Devlin and R.B.Jensen, Marginalia to a theorem of Silver, Logic Conference Kiel 1974, Lecture Notes in Mathematics, Volume 499, Springer-Verlag 1975, pp 115-142.

[3] T.Dodd and R.B.Jensen, The core model, manuscript, 1976, 207 pages.

[4] A.Hajnal, On a consistency theorem connected with the generalised continuum problem, Z.Math. Logik Grundlagen Math. 2 (1956) 131-136.

[5] S-A.Jalali-Naini, The monotone subsets of Cantor space, filters and descriptive set theory, Doctoral dissertation, Oxford, 1976.

[6] A.Kanamori, Some combinatorics involving ultrafilters, to appear.

[7] J.Ketonen, On the existence of P-points in the Stone-Čech compactification of integers, Fundamenta Mathematicae XCII (1976), 91-94.

[8] K.Kunen, Some points in βN, Mathematical Proceedings of the Cambridge Philosophical Society, 80 (1976) 385-398.

[9] R.Laver, On the consistency of Borel's conjecture, Acta Mathematica 137 (1976), 151-169.

[10] A.R.D.Mathias, A remark on rare filters, Colloquia Mathematica Societatis Janos Bolyai, 10 Infinite and Finite sets, to Paul Erdős on his 60th birthday, edited by A.Hajnal, R.Rado and V.T.Sós North Holland 1975, pp 1095-1097.

[11] A.Miller, There are no q-points in Laver's model for the Borel conjecture, typescript, July 1977, 8 pages.

[12] W.Rudin, Homogeneity problems in the theory of Čech compactifcations, Duke Mathematical Journal 23 (1956), 409 - 419.

[13] R.M.Solovay, A non-constructible Δ^1_3 set of integers, Transactions of the American Mathematical Society 127 (1967) 58-75.

[14] M.Talagrand, Compacts de fonctions mesurables et filtres non mesurables, to appear.